1 Propositional Logic

1.1 Laws of Negation

Law 1.1 not false is true and not true is false

$$(\neg\text{true}) \iff \text{false}$$

$$(\neg\text{false}) \iff \text{true}$$

Law 1.2 two negatives make a positive

$$(\neg\neg p) \iff p$$

1.2 Laws of Conjunction

Law 2.1 a proposition conjoined with itself is equivalent to itself

$$(p \wedge p) \iff p$$

Law 2.2

$$(p \wedge \text{true}) \iff p$$

Law 2.3 a proposition conjoined with false will always resolve to false

$$(p \wedge \text{false}) \iff \text{false}$$

Law 2.4 a proposition conjoined with its own negation will always resolve to false

$$(p \wedge (\neg p)) \iff \text{false}$$

Law 2.5 conjunction is commutative

$$(p \wedge q) \iff (q \wedge p)$$

Law 2.4 conjunction is associative

$$(p \wedge q) \wedge r \iff p \wedge (q \wedge r)$$

1.3 Laws of Disjunction

Law 3.1 de Morgan's Laws

$$\neg(p \wedge q) \iff ((\neg p) \vee (\neg q))$$

$$\neg(p \vee q) \iff ((\neg p) \wedge (\neg q))$$

Law 3.2 disjunction is idempotent

$$(p \vee p) \iff p$$

Law 3.3 combination via disjunction or a proposition with false is logically equivalent o the value of the proposition

$$(p \vee \text{false}) \iff p$$

Law 3.4 a proposition combined via disjunction with true will always resolve to true

$$(p \vee \text{true}) \iff \text{true}$$

Law 3.5 disjunction is associative

$$p \vee (q \vee r) \iff (p \vee q) \vee r$$

Law 3.6 conjunction is commutative

$$p \vee q \iff q \vee p$$

Law 3.7 a proposition combined via disjunction with its own negation will always resolve to true

$$((\neg p) \vee p) \iff \text{true}$$

Law 3.8 disjunction distributes through conjunction

$$p \vee (q \wedge r) \iff (p \vee q) \wedge (p \vee r)$$

Law 3.9 conjunction distributes through disjunction

$$p \wedge (q \vee r) \iff (p \wedge q) \vee (p \wedge r)$$

1.4 Laws of Implication

Law 4.1 p implies q is the same as not p, or q

$$(p \implies q) \iff ((\neg p) \vee q)$$

1.5 Laws of Equivalence

Law 5.1 equivalence is associative

$$((p \iff q) \iff r) \iff (p \iff (q \iff r))$$

Law 5.2 equivalence is commutative

$$(p \iff q) \iff (q \iff p)$$

Law 5.3 every proposition is equivalent to itself

$$(p \iff p) \iff \text{true}$$

Law 5.4 no proposition is equivalent to its negation

$$(p \iff (\neg p)) \iff \text{false}$$

Law 5.5 claiming proposition p is equivalent proposition q is the same as claiming that p implies qand q implies p

$$(p \iff q) \iff ((p \implies q) \wedge (q \implies p))$$

2 Set Theory

2.1 Laws of Set membership

Law 1.1 for any set S and any element s

$$\neg(s \in S) \iff s \notin S$$

Law 1.2 for any element x

$$x \in \emptyset \iff \text{false}$$

2.2 Laws of Subsets

Law 2.1 for any sets S and T

$$(S \subseteq T \wedge T \subseteq S) \iff S = T$$

Law 2.2 for any sets S

$$(\emptyset \subseteq S)$$

Law 2.3 all sets are a subset of themselves

$$(S \subseteq S)$$

Law 2.4 for any sets S and T

$$\neg(S \subseteq T) \iff S \not\subseteq T$$

Law 2.5 for any sets S and T

$$S \subseteq T \iff (S \subset T \vee S = T)$$

Law 2.6 for any sets S and T

$$S \not\subset T \iff \neg(S \subset T)$$

Law 2.7 for any set S

$$S \not\subset S$$

Law 2.8 for any sets S and T

$$S \subset T \implies T \not\subset S$$

2.3 Laws of Supersets

Law 3.1 for any sets S and T. Stating S is a superset of T is logically equivalent to stating that T is a subset of S

$$S \supseteq T \iff T \subseteq S$$

2.4 Laws of Set Union

Law 4.1 for any element a, and any sets S and T

$$a \in S \cup T \iff (a \in S \lor a \in T)$$

Law 4.2 combining Set S with the empty set Ø, is equivalent to Set S:

$$S \cup \emptyset = S$$

Law 4.3 The set union of any set S combined with itself is equivalent to itself

$$S \cup S = S$$

Law 4.4 Union is commutative

$$S \cup T = T \cup S$$

Law 4.5 Union is associative

$$R \cup (S \cup T) = (R \cup T) \cup S$$

Law 4.6 The union of two sets is always at least as big as each set considered individually

$$S \subseteq S \cup T$$

2.5 Laws of Set Intersection

Law 5.1 where a given element a is in the intersection of sets S and T is must be an element of both sets

$$a \in S \cap T \iff (a \in S \land a \in T)$$

Law 5.2 the intersection of a given set S with the empty set Ø is always the empty set

$$S \cap \emptyset = \emptyset$$

Law 5.3 the intersection of set S with itself is always S

$$S \cap S = S$$

Law 5.4 Intersection is commutative

$$S \cap T = T \cap S$$

Law 5.5 Intersection is associative

$$R \cap (S \cap T) = (R \cap S) \cap T$$

Law 5.6 The intersection of any given sets is always at least as small as one of the given sets

$$S \cap T \subseteq S$$

Law 5.7 union distributes through Intersection and Intersection distributes through distribution

$$R \cup (S \cap T) = (R \cup S) \cap (R \cup T)$$

$$R \cap (S \cup T) = (R \cap S) \cup (R \cap T)$$

2.6 Laws of Set Difference

Law 6.1 if a is an element of the Set difference of Sets $S \setminus T$ then S is a member of the former and not the latter

$$a \in S \setminus T \iff (a \in S \land a \notin T)$$

Law 6.2 Set S intersected with the empty set is equivocal to set S

$$S \setminus \emptyset = S$$

Law 6.3 The empty set intersected with set S is equivocal to the empty set

$$\emptyset \cap S = \emptyset$$

Law 6.4 The intersection of any set with itself is equal to the empty set

$$S \setminus S = \emptyset$$

Law 6.5 The difference in Set R and the union or sets S and T is equivocal to the union of the difference in set R

and S and R and T. A similar propery holds for Intersection.

$$R \setminus (S \cup T) = (R \setminus S) \cap (R \setminus T)$$

$$R \setminus (S \cap T) = (R \setminus S) \cup (R \setminus T)$$

2.7 Laws of equality

Law 7.1 When two different sets have exactly the same elements, they are equal

$$x \in S \iff x \in T$$

2.8 Laws of cardinality

Law 8.1 the cardinality of the empty set is 0

$$\#\emptyset = 0$$

Law 8.2 the cardinality of the union of S and T is equal to the cardinality of S minus the cardinality of the intersection of S and T

$$\#(S \cap T) = \#S - \#(S \setminus T)$$

Law 8.3 the cardinality of the union of S and T is equal to the cardinality of S plus the cardinality of T minus the cardinality of the intersection of S and T

$$\#(S \cup T) = \#S + \#T - \#(S \cap T)$$

Law 8.4 the Cardinality of the intersection of S and T is equal to the cardinality of S minus the cardinality of the intersection of S and T

$$\#(S \setminus T) = \#S - \#(S \cap T)$$

2.9 Laws of Power Sets

Law 9.1 Set S is an element of the power set of T if and only if S is a subset of T

$$S \in \mathbb{P}(T) \iff S \subseteq T$$

Law 9.2 the empty set is an element of the power set of a any given set S

$$\emptyset \in \mathbb{P}(S)$$

Law 9.3 for any given set S, S is an element of the power set of itself

$$S \in \mathbb{P}(S)$$

Law 9.4 the power set of set S is equal to two to the power of the cardinality of S

$$\#(\mathbb{P}(S)) = 2^{\#(S)}$$

2.10 Laws of Generalised Operations

Law 10.1 (Generalised Union) for any set of sets A and any element a, $a \in \bigcup A$ if, and only if, there is some set $S \in A$ such that $a \in S$

$$a \in \bigcup A \iff S \in A | a \in S$$

Law 10.2: (Generalised Intersection) for any set of sets A and any element a, $a \in \bigcap A$ if, and only if, for every set $S \in A$ it is the case that $a \in S$

$$a \in \bigcap A \iff \{\forall S \in A | a \in S\}$$