Foundation of Probabilities

Data Science Immersive

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FLATIRON
SCHOOL

Foundations of Probabilities

- Agenda today
  - Set & Set Theory
  - Independent Probabilities
  - Permutations & Combinations

After today, you'll be able to...

- Understand and explain the difference between combinations and permutations
- Gain an intuitive understanding of why we need to learn probabilities
- Calculate and compute conditional probabilities in given context
- Apply permutations and combinations in specific scenarios

Set Theory

- In probability theory, a set is denoted as a well-defined collection of distinct objects.
- Mathematically, you can define a set by S. If an element x belongs to a set S, then you'd write $x \in S$. On the other hand, if x does not belong to a set S, then you'd write $x \notin S$.
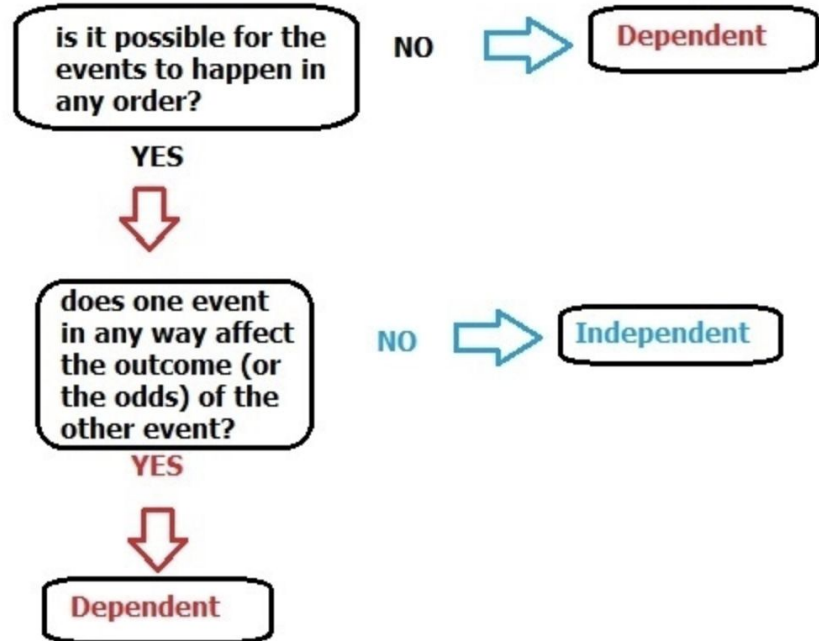
Subset & Set Operations

- Set **T** is a subset of set **S** if every element in set **T** is also in set **S**. The mathematical notation for a subset is **T ⊆ S.**
    - T = {2,3,4}
    - S = {1,2,3,4,5}
    - **T ⊆ S**
- Set Operations:
    - Union of two sets: **A ∪ B**
    - Intersection of two sets: **A ∩ B**

Independent Probabilities

- What are some examples
  of independent events?

- What are some examples
  of dependent events?

Dependent or Independent?

is it possible for the events to happen in any order? **NO** → **Dependent**

YES ↓

does one event in any way affect the outcome (or the odds) of the other event? **NO** → **Independent**

YES ↓

Dependent

Independent Probabilities

- What is probabilities?

  Probability theory is the study on the **frequency** of a given event occurring in all possible events.

Independent Probabilities

- What is the probability of event **A** occurring?
  - Event **A** is known as the **event space**, and all possible events are known as the **sample space**
  - P(A) = A / (all possible events)
  - For example, what is the probability of drawing an Ace in a deck of cards?

Independent Probabilities

- Probability of A and B
    - If the probabilities of A and B are independent, then we can calculate the probability of A and B as P(A) * P(B)
    - For example, what is the probability of getting two heads in a two tosses of a fair coin?
- Probabilities of A or B
    - If the probabilities are disjoint or mutually exclusive, the probability of P(A or B) is P(A) + P(B)
        - What is the probability of getting an ace of spade or queen of heart from a standard deck of cards?
    - If the probabilities are not mutually exclusive, the probability of (P or B) = P(A) + P(B) - P(A and B)
        - Eg. What is the probability that a card chosen from a standard deck will be a Jack or a Heart?

Independent Probabilities

- Ok, but why do we need to care about probabilities?
    - The Monte Carlo Fallacy
    - Allows us to make informed decision
        - Should you make a living by purchasing lottery?
    - Allows us to speak about uncertainty in an informed way
        - More on this when we learn statistical distributions

Combinatorics

❖ We throw around combinatorics language in our daily life, sometimes incorrectly. For example, we ask what is our safe combination, even though in reality we are really asking what is our safe permutations.

❖ So what is the difference?

➢ Ordering matters in Permutation

➢ Ordering doesn't matter in Combination

Permutations

$$P_k^n = \frac{n!}{(n-k)!}$$

Permutations

Example 1 - how many different ways can you give a 4 digit passwords on an iphone assuming that digit cannot be repeated?

What about when digits can be repeated?

Example 2 - how many different ways can you arrange the letter **MISSISSIPPI**?

$$\frac{n!}{n_1! \, n_2! \dots n_k!}$$

Combination

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Combination

Example 1 - How many different ways can I randomly give 3 Amazon gift cards to the 26 students here?

Combinatorics - Examples

- What are the number of different ways in which you can get 4 cards from a standard deck of cards?

- Is this an example of combination or permutation?

Combinatorics - Example

- What is the probability of getting exactly 4 heads out of 6 tosses?

Combinatorics - Example

- What is the probability of getting at least 3 heads out of 6 tosses?