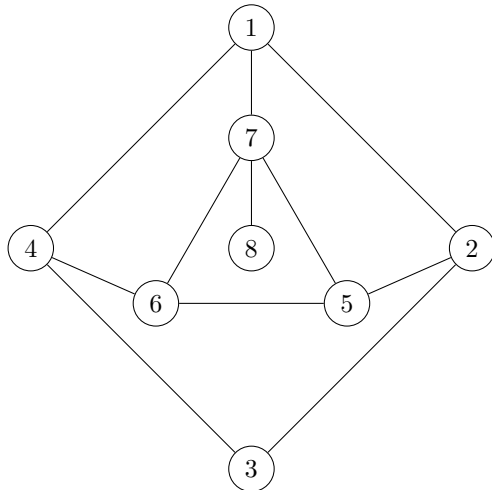


9 Graph Theory

9.1

Consider the following graph.



1. Is it directed, connected, and/or cyclic?
2. List source, sink, and transfer nodes as sets
3. Give the min. and max. degree of the graph
4. What is the degree of node 4?
5. List the loops in this graph as a set
6. Write a simple cycle in this graph as a list
7. Write a simple path that connects every node in this graph as a list
8. Write a shortest path connecting every even-numbered node as a list

9.2

A graph G is defined as $G = \{V, E\}$, where $V = \{a, b, c\}$, and $E = V \times V$

1. Is it directed, connected, and/or cyclic?
2. How many nodes and edges does it contain?
3. Give the min. and max. degree of the graph

9.3

A formal string-rewriting system is defined with the rules below. The starting string is a

- $a \rightsquigarrow aba$
- $bab \rightsquigarrow b$

1. Draw a graph of the first 5 unique strings that can be constructed in this system, where edges represent applications of one of the grammatical rules

9.4

Below is a database for a Twitter-like social network

Username	Following	Posts
Alice	{Bob, Eve}	24
Bob	{Alice, Eve}	3
Eve	{Alice, Mary}	124
Mary	{Eve}	10

We can produce a graph of this social network $G = (V, E)$ where $V = \{x : \text{Username} \mid \text{true}\}$, and E is a set of directed edges (a, b) where A 'follows' B .

1. Give an extensional definition of E
2. Give an intensional definition of E using set comprehension
3. List of followers: Give a set comprehension that returns a tuple (a, b) , where a is a username and b is a person following that user
4. Follow recommendation: Give a set comprehension that returns a tuple (a, b) , where a is a username and b is a username of someone a might like to follow (a might 'like to follow' b if and only if b is followed by someone a follows, a does not already follow them, and a and b are not the same person).