1 Propositional Logic

The following laws of propositional logic are compiled and numbered for the benefit of this course. This is not an exhaustive list.

1.1 Laws of Negation

Law 1.1 not false is true and not true is false

$$(\neg \text{true}) \iff \text{false}$$

$$(\neg \text{false}) \iff \text{true}$$

Law 1.2 two negatives make a positive

$$(\neg\neg p) \iff p$$

1.2 Laws of Conjunction

Law 2.1 a proposition conjoined with itself is equivalent to itself

$$(p \wedge p) \iff p$$

Law 2.2 a proposition conjoined with true is equivalent to itself

$$(p \wedge \text{true}) \iff p$$

Law 2.3 a proposition conjoined with false is equivalent to false

$$(p \wedge \text{false}) \iff \text{false}$$

Law 2.4 a proposition conjoined with its own negation is equivalent to false

$$(p \wedge (\neg p)) \iff \text{false}$$

Law 2.5 conjunction is commutative

$$(p \wedge q) \iff (q \wedge p)$$

Law 2.4 conjunction is associative

$$(p \wedge q) \wedge r \iff p \wedge (q \wedge r)$$

1.3 Laws of Disjunction

Law 3.1 de Morgan's Laws

$$\neg(p \wedge q) \iff ((\neg p) \vee (\neg q))$$

$$\neg(p \vee q) \iff ((\neg p) \wedge (\neg q))$$

Law 3.2 disjunction is idempotent

$$(p \vee p) \iff p$$

Law 3.3 a proposition disjoined with false is equivalent to itself

$$(p \vee \text{false}) \iff p$$

Law 3.4 a proposition combined via disjunction with true is equivalent to true

$$(p \vee \text{true}) \iff \text{true}$$

Law 3.5 disjunction is associative

$$p \vee (q \vee r) \iff (p \vee q) \vee r$$

Law 3.6 conjunction is commutative

$$p \vee q \iff q \vee p$$

Law 3.7 a proposition combined via disjunction with its own negation is equivalent to true

$$((\neg p) \vee p) \iff \text{true}$$

Law 3.8 disjunction distributes through conjunction

$$p \vee (q \wedge r) \iff (p \vee q) \wedge (p \vee r)$$

Law 3.9 conjunction distributes through disjunction

$$p \wedge (q \vee r) \iff (p \wedge q) \vee (p \wedge r)$$

1.4 Laws of Implication

Law 4.1 p implies q is the same as 'not p, or q'

$$(p \implies q) \iff ((\neg p) \vee q)$$

1.5 Laws of Equivalence

Law 5.1 equivalence is associative

$$((p \iff q) \iff r) \iff (p \iff (q \iff r))$$

Law 5.2 equivalence is commutative

$$(p \iff q) \iff (q \iff p)$$

Law 5.3 every proposition is equivalent to itself

$$(p \iff p) \iff \text{true}$$

Law 5.4 no proposition is equivalent to its negation

$$(p \iff (\neg p)) \iff \text{false}$$

Law 5.5 claiming 'p is equivalent to q' is the same as claiming that p implies q and q implies p

$$(p \iff q) \iff ((p \implies q) \wedge (q \implies p))$$