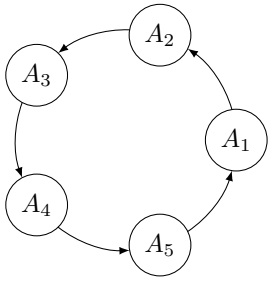
1 Formal Systems

1.1

A formal system has states $A_1, \ldots, A_5$ and rules R. Below is a graph showing all possible transitions between states.



1. Is the derivation $A_1 \Rightarrow_R^* A_5$ possible within this system?
2. If so, provide a derivation.

1.2

A formal system has states $X_1, \ldots, X_7$. The rules R are such that from a state X_i it is possible to derive the state X_{i+1}

1. List the steps in the derivation $X_1 \Rightarrow_R^* X_7$
2. What is the length of this derivation?
3. Is there a derivation $X_4 \Rightarrow_R^* X_3$?

1.3

A string rewriting system has the following rules R.

- $aba \curvearrowright b$
- $aaa \curvearrowright$

Let A be the string *aaaababbbbabababaaaababa*

1. List all the strings B that can be derived from A in one step: $A \Rightarrow_R B$
2. List the strings in one possible derivation of T, $A \Rightarrow_R^* T$, where T is a terminal state (where no further rule applications are possible)

1.4

A string rewriting system is defined with the following rules.

- $(fof) \curvearrowright f$
- $(tof) \curvearrowright t$
- $(fot) \curvearrowright t$
- $(tot) \curvearrowright t$

For each of the following, derive a terminal string (where no further rule applications are possible)

1. $((fot)ot)$
2. $((fof)o(tot))$
3. $(((tof)of)o(fo(fof)))$
4. Give a string from which the string f is derivable by a derivation of length 3

1.5

For the following binary relations, state whether they are: Reflexive, Symmetric, Transitive

1. $=$
2. $>$
3. $\leq$
4. $\neq$
5. older than
6. can be rotated (by some angle) to get

1.6

For the following table of binary operators, state whether they are: Commutative, Associative

1. $+$
2. $\times$
3. $\div$
4. $\times \pmod n$
5. $\div \pmod n$