5 Propositional Logic

5.1

When p is true and q is false, state whether the following statements are true or false

1. $\neg\neg\neg q$
2. $(p \wedge q) \vee p$
3. $(p \vee \neg q) \implies p$
4. $q \implies (p \iff q)$
5. $p \wedge q \iff \neg p \vee q$

5.2

Let p be a true proposition and q be any proposition. Which of the following are true, and which of the following are false? (others might be true or false)

1. $p \implies q$
2. $q \implies p$
3. $\neg q \implies p$
4. $\neg p \implies q$

5.3

Construct a truth table for each of the following. For each, state whether the statement is always true, or if not give a case where it is false.

1. $p \implies (q \implies (r \implies p))$
2. $p \vee \neg r \implies (\neg(r \wedge p)$

5.4

Identify the atomic propositions in the following sentences and assign them each a letter (e.g. b = "the bus is late") Then express this as a statement of propositional logic using the notation taught in class

1. If my bike is not working or the bus is late, then I am late for class
2. I am happy if and only if I am riding my bike

5.5

For the following, state whether they are tautologies, contraditions, or contingiencies

1. $p \implies (\neg p \vee p)$
2. $p \vee q \implies p \wedge q$
3. $\neg p \vee \neg\neg p$
4. $p \vee \neg(p \vee \neg p)$

5.6

Do the following properties hold of implication? You may wish to use either a truth table or Equational Reasoning to arrive at your answer.

1. Implication distributes over conjunction

   $$(p \implies (q \wedge r)) \iff (p \implies q) \wedge (p \implies r)$$

2. Implication distributes over disjunction

   $$(p \implies (q \vee r)) \iff (p \implies q) \vee (p \implies r)$$

3. Implication distributes over implication

   $$(p \implies (q \implies r)) \Leftrightarrow (p \implies q) \implies (p \implies r)$$

5.7

Prove the following by Equational Reasoning. Format your proof as in the lecture slides

1. $p \implies (p \vee \neg p)$
2. $\neg p \wedge \text{true} \iff \neg p$

For this, use only the laws given in the lecture slides and the handout.