

# Formal Semantics

The structure of a language defines its syntax, but what defines semantics or meaning?

⇒ Behavior!

The most straight forward way to define semantics is to provide a simple interpreter for the programming language that highlights the behavior of the language,

⇒ Operational Semantics

# Operational Semantics

Let's develop an operational semantics for a simple programming language called *ONE*;

*ONE*:  
 $\langle \text{exp} \rangle^* ::= \langle \text{exp} \rangle + \langle \text{mulexp} \rangle \mid \langle \text{mulexp} \rangle$   
 $\langle \text{mulexp} \rangle ::= \langle \text{mulexp} \rangle * \langle \text{rootexp} \rangle \mid \langle \text{rootexp} \rangle$   
 $\langle \text{rootexp} \rangle ::= (\langle \text{exp} \rangle) \mid \langle \text{constant} \rangle$   
 $\langle \text{constant} \rangle ::= \text{all valid integer constants}$

Note: The grammar is unambiguous, both precedence and associativity rules of “standard” arithmetic are observed.

Do the following sentences belong to  $L(\text{ONE})$ ? Why? Why not?

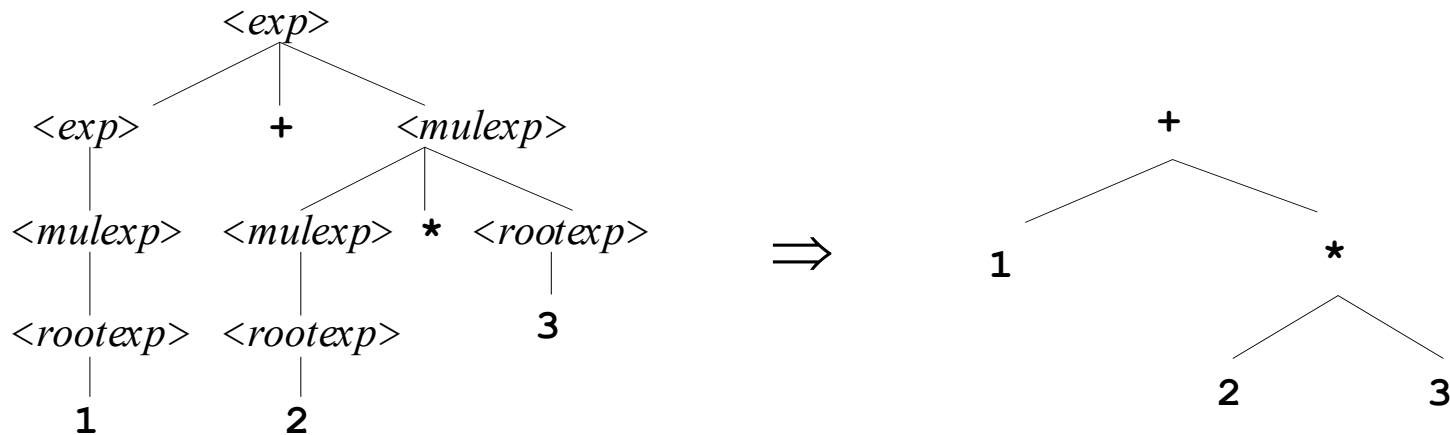
$s = 1 + 2 * 3$

$s = (1 + 2) * 3$

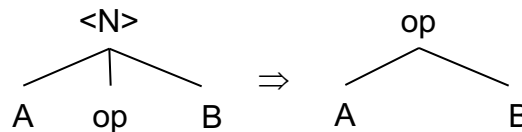
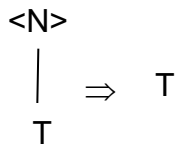
$s = a + 3$

# Abstract Syntax Trees

We want to define an operational semantics, i.e., an abstract interpreter for the language, but parse trees are not very convenient, too many non-terminal symbols  $\Rightarrow$  Abstract Syntax Tree (AST)



Transformation Rules:



**Note:** This rule also applies to unary operators and operators with arity  $> 2$ .

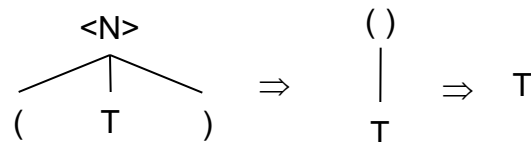
# Observations

Definition: An abstract syntax tree is a finite, labeled, directed tree, where the internal nodes are labeled by operators, and the leaf nodes represent the operands of the node operators. -Wikipedia, 2006

Observation: The abstract syntax tree is a simplified form of the parse tree: same order as the parse tree, but no non-terminals.

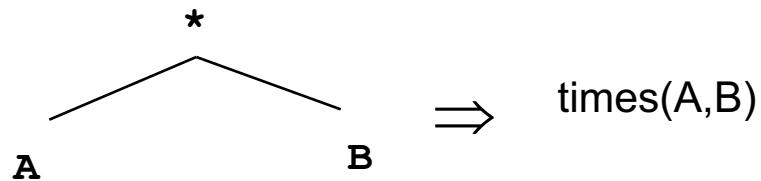
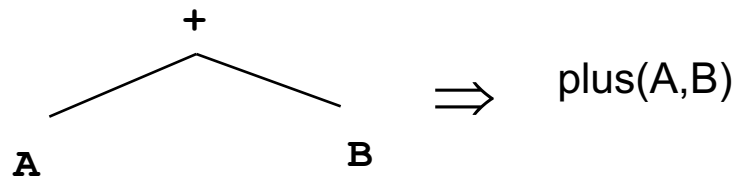
# ASTs & Parentheses

- What happens to parentheses in the AST representation of a program?
- They are not needed!
- ASTs naturally represent associativity and precedence relations.
- Consider:  $(1 + 2) * 3$
- Parentheses do not contribute to computations, therefore the following tree transformations can be applied:

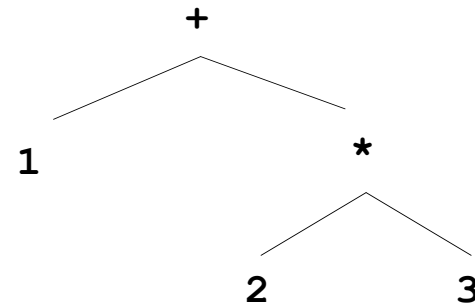


# Prolog ASTs

We can represent ASTs in Prolog:



`c` (constant)  $\Rightarrow$  `const(c)`



`plus(const(1),times(const(2),const(3)))`

# ONE: Prolog Interpreter

A simple interpreter that computes a semantic value for syntactic constructs, the computation of this semantic value can be interpreted as the behavior: myeval / 2, AST input and semantic value as output.

```
myeval(plus(X,Y),Value) :-  
    myeval(X,XValue),  
    myeval(Y,YValue),  
    Value is XValue + YValue.
```

```
myeval(times(X,Y),Value) :-  
    myeval(X,XValue),  
    myeval(Y,YValue),  
    Value is XValue * YValue.
```

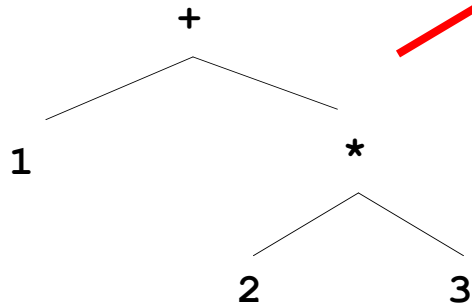
```
myeval(const(X),Value) :- Value = X.
```

```
?- myeval(const(1),X) .  
  
X = 1  
  
Yes  
?- myeval(plus(const(1),const(2)),X) .  
  
X = 3  
  
Yes  
?- myeval(plus(const(1),times(const(2),const(3))),X) .  
  
X = 7  
  
Yes
```

# Semantics of Expressions

- With our semantics (interpreter) in place we can now compute the semantic value of any expression program, e.g.

1+2\*3



plus(const(1),times(const(2),const(3)))



```
?- myeval(plus(const(1),times(const(2),const(3))),X).
```

X = 7



# Exercises

- Extend the grammar for language ONE with the subtraction operator
- Extend the operational semantics appropriately, e.g.,
  - $6 - 3$  should give the value 3Assume that the abstract syntax of this operator is  $\text{sub}(x,y)$ .
- Compute the semantic value for the following expressions:
  - $\text{sub}(3,1)$
  - $\text{sub}(4,2)$

# Exercises

- Extend the grammar for language ONE with the ‘!’ factorial operator
- Extend the operational semantics appropriately, e.g.,
  - 3! should give the value 6Assume that the abstract syntax of this operator is `fact(x)`.
- Compute the semantic value for the following expressions:
  - `fact(3)`
  - `fact(4)`