6 Set Theory

6.1

Define the following sets by extension

1. The set of natural numbers between 6 and 11 (not inclusive)
2. The set of letters in the phrase "formal systems, logic, and semantics"
3. The set of sets with exactly one subset

6.2

State whether each of the following is a singleton, the empty set, or neither of these

1. The set of Real numbers, less the Natural numbers
2. The set of even primes
3. The set of digits used in binary
4. The set black hearts in a standard deck of cards
5. A set that has no proper subsets
6. A set with exactly two subsets

6.3

Give the extension of the following sets

1. $\{a, b, c\} \cap \{c, d\}$
2. $\{a, c\} \cup \{b, c, d\}$
3. $\{a, b, c\} \setminus \{b, c, d\}$
4. $\{a\} \cap (\{b\} \cup \{a, b, c\})$
5. $(\{c\} \cup \{b\} \cup \{a\}) \cap (\{b, c, a\} \cup \emptyset)$

6.4

For the given sets, state whether the following propositions are true or false

$A = \{a, b, c\}$
$B = \{a, d\}$
$C = \{c\}$

1. $A \supset C$
2. $(C \cup A) \subseteq B$
3. $c \in (A \cap B)$
4. $a \notin (A \setminus B)$
5. $(B \cap \{d, a\}) \supseteq B$

6.5

For the given sets, give the extension of the following sets

$A = \{a, b, c\}$
$B = \{a, d\}$
$C = \{c\}$

1. $(A \setminus B) \cap C$
2. $(B \cap C) \cup A$
3. $C \cup (A \setminus (B \cap A))$

6.6

Use equational reasoning to prove the following. Reference the laws of set theory and propositional logic introduced in the lectures and on the handout.

1. $S \subseteq (T \cup (S \cap S))$
2. $(S = T) \wedge (T \not\subseteq S) \iff \text{false}$
3. $\emptyset \not\subset ((S \cap S) \setminus (S \setminus \emptyset))$