2 Set Theory

The following laws of set theory are compiled and numbered for the benefit of this course. This is not an exhaustive list.

2.1 Laws of Set membership

Law 1.1 for any set S and any element s

$$\neg(s \in S) \iff s \notin S$$

Law 1.2 for any element x

$$x \in \emptyset \iff \text{false}$$

2.2 Laws of Subsets

Law 2.1 for any sets S and T

$$(S \subseteq T \wedge T \subseteq S) \iff S = T$$

Law 2.2 for any sets S

$$(\emptyset \subseteq S)$$

Law 2.3 all sets are a subset of themselves

$$(S \subseteq S)$$

Law 2.4 for any sets S and T

$$\neg(S \subseteq T) \iff S \nsubseteq T$$

Law 2.5 for any sets S and T

$$S \subseteq T \iff (S \subset T \vee S = T)$$

Law 2.6 for any sets S and T

$$S \not\subset T \iff \neg(S \subset T)$$

Law 2.7 for any set S

$$S \not\subset S$$

Law 2.8 for any sets S and T

$$S \subset T \implies T \not\subset S$$

2.3 Laws of Supersets

Law 3.1 for any sets S and T. Stating S is a superset of T is logically equivalent to stating that T is a subset of S

$$S \supseteq T \iff T \subseteq S$$

2.4 Laws of Set Union

Law 4.1 for any element a, and any sets S and T

$$a \in S \cup T \iff (a \in S \vee a \in T)$$

Law 4.2 combining Set S with the empty set ∅, is equivalent to Set S:

$$S \cup \emptyset = S$$

Law 4.3 The set union of any set S combined with itself is equivalent to itself

$$S \cup S = S$$

Law 4.4 Union is commutative

$$S \cup T = T \cup S$$

Law 4.5 Union is associative

$$R \cup (S \cup T) = (R \cup T) \cup S$$

Law 4.6 The union of two sets is always at least as big as each set considered individually

$$S \subseteq S \cup T$$

2.5 Laws of Set Intersection

Law 5.1 where a given element a is in the intersection of sets S and T is must be an element of both sets

$$a \in S \cap T \iff (a \in S \land a \in T)$$

Law 5.2 the intersection of a given set S with the empty set Ø is always the empty set

$$S \cap \emptyset = \emptyset$$

Law 5.3 the intersection of set S with itself is always S

$$S \cap S = S$$

Law 5.4 Intersection is commutative

$$S \cap T = T \cap S$$

Law 5.5 Intersection is associative

$$R \cap (S \cap T) = (R \cap S) \cap T$$

Law 5.6 The intersection of any given sets is always at least as small as one of the given sets

$$S \cap T \subseteq S$$

Law 5.7 union distributes through Intersection and Intersection distributes through distribution

$$R \cup (S \cap T) = (R \cup S) \cap (R \cup T)$$

$$R \cap (S \cup T) = (R \cap S) \cup (R \cap T)$$

2.6 Laws of Set Difference

Law 6.1 if a is an element of the Set difference of Sets $S \setminus T$ then S is a member of the former and not the latter

$$a \in S \setminus T \iff (a \in S \land a \notin T)$$

Law 6.2 Set S intersected with the empty set is equivocal to set S

$$S \setminus \emptyset = S$$

Law 6.3 The set difference of the empty set with a set S is the empty set

$$\emptyset \setminus S = \emptyset$$

Law 6.4 The difference in any set S and itself produces the empty set

$$S \setminus S = \emptyset$$

Law 6.5 The difference in Set R and the union or sets S and T is equivocal to the union of the difference in set R

and S and R and T. A similar propery holds for Intersection.

$$R \setminus (S \cup T) = (R \setminus S) \cap (R \setminus T)$$

$$R \setminus (S \cap T) = (R \setminus S) \cup (R \setminus T)$$

2.7 Laws of equality

Law 7.1 When two different sets have exactly the same elements, they are equal

$$x \in S \iff x \in T$$