DAGS and Topological Sort

A graph is *acyclic* if does not contain a cycle. A *d*irected *a*cyclic *g*raph is called a *DAG*.

Here is an example of a DAG. Let the nodes be the courses that can be taken to satisfy a computer science (or other) major. Draw a directed edge from course c1 to course c2 if c1 is a prerequisite for c2. This graph better not have a cycle! Also, one can analyze the graph to find the longest prerequisite chain —it should not be more than 8, and preferably less, so students can graduate in eight semesters.

CS3110

CS1110

CS2110

CS2800

Another prime use of a DAG is to help determine the layout of tasks on a factory floor.

DAGs have many uses. In another pdf file on DAGs, we develop an algorithm called *topological sort*, which can determine whether a directed graph is a DAG.

**Topological sort**

Suppose we want to number the nodes of a DAG with integers 0, 1, 2, … so that n1 < n2 if there is a directed edge from n1 to n2. With the prerequisite graph shown above, there are two possible numberings, given by these orderings:

CS1110, CS2110, CS2800, CS3110  
 CS1110, CS2800, CS2110, CS3110

Thus, CS1110 is number 0,

In a DAG, there must be a node with indegree 0 —if not, there would be a cycle and it would not be a DAG. We can use this fact to produce a topological ordering of the DAG. Choose a node of indegree 0, give it

// Invariant: k nodes have been given numbers in 0..k-1.  
 // these nodes

for (int k= 0; k < n; k= k+1) {

}

**Planarity**

A graph is *planar* if it can be drawn in the plane without any edge crossing.

Be careful with this definition! The first graph on the right doesn’t look planar because edges (D, C) and (A, B) cross. But edge (D, C) can be redrawn so that the edges don’t cross, so this graph *is* planar.

Pdf file *PlanarGraphs* discusses planar graphs in more detail.

**Bipartite graphs**

A directed or undirected graph is *bipartite* if its nodes can be partitioned into two sets such that no edge connects two nodes in the same set. An example of a bipartite graph appears to the right, with the two sets being {A, D, B } and {1, 2}.

The following three properties are equivalent.

1. Graph G is bipartite
2. Graph G is 2-colorable
3. Graph G has no cycles of odd length.