The well-known Fibonacci numbers are defined recursively in the box to the right. The sequence of Fibonacci starts off like this:

**Fibonacci numbers**

F(0) = 0. F(1) = 1.

For n > 1, F(n) = F(n-1) + F(n-2)

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, …

The golden ratio and its conjugate, the silver ratio, are defined like this:

ϕ = (1 + √5)/2 = 1.6180339887…. Φ = (1 – √5)/2 = –.6180339887…

They are the roots of the polynomial x2 – x – 1. Also, limn­–>∞ F(n+1)/F(n) = ϕ.

**The golden rectangle**

We show you how to construct a golden rectangle with a rule and compass. First, draw a square of length 1. Second, find the middle of the bottom line. Third, use your compass to draw an arc, as shown in the third diagram; the place where the arc crosses the bottom line marks the end of the bottom line of the golden rectangle.

1 sqrt(1 + ¼) = sqrt(5/4) = √5/2. The length the lower line is therefore (1 + √5)/2 ---the golden ratio!

21 21/2 sqrt(21\*21 + 21\*21/4) = sqrt(21\*21\*5/4) = 21√5/2

Now suppose the square has vertical side length a instead of 1, where a is a Fibonacci number, say 21. Then the bottom side has length a+b where b = 21√5/2