10 Probability

10.1

You have 3 fair six-sided dice. What is the probability of rolling

1. 3 sixes
2. At least 2 sixes
3. An ascending sequence $\{n, n+1, n+2\}$ in order (e.g. 1,2,3 or 3,4,5)

10.2

Let $P(A) = 0.25$, $P(B) = 0.5$, and $P(C) = 0.75$. You know that events B and C are independent, and that $P(A \mid B) = 0.7$. Calculate the following

1. $P(A \cap B)$
2. $P(B \cup A)$
3. $P(B \cup C)$
4. $P(A')$
5. $P(C \mid B)$
6. $P(B \mid A)$

10.3

Calculate the following

1. 4!
2. 0!
3. ${}^4P_3$
4. ${}^8C_7$

10.4

1. How many combinations of four unique 8-bit binary numbers are possible?
2. I define an ordering over the set {a, b, c, d}. How many possible orderings can I define?
3. A industrial robot is designed to carry 4 objects simultaneously. Each object is carried by one of 4 differently positioned and proporitioned arms. Each object can be in one of 6 weight categories. To exhaustively test that the robot will never overbalance on any set of weights, how many tests must be run?

10.5

A_1, and A_2 partition a probability space. B_1, B_2, and B_3 partition the event A_1. There is an event C, such that $P(C) = \frac{1}{2}$, and $P(C \mid A_2) = 0$

1. What is $\sum_{i=1}^{3} P(C \mid B_i)$?

10.6

You have 3 bags of balls that each contain 80 balls, their colours are described in the table below.

Bag	Red Balls	Black Balls
Bag 1	65	15
Bag 2	23	57
Bag 3	23	57

You pick a bag at random and pick a random ball from that bag. What is the probability that you pick a red ball?

10.7

What is the probability that a student who passed their exam attended lectures, given: 80% of students attend lectures; 80% of students pass their exam; and of those who attend lectures, 95% pass their exam

10.8

You have 3 hypotheses that are disjoint and collectively exhaustive, H_1, H_2, H_3. The prior probability of each being true, and conditional probabilities that each is true given the event E are given in the table below.

H_n	$P(H_n)$	$P(E \mid H_n)$
H_1	3/8	2/6
H_2	1/8	2/6
H_3	4/8	1/6

Calculate posterior probabilities $P(H_n \mid E)$ for all three hypotheses.