8 Set Theory 2

8.1

Give the cardinality of the following sets (state if infinite), where $A = \{a, b\}, B = \{a, b, d\}, C = \{a, b, d, e\}$

1. $\{7, 8, 8\}$
2. $\bigcap\{A, B, C\}$
3. $\mathbb{P}(\{a, b, c\})$
4. $\mathbb{N}^0 \setminus \mathbb{N}^+$
5. $\mathbb{Q} \setminus \mathbb{R}$
6. $\bigcup\{A, B, C\}$

8.2

Give the cardinality of the following sets, given:

- $\#A = 4$
- $\#B = 6$
- $\mathbb{P}(C) = 8$
- $\#(A \cap B) = 1$
- $C \subseteq A$
- $\#(C \cap B) = \emptyset$

1. $A \cup B$
2. $\mathbb{P}(A \setminus B)$
3. $C \cap A$
4. $B \cup C$
5. $A \times B$
6. $B \setminus C$

8.3

Give the extension of the following sets, , given $A = \{a, b, c, d\}$, $B = \{a, b, c\}$

1. $\mathbb{P}(A)$
2. $\bigcup\mathbb{P}(B)$
3. $\{a, b\} \times A$
4. $\mathbb{P}(\emptyset)$
5. $(\{1\} \times \{a, b\}) \cap (\{1, 2\} \times \{b, a\})$

8.4

Give the extension of the following sets

1. $\{n : \mathbb{N} \mid n \mod 3 = 1 \land n < 13\}$
2. $\{n : \mathbb{N} \mid 4 < n < 7\}$
3. $\{n : \mathbb{N} \bullet n \mod 7\}$
4. $\{a : \mathbb{R}; b : \mathbb{R} \mid a^2 = b \land b^2 = a \bullet a\}$
5. $\{a : \mathbb{N}^+; b : \mathbb{N}^+ \mid a + b < 3 \bullet (a, b)\}$

8.5

Define by extension the set containing the smallest four elements of the following sets:

1. $\{n : \mathbb{N}^+ \bullet n^n\}$
2. $\{n : \mathbb{N}^+ \bullet \frac{n}{n+1}\}$
3. $\{n : \mathbb{Z} \mid n^2 \leq 16\}$

8.6

Given the table below, with types Name, Age, and Group, give the extensions of the following sets

Name	Age	Group
Alice	18	A
Bob	17	B
Eve	19	A
Mary	22	B

1. $\{x : Group \times Name \mid \text{ true}\}$
2. $\{x : Name \times Age \mid x.2 \geq 18 \bullet x.1\}$
3. $\{x : Name \times Age \times Group \mid x.3 = A\}$
4. $\{a : Name \times Group; b : Name \times Group \mid a.2 = b.2 \bullet \{a.1, b.1\}\}$

8.7

For the table given above, define the following sets by set comprehension:

1. The set of all names
2. The set of groups containing someone under the age of 18
3. The set of Name, Age tuples for someone whose age is a multiple of 3
4. The set of Name, Age, Group tuples for everyone who isn't called Alice