The first column to the right shows the first 33 *natural numbers* written in the *decimal* or *base-10* number system. You know the system. There are 10 different symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. In going from one integer the next higher one, use them in that order. When there are no more, put a 1 before them: 10, 11, 12, …, 19. Then 20, 21, …, 29. After 99 comes 100, and so forth.

Deci- Binary Octal Hexa-  
mal decimal

0 0 0 0

1 1 1 1

2 10 2 2

3 11 3 3

4 100 4 4

5 101 5 5

6 110 6 6

7 111 7 7

8 1000 10 8

9 1001 11 9

10 1010 12 A

11 1011 13 B

12 1100 14 C

13 1101 15 D

14 1110 16 E

15 1111 17 F

16 10000 20 10

17 10001 21 11

18 10010 22 12

19 10011 23 13

20 10100 24 14

21 10101 25 15

22 10110 26 16

23 10111 27 17

24 11000 30 18

25 11001 31 19

26 11010 32 1A

27 11011 33 1B

28 11100 34 1C

29 11101 35 1D

30 11110 36 1E

31 11111 37 1F

32 100000 40 20

In the *binary* or *base-2* number system, two symbols are used: 0 and 1. They are called *bits*. The same method of writing successive integers is used, using the two bits. You can see this in the second column in the table to the right; each entry contains the binary representation of the decimal number to its left. Thus,

1610 = 100002

In this equation, a subscript gives the base in which the integer is written. So, read the above line as “16 in the base-10 system equals 10000 in the base-2 system.”

The octal system is similar but it uses 8 symbols. That’s the third column in the table.

The hexadecimal system is similar but it uses 16 symbols: 0, 1, …, 9, A, B, C, D, E, F. That’s the fourth column in the table.

**Use in computers**

Digital computers generally use the base-2 system because it’s easy to represent bits 0 and 1 by physical devices. For example, typically,

off means 0, on means 1  
not magnetized means 0, magnetized means 1  
not charged electrically means 0, charged means 1

**Relation between binary, octal, and hexadecimal**

Take a base-2 integer, e.g. 10110110101. Put a space before each three bits, starting on the right: 10 110 110 101. Now replace each part by its octal equivalent from the table above: 2665. Therefore,

101101101012 = 26658

To write any binary integer in hexadecimal, do the same thing but break the binary integer into 4-bit parts. For example, write 10110110101 as 101 1011 0101 and put each part into hexadecimal:

101101101012 = 5B516

So any binary integer can easily be written in a more compact representation in octal or hexadecimal. On the IBM7090, in the 1950’s and 60’s, one would get a “dump” of memory if a program crashed, showing the contents of every memory location, and each would be written in octal.

Today, Unicode characters are written in hexadecimal, though of course in the computer they are in binary. For example, in Java, you can write the character 'a' as '\u0061' where 0061 is in hexadecimal, and

006116 = 00000000011000012 = 1418 = 9710

For more information on Unicode and character representations, look at JavaHyperText entry “Unicode”.

**Powers of 2**

The powers of 2 are: 1, 2, 4, 8, 16, 32, … In binary, the decimal integer 2*k* is a 1 followed by *k* 0’s. For example, 25 = 3210 = 1000002. Also, any power of 2 written in octal or hexadecimal is all 0’s except for the leftmost digit.

**General base-b or radix b system**

Above, we discussed the decimal, binary, octal, and hexadecimal number systems. These are called the base-10, base-2, base-8, and base-16 number systems. The base is also called the *radix*, so base-2 uses radix 2. We mention this only because you may come across the term *radix* and wonder what it means.

In general, for any integer *b* > 1, there is the base-*b* or radix *b* number system. It requires *b* symbols, and it follows the pattern shown for the four systems we have looked at.

In general, an integer > 0 is written in the base-*b* system with no leading 0’s in the form:

(1) d*n*d*n*-1…d1d0 where each d*i* is in 0..*b*-1and the leading number, d*n* > 0.

The decimal integer 426 has the value 4\*102 + 2\*101 + 6\*100.

In the same way, the binary integer 11012 has the value 1\*23 + 1\*22 + 0\*101 + 1\*100.

The value of (1) is given by

(2) d*n*\**b*n + d*n*-1\**bn*-1 + … + d1\**b*1 + d0\**b*0 = d*kbk*, the sum being over *k* in 0..*n*.

**Transforming an integer to base-b.**

Given a base *b* number in an array d, formula (2) above shows you how to calculate its value.

On the other hand, you can use this formula to go the other way: Given an **int** value *v*, use the formula to calculate and store its base *b* representation in an array. For example, suppose we want to calculate the base-2 representation of *v*. The formula is:

*v* = d*n*\*2*n* + dn-1\*2n-1 + … + d1\*21 + d0\*20 where each d*k* is in 0..1

Factor out 2 in all but the last term and simplify the last term —remember, 20 = 1:

v = 2\*(d*n*\*2*n*-1 + dn-1\*2*n*-2 + … + d1\*20) + d0

We see that d0 =*v* %2, and the value of the expression within parentheses is v/2. Thus, d0 is easy to calculate. Further the expression within parenthesis has the form of the original expression but with one less term, so this process can be repeated, using a loop, to pick off one bit d*k* at each iteration. We leave the formulation of the loop invariant and loop to you.

**History**

Wikipedia (https://en.wikipedia.org/wiki/Hindu–Arabic\_numeral\_system) tells you that the decimal number system is the most common system for representation of numbers in the world. Further, “It was an ancient Indian numeral system which was re-introduced in the *book On the Calculation with Hindu Numerals* written by the medieval-era Iranian mathematician and engineer al-Khwarizmi, whose name was latinized as Algoritmi. The system later spread to medieval Europe by the High Middle Ages.” Of course, the Indians did not use the actual symbols 0, 1, 2, … which were introduced later. They used other symbols.

Positional number systems like the decimal and binary systems would not have been possible without the concept of and a symbol for zero. The concept of zero was found in India in a text known as the *Bakshali manuscript*, estimated to have been written in the third or fourth century A.D. or perhaps earlier. Zero is represented by a small dot. See this page: news.nationalgeographic.com/2017/09/origin-zero-bakhshali-manuscript-video-spd/.