2.8 Laws of cardinality

Law 8.1 the cardinality of the empty set is 0

$$\#\emptyset = 0$$

Law 8.2 the cardinality of the union of S and T is equal to the cardinality of S minus the cardinality of the intersection of S and T

$$\#(S \cap T) = \#S - \#(S \setminus T)$$

Law 8.3 the cardinality of the union of S and T is equal to the cardinality of S plus the cardinality of T minus the cardinality of the intersection of S and T

$$\#(S \cup T) = \#S + \#T - \#(S \cap T)$$

Law 8.4 the Cardinality of the intersection of S and T is equal to the cardinality of S minus the cardinality of the intersection of S and T

$$\#(S \setminus T) = \#S - \#(S \cap T)$$

Law 8.5 the Cardinality of the cartesian product of S and R is the product of the cardinalities of S and R

$$\#(S \times T) = \#S \times \#T$$

2.9 Laws of Power Sets

Law 9.1 Set S is an element of the power set of T if and only if S is a subset of T

$$S \in \mathbb{P}(T) \iff S \subseteq T$$

Law 9.2 the empty set is an element of the power set of a any given set S

$$\emptyset \in \mathbb{P}(S)$$

Law 9.3 for any given set S, S is an element of the power set of itself

$$S \in \mathbb{P}(S)$$

Law 9.4 the cardinality of the power set of set S is equal to two to the power of the cardinality of S

$$\#(\mathbb{P}(S)) = 2^{\#(S)}$$

Law 9.5 for a given set R which is an element of the power set of S, the intersection of R and S is equal to R.

$$R \cap S = R, \text{ where } R \in \mathbb{P}(S)$$

2.10 Laws of Complements

Law 10.1 For a given set T $\subseteq$ S, the compliment of T is equal to the set difference in S and T

$$T^{-} = S \setminus T, \text{ where } T \in \mathbb{P}(S)$$

Law 10.2 For a given set T $\subseteq$ S, The union of T and its compliment is equal to the S

$$T \cup T^{-} = S, \text{ where } T \in \mathbb{P}(S)$$

Law 10.3 For a given set T $\subseteq$ S, The intersection of T and its compliment is equal to the empty set.

$$T \cap T^{-} = \emptyset, \text{ where } T \in \mathbb{P}(S)$$

2.11 Laws of Generalised Operations

Law 11.1 for any set of sets A and any element a, $a \in \bigcup A$ if, and only if, there is some set $S \in A$ such that $a \in S$

$$a \in \bigcup A \iff \exists S \in A . a \in S$$

Law 11.2 for any set of sets A and any element a, $a \in \bigcap A$ if, and only if, for every set $S \in A$ it is the case that $a \in S$

$$a \in \bigcap A \iff \forall S \in A . a \in S$$