ODE model of the spread of measles in secondary schools

David Gurevich

The following introduces a system of differential equations that is used to model the spread of measles through a secondary school.

List of Symbols

β - Probability of disease transmission following an interaction
σ - Rate of maturation from exposed class to infected class
α - Rate at which symptoms appear in an infected individual
γ - Rate at which infected individuals recover following the first appearance of symptoms
ω - Probability of an individual visiting the bathroom
q - Quantity of virus "shed" per minute
Q - Ventilation of air inside washroom (per cubic meter)
V - Volume of each washroom (cubic meters)
P - Pulmonary ventilation rate (cubic meters per minute)
T - Average duration of washroom visit
$\mathcal{F}$ - Symmetrical contact matrix for friends between grades
$\mathcal{C}$ - Symmetrical contact matrix for classmates between grades

SVEIR Model

S_i - Proportion of population i that is susceptible
V_i - Proportion of population i that is vaccinated
E_i - Proportion of population i that is exposed
I_i - Proportion of population i that is infected
H_i - Proportion of population i that is at home because of infection
R_i - Proportion of population i that has recovered
W - Sum of all concentrations of measles virus in the air in washrooms

τ - Day state: 1 if students are at home, 2 if students are in the halls, 3 if students are in class

$$\frac{dS_i}{dt} = \begin{cases} 0 & \text{if } \tau = 1 \\ -\Big[\Big(\sum_{j=1}^{4} \frac{2}{20}\beta\mathcal{F}_{i,j}I_j\Big) + \frac{1}{6}\omega\beta WP\Big](\frac{S_i}{N_i}) & \text{if } \tau = 2 \\ -\Big[\Big(\sum_{j=1}^{4} \frac{2}{20}\beta\mathcal{C}_{i,j}I_j\Big) + \frac{1}{6}\omega\beta WP\Big](\frac{S_i}{N_i}) & \text{if } \tau = 3 \end{cases} \tag{1}$$

$$\frac{dV_i}{dt} = 0 \tag{2}$$

$$\frac{dE_i}{dt} = \begin{cases} 0 & \text{if } \tau = 1 \\ \Big[\Big(\sum_{j=1}^{4} \frac{2}{20}\beta\mathcal{F}_{i,j}I_j\Big) + \frac{1}{6}\omega\beta WP\Big](\frac{S_i}{N_i}) - \sigma E_i & \text{if } \tau = 2 \\ \Big[\Big(\sum_{j=1}^{4} \frac{2}{20}\beta\mathcal{C}_{i,j}I_j\Big) + \frac{1}{6}\omega\beta WP\Big](\frac{S_i}{N_i}) - \sigma E_i & \text{if } \tau = 3 \end{cases} \tag{3}$$

$$\frac{dI_i}{dt} = \sigma E_i - \alpha I_i \tag{4}$$

$$\frac{dH_i}{dt} = \alpha I_i - \gamma H_i \tag{5}$$

$$\frac{dR_i}{dt} = \gamma H_i \tag{6}$$

$$\frac{dW}{dt} = \begin{cases} \frac{-Q}{V} & \text{if } \tau = 1 \\ \frac{-Q}{V} + \frac{1}{6}\omega qT\sum_{j=1}^{4} I_j & \text{if } \tau = 2, 3 \end{cases} \tag{7}$$

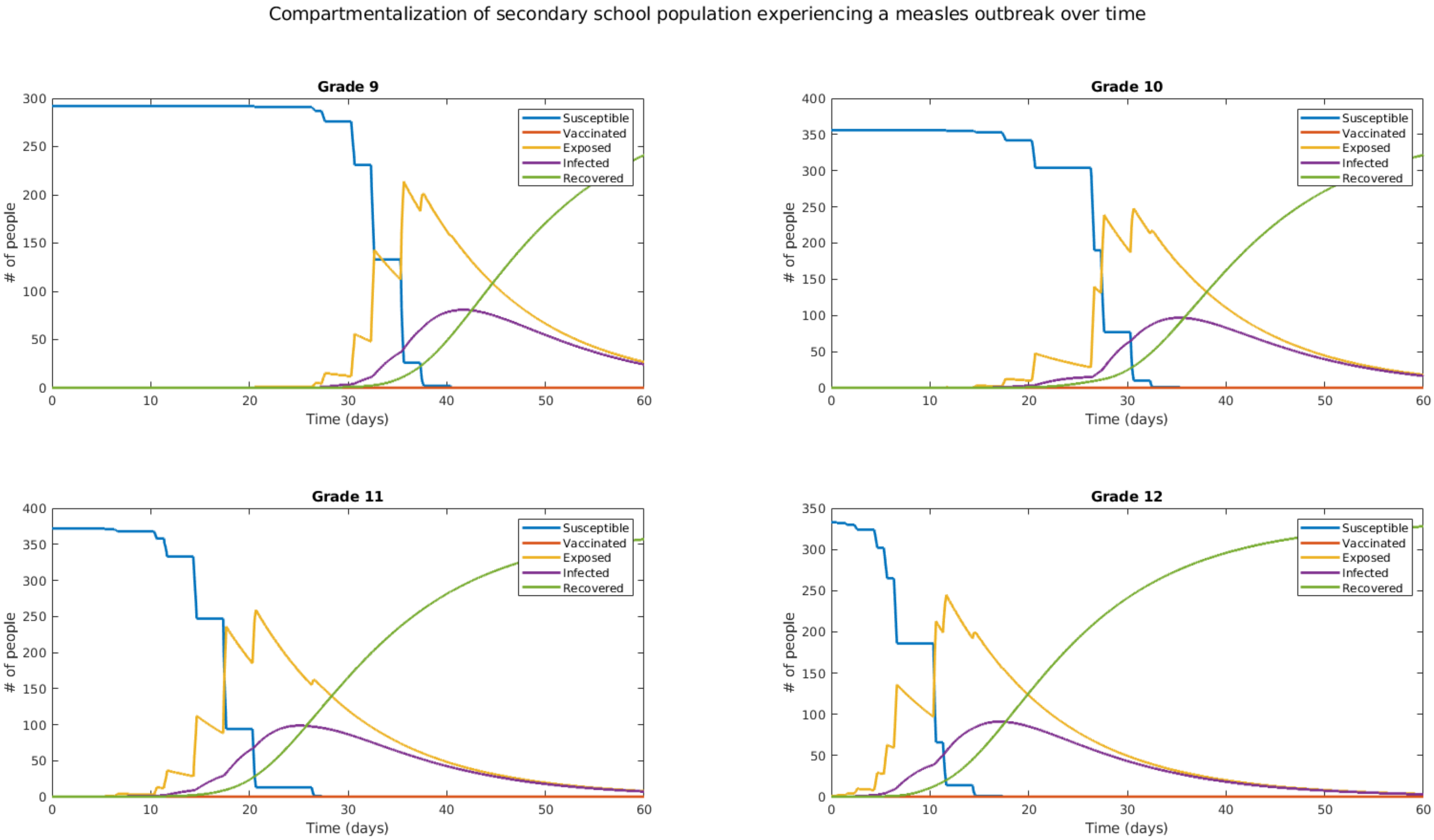

Figure 1: Plot of ODE solution to model

$$\beta = 0.91 \qquad \frac{1}{\gamma} = 4 \qquad Q = 5 \qquad T = 2.2$$

$$\frac{1}{\sigma} = 11 \qquad \omega = 0.00047 \qquad V = 32$$

$$\frac{1}{\alpha} = 4 \qquad q = 144 \qquad P = 0.00556$$

$$\mathcal{F} = \begin{bmatrix} 0.97 & 0.03 & 0 & 0 \\ 0.03 & 0.97 & 0.03 & 0 \\ 0 & 0.03 & 0.097 & 0.03 \\ 0 & 0 & 0.03 & 0.97 \end{bmatrix} \qquad \mathcal{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$