Signals and Systems MATLAB HW2

Deadline: 2018/05/14 before 23:59

1. Discrete Fourier Transform

The objective of this section is to learn how to use **fft** and **ifft** function.

1.1 Background

In order to analyze the frequency domain of a finite duration and discrete-time signal $x[n]$, *n*=1,2,…*N*, its discrete Fourier transform (DFT) is defined as

$$X_k = \sum_{n=1}^{N} x[n] e^{-j\frac{2\pi}{N}(n-1)(k-1)}, k = 1,2,...,N$$

It is observed that DFT is the sampled Fourier transform of a finite duration signal with $w = \frac{2\pi k}{N}$. On the other hand, the inverse DFT (IDFT) of X_k is defined as

$$x[n] = \frac{1}{N} \sum_{k=1}^{N} X_k e^{j\frac{2\pi}{N}(n-1)(k-1)}, n = 1,2,...,N$$

The fast Fourier transform (FFT) is equivalent to DFT with reduced computational complexity as well as inverse FFT (IFFT) to IDFT. To calculate the DFT of the signal $x[n]$ in MATLAB, you may type:

X = fft(x);

If you want to explicitly specify the length *M*, then you can type:

X = fft(x, M);

The **fftshift** command swaps the first and the second half of the vector X so that the frequency range is in $[\frac{-N}{2}, \frac{N}{2}]$ (assuming *N* is even.)

However, for signals with infinite length, we have to truncate it so that it can be computed with MATLAB. Such truncation causes *Gibbs phenomenon* (pp. 200-201 of the textbook).

Lab Procedure and Questions

1. Let $x(t)$ be a sinc function written as

$$x(t) = \frac{\sin(\pi t)}{\pi t}$$

Now, $x(t)$ is sampled at the rate $T_s = T / N_1$ so that $x[n] = x(nT_s)$,

$n \in \{-N_1, -N_1 + 1, \ldots 0, \ldots N_1 - 1, N_1\}$ and $N=2N_1+1$.

Let N=1001 and T=100. Please write an m-file **fftsinc.m** (not a function) to do the following:

(a) (20%) Plot $x[n]$.

(b) (30%) Plot the magnitude response of the DFT of x during $[-N_1,N_1]$. The zero frequency should be centered in your plot. Observe the *Gibbs phenomenon* in (b).

2. A way of mitigating *Gibbs phenomenon* is to multiply $x(t)$ by a finite-duration signal $w(t)$, i.e., $y(t) = x(t)w(t)$. The signal $w(t)$ is called as the window function. A famous one is *Hanning* window, which is specifically written as

$$w(t) = \begin{cases} \frac{1}{2}[1 + \cos(\frac{2\pi |t|}{T_w})], & |t| \le T_w / 2 \\ 0 & , \text{else} \end{cases}$$

Where T_w denotes the duration of the window function.

Suppose $w(t)$ is also sampled at a rate $T_s = T / N_1$ so that

$w[n] = w(nT_s)$, $n \in \{-N_1, -N_1 + 1, \ldots 0, \ldots N_1 - 1, N_1\}$ and $N=2N_1+1$.

Let $T_w=T/2$, N=1001, and T=100. Please write an m-file **fftHan.m** (not a function) to do the following:

(a) (10%) Plot $w[n]$

(b) (20%) Plot $y[n] = x[n]w[n]$, where $x[n]$ is the signal plotted in 1(a).

(c) (20%) Plot the magnitude response of the DFT of $y[n]$. The zero frequency should be centered in your plot. Observe the *Gibbs phenomenon* here and compare it the that of 1(b).

2. CEIBA Submission

- Please upload a compressed file (.zip, .rar or .tar), which includes your **m-files (fftsinc.m and fftHan.m)** and a **word file (report.doc)**. Please show the relevant plots mentioned above in the word file (report.doc).
- The compressed file name should be **ID_MATLAB2 (ex: B05901xxx_MATLAB2)**