Supplementary Materials for Time-Evolving Psychological Processes Over Repeated Decisions.

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Simulation Study: Model Selection

We report a simulation study to investigate whether the marginal likelihood favors the data-generating model (at least in large enough simulated data examples). We simulated data from each of the four models: the static (standard), AR, trend, and Markov switching LBA models. For each simulated dataset, we used the IS^2 method to estimate the marginal likelihoods of the four models. The simulated data are based on the experiment of Forstmann et al. (2008) and the models are parameterized as described in the main text.

We simulate data from $S = 19$ participants. For the AR and trend models, which explain time-varying effects at the block level, we investigate two different ways of trading off the size of the block with the number of trials in each block. In both cases, the total number of simulated trials per participant is $N = 2,000$, but this is either split into many smaller blocks ($T = 100$ time periods of $n = 20$ trials each) or fewer larger blocks ($T = 50$ time periods of $n = 40$ trials each). The static and switching models have no differences defined by blocks, so we simulate $N = 2,000$ trials without a block structure. In all cases, we generate simulated data in turn from the four different models. Model selection by appropriate methods (in our case, Bayes factors) penalizes the larger model for complexity. Ideally, this results in the data-generating model being selected in each case, although exceptions to this occur when the sample size or the time-varying effects are very small; neither of which is the case in our simulation study.

When analyzing the simulated data in the model recovery study, we use our new particle Metropolis-within-Gibbs (PMwG) method (see Appendix of the main text) to generate samples from the posterior distributions with the number of particles set to $R = 250$. Three different sampling stages are employed; in the initial stage, the first $1,000$ iterates are discarded as burnin, the next $4,000$ iterates are used in the adaptation stage to construct the efficient proposal densities for the random effects for the final sampling stage, finally, a total of $10,000$ MCMC posterior draws are obtained in the sampling stage. When estimating the AR and trend models, we used the same values of n and T which are used to generate the synthetic data, and when the synthetic data is generated by the static and switching models, we use $n = 20$ and $T = 100$.

We use the IS^2 method to estimate the marginal likelihood for the models; see the Appendix of the main text, using $M = 10,000$ importance samples. The number of particles used to obtain the unbiased estimate of the likelihood was set to $R = 500$, and the Monte Carlo standard errors of the log of the marginal likelihood estimate were obtained by bootstrapping the importance samples. Tran et al. (2021) provide more details about the IS^2 method.

Table 1 summarizes the results of the model recovery simulation, which reports the log of the marginal likelihood for each of the four models (columns) when estimated from data generated by each of the four models (rows). The entries in each row have had the entry for the data-generating model subtracted, so that negative entries indicate less evidence for a model in question than for the data-generating model. In each row, the highest likelihood is zero (for the data-generating model) which implies that model selection via the marginal likelihood recovers the data-generating model in all simulated comparisons.

Of particular importance is the first row of Table 1, which shows that the models do not give "false positive" results. The first row shows model performance when data were simulated from a static LBA model, and in that row the static LBA model was preferred by a large Bayes factor. This was the case even though the simulated data naturally always contain some variability across time periods, due to sampling variability. The model selection results confirm that this is correctly attributed to random sampling variability and not to a dynamic generating process. The results for the static and switching models also suggest that our algorithms can identify the number of latent states, because the static model has a single state and the switching model has two states. In both cases, the marginal likelihood correctly identifies the data-generating model.

Table 1
Model selection results for the simulation study. Rows represent the different data generating processes (DGP), and also different balances between numbers of trials per block (n) and blocks (time periods, T). The columns correspond to different models estimated from the simulated data. Each entry shows the difference in the log of the estimated marginal likelihood for the estimated model relative to the data generating model, with negative values indicating poorer performance than the data generating model, i.e., smaller marginal likelihood estimate. Standard errors of the log of the marginal likelihood estimates were all smaller than 1.1, and are omitted for clarity.

DGP	n	T	Static	AR	Trend	Switching
Static	-	-	0	-667.59	-1214.70	-46.10
AR	20	100	-21581.75	0	-487.96	-5875.70
	40	50	-16263.35	0	-2206.74	-13508.83
Trend	20	100	-15268.88	-295.97	0	-6023.49
	40	50	-28381.01	-336.48	0	-13627.75
Switching	-	-	-348.04	-1019.78	-1573.72	0

Simulation Study: Parameter Recovery for the Markov Switching Model

This section reports a simulation study showing that the proposed PMwG method is able to estimate the Markov switching LBA model in practical sample sizes. We simulated data from switching LBA model with two states, for $S = 100$ participants with $N = 2,000$ trials per participant. The vector of random effects for subject j in state 1 is

$$\left(B_{j,1}, A_{j,1}, v_{j,1}^{(e)}, v_{j,1}^{(c)}, \tau_{j,1}\right), \tag{1}$$

and the vector of random effects for subject j in state 2 is

$$\left(B_{j,2}, A_{j,2}, v_{j,2}^{(e)}, v_{j,2}^{(c)}, \tau_{j,2}\right). \tag{2}$$

The parameters $v^{(e)}$ and $v^{(c)}$ refer to the means of the drift rate distributions for the two accumulators. The superscript (c), for "correct", is for the accumulator where the associated response matches the actual stimulus, and (e), for "error", is for the accumulator where the associated response does not match the actual stimulus. The data-generating ("true") values of the group-level parameters μ and Σ are given in Table 2. The elements of the transition matrix $\xi_{1,1,j}$ and $\xi_{2,2,j}$ are set to 0.9 and 0.2, respectively, for all j. We use our PMwG method to generate samples from the posterior distribution with the same settings as the model selection simulation study.

Table 2 presents the estimated posterior means and 95% credible intervals for the mean group-level parameters (μ) and the group-level covariance matrix (Σ) of the switching LBA model. For nine of the ten mean parameters (μ) the estimated credible interval includes the data-generating parameter – the data-generating value for B_2 is slightly below the 95% credible interval. For eight of the ten between-subject variance parameters (diagonal elements of Table 2) the credible interval includes the data-generating value – the data-generating value for the between subject variance in parameters B_1 and $v_1^{(e)}$ are slightly below the estimated credible interval. The off-diagonal elements of Table 2, which describe the covariance between parameters, are also well recovered (35 of the 45 data-generating values are within the estimated credible intervals). The parameter recovery process is satisfactory given the complexity of the model.

Figures 1 and 2 show the posterior distributions estimated by PMwG for the elements of the transition matrix $\xi_{1,1,j}$ and $\xi_{2,2,j}$, for $j = 1, ..., 50$, with the vertical lines showing the true values. The figures show that the true values are well within the posterior density estimates, suggesting that our methods reliably recover participant-level parameter estimates.

Table 2

Results of the simulation study investigating parameter recovery for the Markov switching LBA model. Each cell represents one parameter. Within each cell, the data-generating value for that parameter is shown at the top and the 95% credible interval for the Markov switching LBA model is shown at the bottom.

μ		B_1	A_1	$v_1^{(e)}$	$v_1^{(c)}$	τ_1	B_2	A_2	$v_2^{(e)}$	$v_2^{(c)}$	τ_2
−1.12 (−1.20,−1.07)	B_1	0.0484 (0.0594,0.1111)									
−0.80 (−0.87,−0.77)	A_1	0.0387 (0.0312,0.0655)	0.0484 (0.0459,0.0794)								
0.49 (0.37,0.50)	$v_1^{(e)}$	0.0387 (0.0445,0.0908)	0.0387 (0.0351,0.0703)	0.0484 (0.0636,0.1143)							
1.45 (1.39,1.49)	$v_1^{(c)}$	0.0387 (0.0326,0.0684)	0.0387 (0.0309,0.0600)	0.0387 (0.0372,0.0732)	0.0484 (0.0468,0.0798)						
−1.74 (−1.77,−1.68)	τ_1	0.0387 (0.0237,0.0527)	0.0387 (0.0269,0.0527)	0.0387 (0.0277,0.0574)	0.0387 (0.0261,0.0510)	0.0484 (0.0419,0.0702)					
0.24 (0.25,0.44)	B_2	0.0387 (0.0110,0.0547)	0.0387 (0.0121,0.0489)	0.0387 (0.0144,0.0605)	0.0387 (0.0108,0.0476)	0.0387 (0.0106,0.0443)	0.0484 (0.0375,0.0943)				
0.20 (0.08,0.28)	A_2	0.0387 (−0.0029,0.0401)	0.0387 (0.0022,0.0394)	0.0387 (−0.0021,0.0430)	0.0387 (0.0005,0.0366)	0.0387 (0.0041,0.0368)	0.0387 (−0.0080,0.0275)	0.0484 (0.0270,0.0721)			
0.49 (0.48,0.59)	$v_2^{(e)}$	0.0387 (0.0133,0.0447)	0.0387 (0.0155,0.0420)	0.0387 (0.0187,0.0510)	0.0387 (0.0155,0.0418)	0.0387 (0.0145,0.0391)	0.0387 (0.0154,0.0537)	0.0387 (0.0028,0.0360)	0.0484 (0.0320,0.0662)		
0.98 (0.97,1.06)	$v_2^{(c)}$	0.0387 (0.0166,0.0449)	0.0387 (0.0194,0.0433)	0.0387 (0.0189,0.0478)	0.0387 (0.0185,0.0421)	0.0387 (0.0184,0.0406)	0.0387 (0.0129,0.0459)	0.0387 (0.0032,0.0345)	0.0387 (0.0117,0.0374)	0.0484 (0.0321,0.0592)	
−0.20 (−0.26,−0.15)	τ_2	0.0387 (0.0257,0.0583)	0.0387 (0.0211,0.0481)	0.0387 (0.0259,0.0588)	0.0387 (0.0237,0.0507)	0.0387 (0.0239,0.0489)	0.0387 (0.0003,0.0306)	0.0387 (0.0017,0.0368)	0.0387 (0.0082,0.0317)	0.0387 (0.0146,0.0369)	0.0484 (0.0416,0.0752)

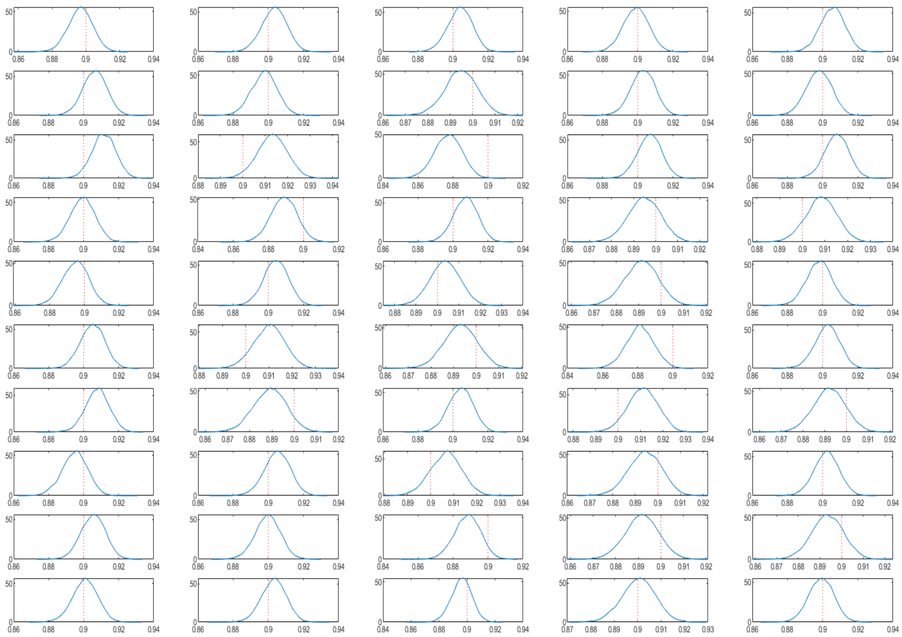

Figure 1. Kernel density estimates of the posterior distributions over an element of the transition matrix ($\xi_{1,1}$) for the first 50 simulated participants in the parameter recovery study of the Markov switching model. Vertical lines show the true (data-generating) values

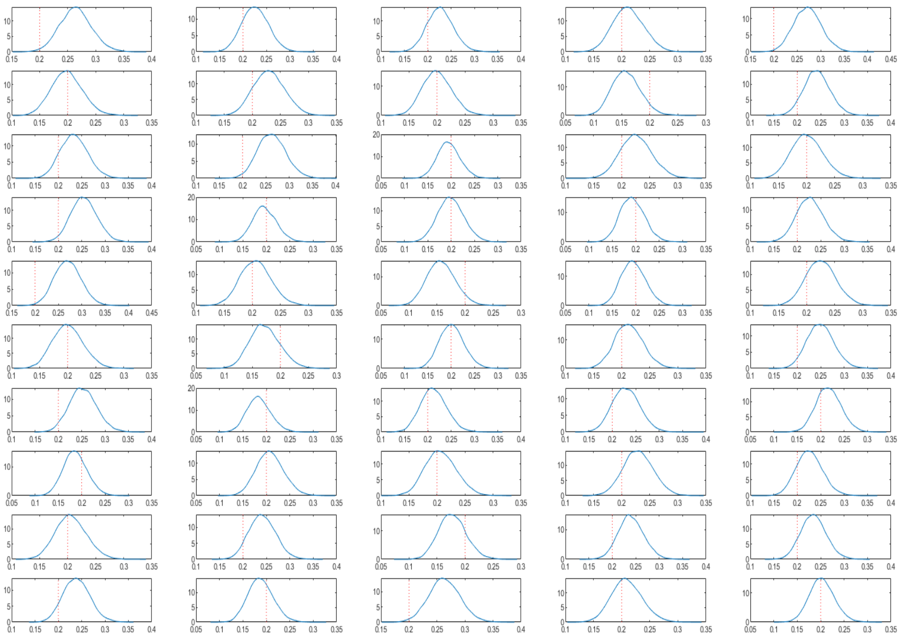

Figure 2. Kernel density estimates of the posterior distributions over an element of the transition matrix ($\xi_{2,2}$) for the first 50 simulated participants in the parameter recovery study of the Markov switching model. Vertical lines show the true (data-generating) values

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