Mathematics and Problem Solving

Lecture 6

Set Theory

“Since, as is well known, God helps those who help themselves, presumably the Devil helps all those, and only those, who don't help themselves. Does the Devil help himself?”

Douglas Hofstadter

What we will do today

- What Are Sets?
  - Singletons
  - Subsets
  - Supersets

- Set operations
  - Union
  - Intersection
  - Difference

What is a Set?

What is a set?

- A set is a collection of objects, which
  - is unordered
  - contains no repetition

Example Sets

- Examples of sets:
  - All the people in the UK
  - All the natural numbers in the set denoted by $\mathbb{N}$
  - All the counties in Northern Ireland

Typed Sets

- Set theories have to restrict this definition somehow, as a naive approach leads to paradox
  - Consider the set of all sets that don’t contain themselves
  - (e.g. a book containing the titles of all books that don’t contain their own title)
- We will see a strictly typed a set theory
  - All the examples we will look at will all include the same types of object
  - Tends to be most useful for CS

Set Definition

Definition by Extension

Extension of a Set

- Rather than describe a set in words, e.g.
  - “*N* is the set containing all the numbers between 4 and 7” (a bit unclear)

  we can list the things it contains (very clear!)

  $$N = \{ 5, 6 \}$$

- This sort of listing is called a **definition by extension**
  - Written using curly brackets { }
  - Every element is seperated with a comma

Exercise 1:

Define the following sets by extension:

1. The numbers between 1 and 10
2. The letters in the alphabet preceeding ‘f’
3. The set of all even primes greater than 2

- **Hint:**
  - Definition by extension
    - Written using curly brackets { }
    - Every element is seperated with a comma
  - e.g. N = { 5, 6 }

Sets

Unordered, unique elements

Repetition

- An element can **only occur once** in a set – all elements are unique
- Sets are **equal** (=) if they contain the same elements
  - Order doesn't matter

- If we express the following collections as sets
  - Collection_1 = (apple, pear, banana, banana, banana, plum)
  - Collection_2 = (apple, banana, apple, pear, pear, banana, plum, plum)
- They would be equal
  - Collection_1= { apple, pear, banana, plum }
  - Collection_2= { apple, banana, pear, plum }

Exercise 2:

Which of the following equalities hold if each side is described as a set?

1. (1, 1, 2, 3, 4, 5) =?= (1, 2, 3, 4, 4, 5, 5)
2. Vowels in the alphabet =?= Vowels in the phrase ‘Rain wind and snow tomorrow’
3. (dog, cat, parrot) =?= (parrot, parrot, cat, parrot, dog, goldfish)
4. Notes (A, A#, etc.) on a piano =?= Notes in an octave

- **Hint:**
  - Sets are unordered
  - All elements are unique

Singletons

Singletons

- A set containing only one member is a **singleton**, e.g.
  - { 1 }
  - The set of prime numbers between 6 and 10

Singletons

Exercise 3:

Which of the following are singletons if described as sets:

1. (1, 1, 1, 1)
2. The vowels in the alphabet between ‘A’ and ‘D’
3. The prime numbers between 1 and 10
4. The set of all symbols used in a number system of radix 2 (binary)

- **Hint:**
  - A singleton is a set containing exactly 1 element

The Empty Set

The Empty Set

- The set with no elements is called the **Empty Set**
- All sets with no elements are equal to one another
  - They all have the same elements (nothing!)
  - Hence calling it *the* empty set
- This is notated in either of the following ways

$$\{\,\} \qquad \varnothing$$

The Empty Set

Exercise 4:

Which of the following describes the empty set?

1. The set of all Natural numbers $\mathbb{N}$ that are not in the set of all integers $\mathbb{Z}$
2. The set of all Integers $\mathbb{Z}$, less that 0.
3. The set of all letters in the alphabet tham come before the letter 'a'

- **Hint:**
  - The empty set is any set containing no elements

Sets Containing Sets

- Sets can contain other sets as members
  - e.g. The set of sets of numbers
- This is not the same as containing the constituent elements in a flat structure, i.e.
  - $\{ \mathbb{Z}, \mathbb{N}, \mathbb{R}, \mathbb{Q} \} \neq \mathbb{Q}$
  - $\{ \{ 1, 2\}, \{1, 3\}, \{ 3, 1, 2 \} \} \neq \{1, 2, 3\}$

Sets Containing the Empty Set

- The Empty Set is a set, and can be a member of another set, e.g.
  - $\{ \varnothing$, tall cats, angry lizards $\}$
  - $\{ \varnothing, \{ \varnothing \}, \{ \varnothing, \{ \varnothing \}, \{ \varnothing \ \{ \varnothing, \{ \varnothing \} \} \} \} \}$
- Critically, $\varnothing \neq \{ \varnothing \} \neq \{ \{ \varnothing \} \}$, etc.
  - Writing $\{ \varnothing \}$ to mean the empty set is a very common error!

Sets Containing Sets

Exercise 5:

Express the following as sets, using set theory notation:

1. The set of hibernating giraffes
2. (Ø, Ø, { Ø }, { { Ø }, Ø }, { Ø })
3. The set of all sets with 0 elements
4. The set with 0 elements

Set Membership

A language to talk about sets

- We want to be able to make statements about the relationships between sets
  - Set A is equal to set B
  - Set A contains set B
  - Set A and Set B contain no elements in common
- We want to make statements about what sets contain
  - Cat is in the set of animals
  - The set of reptiles does not contain dogs
- We want to talk about the sizes of sets
  - The set of students in this class has x members

Set Membership

- We say "x is a member of set Y" using the symbol $\in$

$$x \in Y$$

  For example,

  - $1 \in \mathbb{N}$ – 1 is a member of the set of all natural numbers
  - $-1 \notin \mathbb{N}$ – Negative 1 is not a member of the set of natural numbers

- Note the complimentary use of $\in$ and $\notin$

Set Elements

Law 1.1:

This gives us our first law of set theory:

$$\neg (s \in S) \iff s \notin S$$

- The negation of the statement that '*s* is an element of *S*' is equivalent to stating that '*s* is not an element of *S*'
- Note how we can combine propositional logic and set theory

Set Elements

Law 1.2:

Building on law 1.1, as no elements appear in the empty set:

$$x \in \emptyset \iff \text{false}$$

- The statement that x is an element of Ø is logically equivalent to False, regardless of what x is

Set Elements

Exercise 6:

Which of the following set theory statements are logically equivalent to true and which are logically equivalent to false?

1. $a \in \{ r, a, c, b, l, z, k \}$
2. $p \in \{ r, a, c, b, l, z, k \}$
3. $a \notin \{ r, a, c, b, l, z, k \}$
4. $\mathrm{p} \notin \{ r, a, c, b, l, z, k \}$

True, False, and Undefined

Typed Set Theory

- We’re adopting a strictly typed set theory
- In a typed set theory, every set is associated with a type
  - For now, we will have two types:
    - Set of atomic elements
    - Set of sets
- The operators we will see apply only to sets of the same type
  - Attempting otherwise is a syntactic error
  - The ‘result’ of such an operation is undefined

True, False and undefined

- Let $S = \{1, 2, 3\}$
- Valid in our set theory
  - $2 \in S$ *(true)*
  - $5 \notin S$ *(true)*
  - $7 \in S$ *(false)*
- Invalid in our set theory
  - $\{2\} \in S$ *(undefined)*
  - $\{5\} \notin S$ *(undefined)*

True, False and undefined

Exercise 7:

For the set: Cakes = { cupcake, carrotcake, doughnut }, which of the following are True or False and which ones are undefined?

1. Carrot Cake ∈ Cakes
2. Doughnut ∈ Cakes
3. { cupcake, doghnut } ∈ Cakes
4. Ø ∉ Cakes
5. cupcake ∉ Cakes

- Hint:
  - We have two types:
    - Set of atomic elements
    - Set of sets
  - Operators can only apply to sets of the same type

Subsets

Subsets

- We want to be able to express the idea that "Set A is contained within Set B"
  - We formalise this as **subset**, using the symbol $\subseteq$
- For example, for the sets
  - $S = \{ 2, 5, 7, 9, 13\}$
  - $T = \{ 2, 5, 7, 9\}$
- It is true to state that T is a subset of S, which we notate:
  - $T \subseteq S$

Subsets

- Imagine we have two sets with the same elements:
  - $S = \{2, 5, 7\}$
  - $T = \{2, 5, 7\}$
- It is possible to state both:
  - $T \subseteq S$
  - $S \subseteq T$
- Thus we can also say $S = T$

Subsets

Law 2.1:

If *S* is a subset of *T* and *T* is also a subset of *S*, then *S* and *T* are equal

$$(S \subseteq T) \land (T \subseteq S) \Longleftrightarrow \mathrm{S} = \mathrm{T}$$

Subsets

Law 2.2

The empty set is a subset of every set. Thus we can state:

$$\varnothing \subseteq S$$

Subsets

Law 2.3:

Every set is a subset of itself:

$$S \subseteq S$$

Exercise 8:

Which of the following are True, False, or undefined?

1. $A \subseteq \{ a, b, c \}$
2. $\{ a \} \subseteq \{ a, b, c\}$
3. $\varnothing \subseteq \{ a, b, c \}$
4. $\{ a, b, c \} \subseteq \varnothing$
5. $\varnothing \in \{ a, b, c\}$

Hint:

- Law 2.1: $(S \subseteq T) \wedge (T \subseteq S) \Longleftrightarrow \mathrm{S} = \mathrm{T}$
- Law 2.2: $\varnothing \subseteq S$
- Law 2.3: $S \subseteq S$

Not a Subset

- To express "not a subset", we can use the symbol $\not\subseteq$
- Take 2 sets:

  S = { 1, 2, 3, 4, 5, 6 }

  T = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }
- S is a subset of T:

$$S \subseteq T$$

- T is NOT a subset of S:

$$T \not\subseteq S$$

Subset and Not a Subset

Law 2.4:

The negation of the statement "*S* is a subset of *T*" is equivalent to stating "*S* is not a subset of *T*".

$$\neg(S \subseteq T) \Longleftrightarrow S \nsubseteq T$$

Subsets and Proper Subsets

Proper Subsets

- To revisit these sets from just a few slides ago:
  - S = { 1, 2, 3, 4, 5, 6 }
  - T = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }
- Because S is a subset of T, and T contains elements not in S, S is said to be a **proper subset** of T
  - We use the symbol ⊂ to write proper subset:

    $S \subset T$

Subsets

Law 2.5:

If S is a subset of T, S is either a proper subset of, or is equal to, T

$$S \subseteq T \Longleftrightarrow (S \subset T) \vee (S = T)$$

Subsets

Exercise 9:

Which of the following are True and which are False?

1. $\{ \text{monkey} \} \subseteq \{ \text{monkey, elephant, zebra} \}$
2. $\text{monkey} \in \{ \text{monkey, elephant, zebra} \}$
3. $\varnothing \subset \{\text{elephant, monkey, zebra}\}$
4. $\{ \text{monkey, elephant, zebra} \} \subset \{ \text{monkey, elephant, zebra} \}$

Hint:

- Law 2.1: $(S \subseteq T) \wedge (T \subseteq S) \Longleftrightarrow \mathrm{S} = \mathrm{T}$
- Law 2.2: $\varnothing \subseteq S$
- Law 2.3: $S \subseteq S$
- Law 2.4: $\neg (S \subseteq T) \Longleftrightarrow S \nsubseteq T$
- Law 2.5: $S \subseteq T \Longleftrightarrow (S \subset T) \vee (S = T)$

Subsets

Laws

Subsets

Law 2.6:

The claim that "S is not a proper subset of T" is equivalent to the negation of the statement "S is a proper subset of T"

$$S \not\subset T \iff \neg\,(S \subset T)$$

Subsets

Law 2.7:

It follows that S can never be a proper subset of itself. Thus:

$$S \not\subset S$$

Subsets

Law 2.8:

If set *S* is a proper subset of another set *T* then *T* can not also be a proper subset of *S*.

$$S \subset T \Rightarrow T \not\subset S$$

Subsets

Exercise 10:

Given the set *S*

$$S = \{ a, b, c, d\}$$

1. List all the subsets for *S*
2. List all the proper subsets *S*
3. Can you conceive of a mathematical formula for working out the number of proper subsets of a exist for a given set of *n* elements?

Subsets

Exercise 11:

We can apply the laws of set theory in the same way we learned to apply the laws of propositional logic.

Prove the following assertion with reference to the laws covered so far:

$$S \subseteq \varnothing \Rightarrow S = \varnothing$$

- Hint:
  - PL Law 4.1:

    $$(p \Rightarrow q) \Longleftrightarrow ((\neg p) \lor q)$$

  - PL Law 2.2:

    $$(p \land \text{true}) \Longleftrightarrow p$$

  - ST Law 2.2:

    $$\varnothing \subseteq S$$

  - PL Law 3.7:

    $$((\neg p) \lor p) \Longleftrightarrow \text{true}$$

  - ST Law 2.1:

    $$(S \subseteq T) \land (T \subseteq S) \Longleftrightarrow S = T$$

Example Proof

1 $S \subseteq \varnothing \Rightarrow S = \varnothing$

= (PL Law 4.1)

2 $\neg(S \subseteq \varnothing) \vee (S = \varnothing)$

= (PL Law 2.2)

3 $\neg(S \subseteq \varnothing \wedge \text{true}) \vee (S = \varnothing)$

= (ST Law 2.2)

4 $\neg(S \subseteq \varnothing \wedge \varnothing \subseteq S) \vee (S = \varnothing)$

= (ST Law 2.1)

5 $\neg(S = \varnothing) \vee (S = \varnothing)$

= (PL Law 3.7)

6 true

- PL Law 4.1:

$$(p \Rightarrow q) \Longleftrightarrow ((\neg p) \vee q)$$

- PL Law 2.2:

$$(p \wedge \text{true}) \Longleftrightarrow p$$

- ST Law 2.2:

$$\varnothing \subseteq S$$

- PL Law 3.7:

$$((\neg p) \vee p) \Longleftrightarrow \text{true}$$

- ST Law 2.1:

$$(S \subseteq T) \wedge (T \subseteq S) \Longleftrightarrow S = T$$

Supersets

Supersets

- “Superset” is the inverse of “Subset”
  - A superset contains all the elements of its subset

    Uses the symbol ⊇ (the subset symbol backwards)

- If S is a subset of T

  $S \subseteq T$

  then T is a superset of S

  $T \supseteq S$

Supersets

- Proper Supersets is like the inverse of Proper Subset
  - Contain all the elements of their subset, and more
  - Proper subset is written with a backwards proper subset symbol: $\supset$
- We can also negate superset and proper superset
  - Not Superset: $\not\supseteq$
  - Not Proper Superset: or $\not\supset$

Supersets

Laws

Supersets

Law 3.1:

For any sets S and T. Stating that S is a superset of T is logically equivalent to stating that T is a subset of S.

$$S \subseteq T \Longleftrightarrow T \supseteq S$$

Supersets

Exercise 12:

Work out the truth value of the following statements given set S = {Berlin, Belfast, Bratislava, Burnley}:

1. { Berlin } $\subset S$
2. {Berlin} $\subseteq S$
3. $S \supseteq$ {Berlin, Bratislava, Burnely}
4. $\varnothing \supseteq S$
5. $S \supseteq \varnothing$
6. {Berlin, Belfast, Burnely} $\not\supset S$

- **Hint**
  - Law 3.1: $S \subseteq T \Longleftrightarrow T \supseteq S$

Set Operations

Set operators

- Set operators are as follows:
  - Union: ∪
  - Intersection: ∩
  - Difference: \
  - Cartesian Product: ×
- We're going to look at the first three of these and leave the last operator to next week

Set Union

Set union

- Set Union is a binary operation on sets
  - It uses the symbol U
- The union operator combines two sets to form a set containing all of the elements of each set

$S = \{ 1, 2, 3, 5, 7, 9 \}$

$T = \{1, 3, 4, 5, 6, 7, 8, 10\}$

$S \cup T = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Set union

Exercise 13:

Calculate the following:

1. { 1, 3, 4 } U { 1, 2, 3, 5, 7, 9 }
2. {rich tea, digestive, jammy dodger} U Ø
3. { a, b, c, d } U { a, b, c, d }
4. { a, b, c, d } U {1, 2, 3, 4 }

Set Union

Laws

Set Union

Law 4.1:

For any element *a* and sets *S* and *T* we can say that the statement *e* is an element of the union of *S* and *T* is logically equivalent to stating that *a* is an element of *S* or *a* is an element of *T*

$$a \in S \cup T \iff (a \in S \vee a \in T)$$

Set Union Identity

Law 4.2:

S combined with the empty set Ø is logically equivalent to *S*. Thus:

$$S \cup \emptyset = S$$

Set Union Identity

Law 4.3:

S combined with itself is logically equivalent to *S*

$$S \cup S = S$$

Commutativity of Set Union

Law 4.4:

The union of *S* and *T* is equivalent to the union of *T* and *S*

$$S \cup T = T \cup S$$

Associativity of Set Union

Law 4.5:

Union is associative. Thus for Sets R, S and T:

$$(R \cup S) \cup T = (R \cup T) \cup S$$

Set Union

Law 4.6:

The Union of two or more sets always at least as big as each set considered individuals. Sets *S* and *T*:

$$S \subseteq S \cup T$$

Set Intersection

Set Intersection

- Set Intersection is a binary operator on sets
  - Uses the symbol ∩
- It produces a set containing only the elements contained in both sets
- The intersection of S and T is the set of elements which are in both set S and set T
  - $S = \{ 1, 2, 3, 4, 5, 6 \}$
  - $T = \{ 4, 5, 6, 7, 8, 9 \}$
  - $S \cap T = \{4, 5, 6 \}$

Set Intersection

Exercise 14:

Calculate the following:

1. $\{ 1, 3, 4 \} \cap \{ 1, 2, 3, 5, 7, 9 \}$
2. { rich tea, digestive, jammy dodger } $\cap \varnothing$
3. $\{ a, b, c, d \} \cap \{ a, b, c, d \}$
4. $\{ a, b, c, d \} \cap \{ d, c, b, a \}$

Set Intersection

Laws

Set Intersection

Law 5.1:

If a given element is in the output of a set intersection operation then it must appear in all sets in the operation. Therefore where *a* is an element of the intersection of sets *S* and *T* a must be an element of *S* and must also be an element of *T*. Thus:

$$a \in (S \cap T) \Longleftrightarrow (a \in S) \land (a \in T)$$

Null Law

Law 5.2:

The intersection of set S and the empty set $\varnothing$ is always the empty set.

$$S \cap \varnothing = \varnothing$$

Intersection Identity

Law 5.3:

The intersection of set S with itself is always S. Thus:

$$S \cap S = S$$

Commutativity of Intersection

Law 5.4:

Intersection is commutative. Thus for set S and T:

$$S \cap T = T \cap S$$

Associativity of Intersection

Law 5.5:

Intersection is associative. Thus for set *R*, *S* and *T*:

$$R \cap (S \cap T) = (R \cap S) \cap T$$

Set Intersection

Law 5.6:

The intersection of any given sets is always at least as small as one of the given sets. Thus for set *S* and *T*:

$$S \cap T \subseteq S$$

Set Intersection

Law 5.7:

Intersection distributes through Union and Union distributes through intersection. Thus for set *S* and *T*:

$$R \cup (S \cap T) = (R \cup S) \cap (R \cup T)$$

$$R \cap (S \cup T) = (R \cap S) \cup (R \cap T)$$

Set Intersection

Exercise 15:

Confirm that Law 5.7 Holds. Where set R, S, and T are defined by extension as:

- $R = \{ a, b, c \}$
- $S = \{ a, b, c, d, e \}$
- $T = \{ c, d, e, f \}$

- By applying it to calculate the following relations:
  - $R \cup (S \cap T) = (R \cup S) \cap (R \cup T)$

- Hint:
  - If you calculate the left side of the = and then the right side and the results match then you've proven that the Law holds.

Set Difference

Set Difference

- Set difference operation takes two inputs and returns as an output the set elements that exists the first set, but not in the latter: Thus for sets *S* and *T*:

  $S = \{ a, b, c, d, e, f, g, h, i, j \}$

  $T = \{ f, g, h, i, j, k\ l, m, n\ o \}$

  $S \setminus T = \{ a, b, c, d, e \}$

Set Difference

Exercise 16:

Calculate the following:

1. $\{ 1, 3, 4 \} \setminus \{ 1, 2, 3, 5, 7, 9 \}$
2. { richtea, digestive, jammy dodger } \ Ø
3. $\{ a, b, c, d \} \setminus \{ a, b, c, d \}$
4. $\{ a, b, c, d \} \setminus \{ d, c, b, a \}$

Set Difference

Laws

Set difference

Law 6.1:

If a is an element of the set difference is $S \setminus T$ then a is a member of S and a is not a member of T

$$a \in S \setminus T \iff (a \in S \land a \notin T)$$

Set difference

Law 6.2:

The difference in Set S and the empty Set Ø is equal to Set S

$$S \setminus \varnothing = S$$

Set difference

Law 6.3:

The inverse of the previous law: the difference in the empty Set Ø and Set *S* is equal to the empty set Ø

$$\varnothing \setminus S = \varnothing$$

Set difference

Law 6.4:

The difference in Set S and itself produces the empty set as there is nothing in the former that is not in the latter

$$S \setminus S = \varnothing$$

Set difference

Law 6.5:

The difference in Set R and the union or sets *S* and *T* is equivocal to the union of the difference in set *R* and *S* and *R* and *T*. A similar propery holds for Intersection. Thus:

$$R \setminus (S \cup T) = (R \setminus S) \cap (R \setminus T)$$

$$R \setminus (S \cap T) = (R \setminus S) \cup (R \setminus T)$$

Set difference

Law 6.6:

The difference in Set S and T will result in a set at least as small as set S. Thus:

$$S \setminus T \subseteq S$$

Set Intersection

Exercise 17:

Confirm that Law 6.5 Holds. Where set R, S and T are defined by extension as:

$R = \{ a, b, c \}$

$S = \{ a, b, c, d, e \}$

$T = \{ c, d, e, f \}$

By applying it to calculate the following relations:

1. $R \setminus (S \cap T) = (R \setminus S) \cup (R \setminus T)$

- Hint:
  - If you calculate the left side of the = and then the right side and the results match then you've proven that the Law holds.

Set Equality

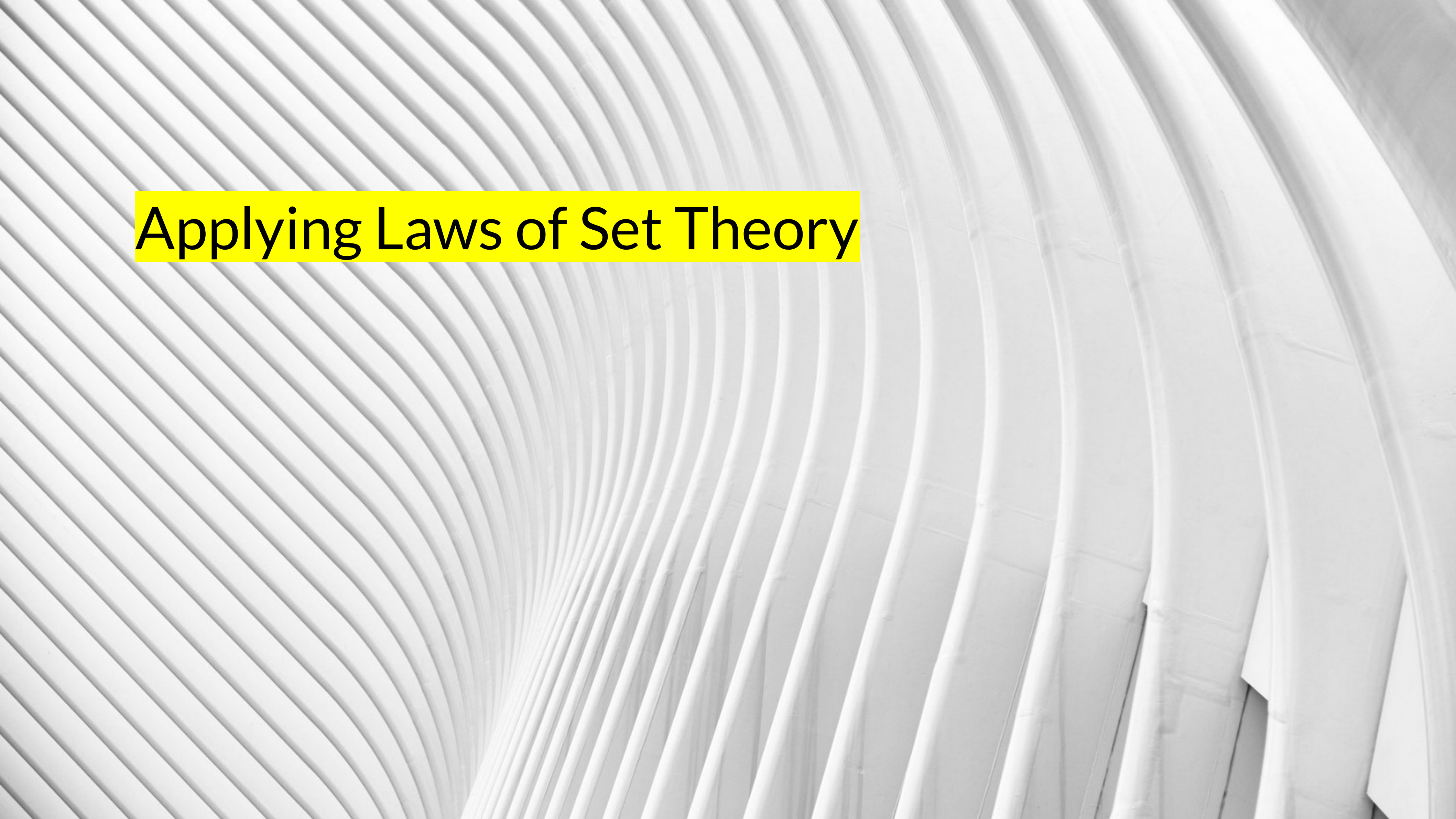
Set Equality

Law 7.1:

When two different sets have exactly the same elements, they are considered equal if for every x is S is equivalent to every x in T

$$x \in S \iff x \in T$$

Applying Laws of Set Theory



Applying the Laws of Set theory

Exercise 18:

Use a combination of the laws of propositional logic and the laws of set theory to prove the following:

1. $S \subseteq (T \cup S) \Rightarrow S \subseteq T \vee S \subseteq S$
2. $T \nsubseteq (\varnothing / S) \cup T \Rightarrow S \subset \varnothing$

- Law 1.1: $\neg(s \in S) \Longleftrightarrow s \notin S$
- Law 1.2: $x \in \emptyset \Longleftrightarrow false$
- Law 2.1: $(S \subseteq T \wedge T \subseteq S) \Longleftrightarrow S = T$
- Law 2.2: $(\varnothing \subseteq S)$
- Law 2.3: $(S \subseteq S)$
- Law 2.4: $\neg(S \subseteq T) \Longleftrightarrow S \nsubseteq T$
- Law 2.5: $S \subseteq T \Longleftrightarrow (S \subset T \vee S = T)$
- Law 2.6: $S \not\subset T \Longleftrightarrow \neg(S \subset T)$
- Law 2.7: $S \not\subset S$
- Law 2.8: $S \subset T \Rightarrow T \not\subset S$
- Law 3.1: $S \supseteq T \Longleftrightarrow T \subseteq S$

- Law 4.1: $a \in S \cup T \Longleftrightarrow (a \in S \vee a \in T)$
- Law 4.2: $S \cup \emptyset = S$
- Law 4.3: $S \cup S = S$
- Law 4.4: $S \cup T = T \cup S$
- Law 4.5: $R \cup (S \cup T) = (R \cup T) \cup S$
- Law 4.6: $S \subseteq S \cup T$
- Law 5.1: $a \in S \cap T \Longleftrightarrow (a \in S \wedge a \in T)$
- Law 5.2: $S \cap \emptyset = \emptyset$
- Law 5.3: $S \cap S = S$
- Law 5.4: $S \cap T = T \cap S$
- Law 5.5: $R \cap (S \cap T) = (R \cap S) \cap T$
- Law 5.6: $S \cap T \subseteq S$
- Law 5.7: $\boldsymbol{R} \cup (\boldsymbol{S} \cap \boldsymbol{T}) = (\boldsymbol{R} \cup \boldsymbol{S}) \cap (\boldsymbol{R} \cup \boldsymbol{T})$

  $\boldsymbol{R} \cap (\boldsymbol{S} \cup \boldsymbol{T}) = (\boldsymbol{R} \cap \boldsymbol{S}) \cup (\boldsymbol{R} \cap \boldsymbol{T})$

- Law 6.1: $\boldsymbol{a} \in S \backslash T \Longleftrightarrow (a \in S \wedge a \notin T)$
- Law 6.2: $S \backslash \emptyset = S$
- Law 6.3: $\emptyset \backslash S = \emptyset$
- Law 6.4: $S \backslash S = \emptyset$
- Law 6.5: $R \backslash (S \cup T) = (R \backslash S) \cap (R \backslash T)$

  $R \backslash (S \cap T) = (R \backslash S) \cup (R \backslash T)$
- Law 7.1: $x \in S \Longleftrightarrow x \in T$

  When two sets have the same elements they are equal

Summary

- What Are Sets?
  - Singletons
  - Subsets
  - Supersets
- Set operations
  - Union
  - Intersection
  - Difference