Supplementary Material: New Estimation Approaches for the Linear Ballistic Accumulator Model

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1. Boundedness of the LBA Density

Given a choice between two alternatives, the density of the first accumulator reaching the threshold at time t and the second accumulator not reaching the threshold at that time is given by

$$\mathrm{LBA}\left(1, t|b, A, v, s, \tau\right) = f_1\left(t-\tau\right)\left(1 - F_2\left(t-\tau\right)\right). \tag{1}$$

To show that $\mathrm{LBA}\left(1, t|b, A, v, s, \tau\right)$ is bounded in its argument, it is sufficient to show that $f_1\left(t-\tau\right)$, specified in Section 2 of the main text, is bounded as $F_2\left(t-\tau\right)$ is a cdf and hence always bounded. Now, $f_1\left(t-\tau\right)$ will be bounded if $-\frac{v^1}{A}\Phi\left(\frac{b-A-tv^1}{ts}\right)$ and $\frac{v^1}{A}\Phi\left(\frac{b-tv^1}{ts}\right)$ are bounded for all $v^1 > 0$ and the required boundedness follows from the inequality $v\Phi\left(-v\right) \leq \phi\left(v\right)$ for all $v > 0$.

2. Illustrative Applications

Here we provide additional results for the estimates of the individual random effects parameters and the Integrated Autocorrelated Time (IACT) for the random effects parameters, for the applications to simulated and real data.

2.1. Application to Simulated Data

Figure 1 shows kernel density estimates of some of the LBA individual random effects for the simulated data used in Section 4.1 in the main paper: PMwG estimate (blue); density tempered SMC estimate (red); the vertical lines show the true parameter values. Thee figures show that the density tempered SMC estimates (red) are very close to the PMwG estimates (blue) for all the parameters, providing a reassuring check. The true values are within the 95% credible interval estimates. The third posterior distribution (in yellow), which is obtained by assuming the random effects are independent apriori, is quite different to the two other posteriors in most cases.

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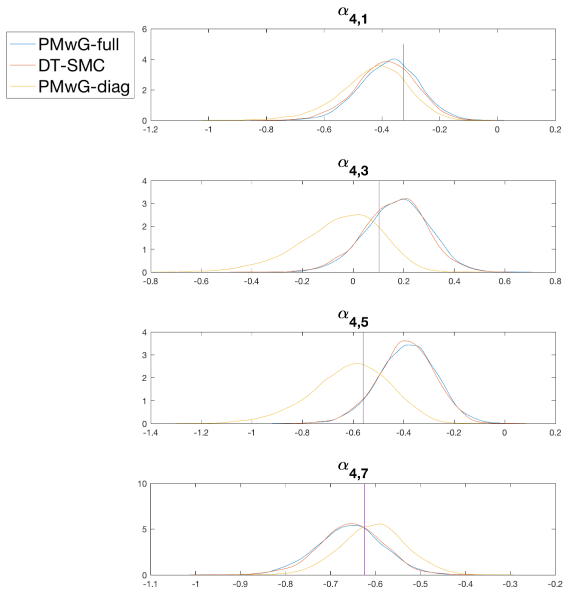

Figure 1: Kernel density estimates of the posterior distributions of some of the individual random effects parameters in the application to simulated data for the three sampling schemes: PMwG (blue); density tempered (red); and PMwG with a diagonal covariance matrix (no between parameter correlations, yellow); the vertical lines show the true (data generating) values. There are $S = 100$ subjects and $N = 1000$ trials.

We evaluated the efficiency of the PMwG sampling method using the IACT. Figure 2 shows the distributions of IACT estimates for each subject's random effect parameters. Similar conclusions can be drawn from these random effects parameters as for the group-level parameters, which are discussed in the main text.

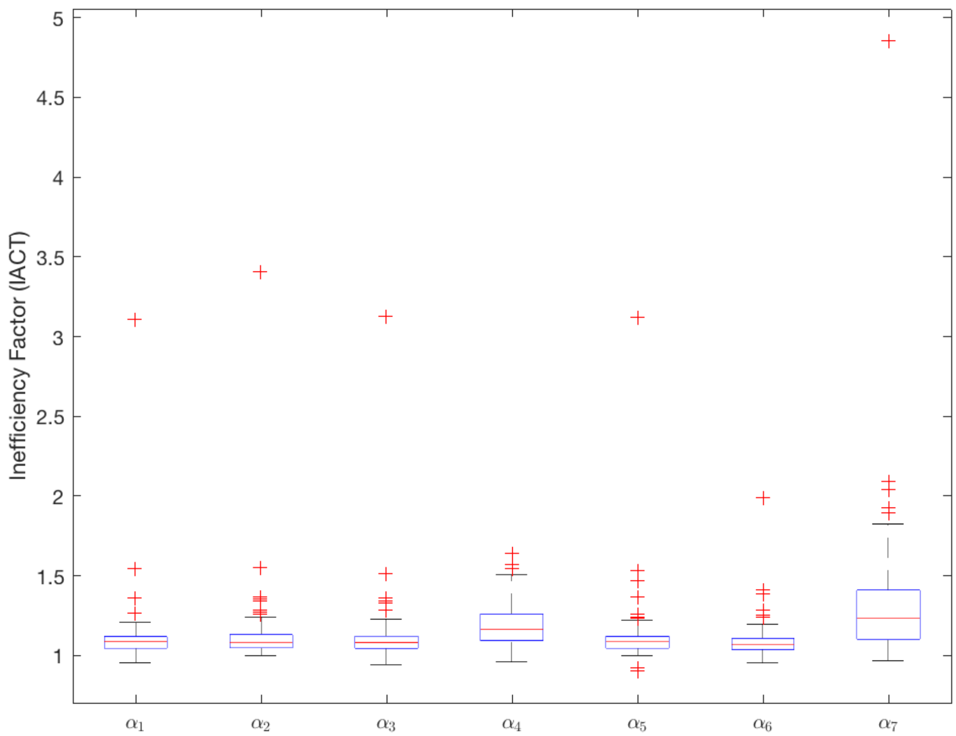

Figure 2: The inefficiency factors (IACT) of the individual level random effects parameters. Each boxplot shows the distribution of IACT across participants, for one of the model parameters, estimated using PMwG applied to the simulated data.

2.2. Application to Real Data

Figure 3 show the kernel density estimates of marginal posterior densities of some of the LBA individual random effects based on PMwG and density tempered SMC samples. The figure shows that the density tempered SMC estimates (in red) are very close to the PMwG estimates (in blue) for all parameters. The third posterior distribution (in yellow), which is obtained by assuming the random effects are independent apriori, is quite different to the two other posteriors in most cases. Figure 4 shows the inefficiency factors (IACT) for the individual level random effects in the model. Again, all IACT values are small, which indicates that chains mixed well and that the performance of the sampler did not deteriorate markedly when moving from simulated data to real data.

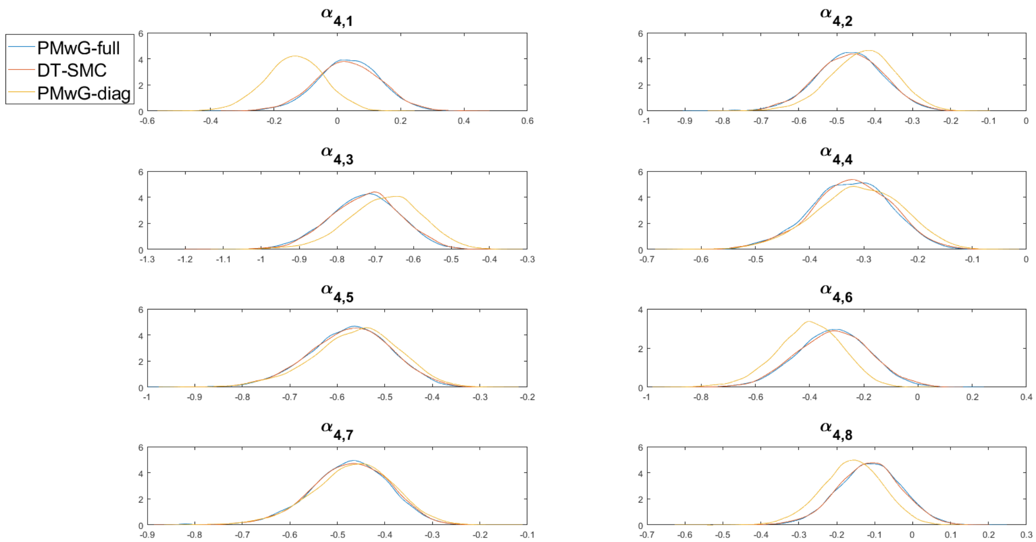

Figure 3: Kernel density estimates of the posterior distributions of some of the individual random effects parameters in the application to the Forstmann (2008) data: PMwG (blue); density tempered SMC (red); and PMwG with a diagonal covariance matrix (no between parameter correlations, yellow).

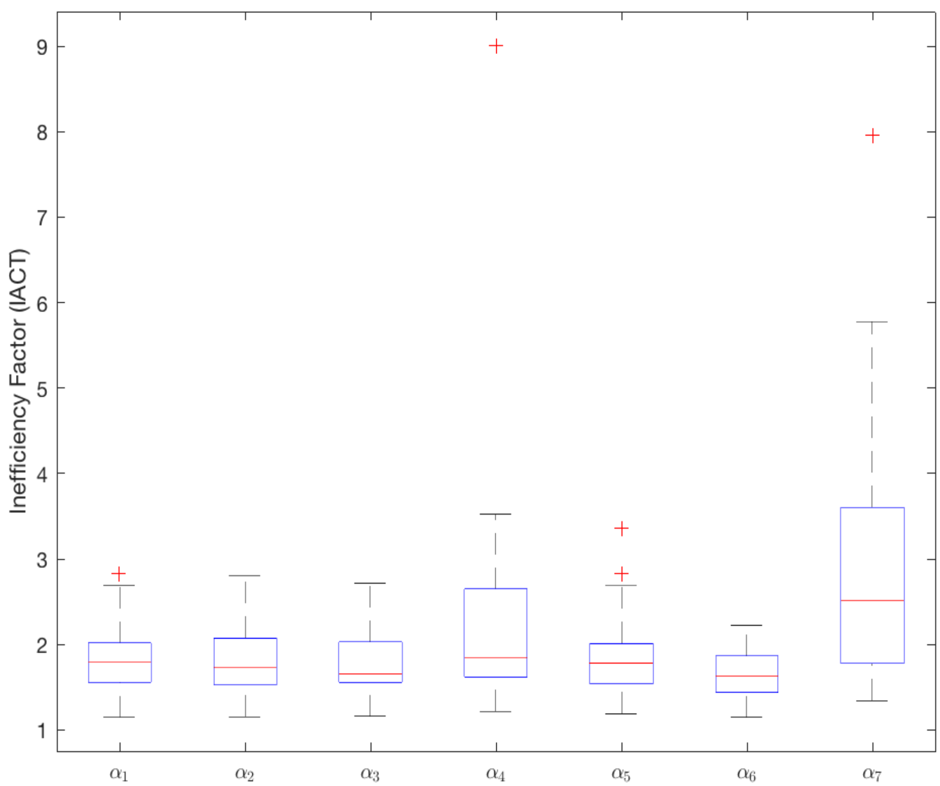

Figure 4: The Inefficiency Factors (IACT) of the individual level random effects parameters from the full model, estimated using the PMwG samples in the application to real data.