A Type is a Set of Values

Consider the Asteroid statement:

let n:%integer = 3;

Here we constrain n to take on any value from the set of all integer values.

Def: A type is a set of values.

Def: A <u>primitive type</u> is a type that is built into the language, e.g., integer, string.

Def: A <u>constructed type</u> is a user defined type, e.g., any type introduced by the user. In Asteroid this is done through the 'structure' statement.

Example: Java, primitive type

float q;

type float ⇒ set of all possible floating point values

q is of type float, only a value that is a member of the set of all floating point values can be assigned to q.

Example: Java, constructed type

```
class Rectangle { int xdim; int ydim; };
```

Rectangle r = new Rectangle();

Now the variable r only accepts values that are members of type Rectangle; <u>object instantiations</u> of class Rectangle.

Example: Asteroid, constructed type

an element of type Rectangle.

In statically typed languages arrays are also considered 'constructed types'

Example: C, constructed type

the variable a will accept values which are arrays of 3 integers.

e.g.: int a[3] =
$$\{1,2,3\}$$
;
int a[3] = $\{7,24,9\}$

That is, 'int a[3]' defines the set of all integer arrays of size three.

Subtypes

Def: a <u>subtype</u> is a <u>subset</u> of the elements of a type.

Example: Java The notation A < B means

A is a subtype of B.

Short is a subtype of int: short < int

Observations:

(1) converting a value of a subtype to a values of the super-type is called <u>widening</u> type conversion. (safe)

(2) converting a value of a supertype to a value of a subtype is called <u>narrowing</u> type conversion. (not safe)

Example: Java

float < double

Subtypes give rise to type hierarchies and type hierarchies allow for automatic type coercion – widening conversions!

Subtypes & Type Hierarchies

- In type hierarchies it is always safe to move from subtype to supertype – widening conversion (coercion)
 - E.g. short < int ✓
- Never the other way around
 - E.g. int < short X

char < short < int < float < double

Part of the Java type hierarchy

Subtypes

- The Asteroid type hierarchy
 - boolean < integer < real < string
 - list < string</p>
 - tuple < string
 - none
 - constructed types

Why do we use types?

- Types allow the computer/language system to assist the developer write <u>better programs</u>.
 Type <u>mismatches</u> in a program usually indicate some sort of <u>programming error</u>.
 - Static type checking check the types of all statements and expressions at compile time.
 - <u>Dynamic type checking</u> check the types at runtime.

Type Equivalence

I. Name (nominal) Equivalence – two objects are of the same type if and only if they share the same type name.

Example: Rust

```
struct Type1 {x:i64, y:i64}
struct Type2 {x:i64, y:i64}

fn main () {
    let x: Type1 = Type1{x:1,y:2};
    let y: Type2 = x;
    println!("{:?}",y);
    Error; even though the types look the same, their names are different, therefore, Rust will not compile.
```

Rust uses name equivalence

Type Equivalence

II. <u>Structural Equivalence</u> – two objects are of the same type if and only if they share the same <u>type structure</u>.

Example: Haskell

```
type Type1 = (Integer, Integer)
type Type2 = (Integer, Integer)

x :: Type1
y :: Type2

x = (1,2)
y = x
```

Even though the type names are different, Haskell correctly recognizes this statement.

Haskell uses <u>structural equivalence</u>.

- Type inference refers to the automatic detection of the data type of an expression in a programming language.
- To see how this might work let's work through an example.

 Assume we have the following statements in a programming language like Asteroid:

```
let x : integer = 3.
let y : integer = 2 * x.
```

- We want to make sure that all the assignments are legal.
- We will use the type notation '3.{integer}' indicating that this syntactic unit has the type integer.

 We start at the primitives on the right side of the assignments of the first statement and then stepping through all the remaining statements

```
let x:\%integer = 3.
let x:%integer = 3.{integer}.
                                let y:%integer = 2.{integer} * x.{integer}.
let y:%integer = 2 * x.
                                let x:%integer = 3. ✓
let x:%integer = 3.{integer}.
                                let y:%integer = 2 *.{(integer,integer)→integer} x.
let y:%integer = 2 * x.
                                let x:%integer = 3. ✓
                                let y: \%integer = 2 *.{(integer,integer)\rightarrowinteger} x.
                               let x:%integer = 3. \checkmark
                               let y:%integer = 2 * x. ✓
```

Let's try a program with a bug in it:

let x:%boolean = 3.

let x:%boolean = 3.{integer}.



let x:%boolean = 3.{integer}.



Recall Asteroid's type hierarchy: boolean < integer < real < string

We are not allowed to assign a supertype to a subtype!

Types & Objects

- In any OO language class definitions create new types
- Objects are the values in those types
- In OO languages that support inheritance, inheritance creates a subtype-supertype relationship in the class hierarchy

Types & Objects

Example: Java

```
class Cup { ... };
class CoffeeCup extends Cup { ... };
class TeaCup extends Cup { ... };
```

Which ones of the following statements are safe and which ones are not?

- 1. Cup x = new Cup();
- 2. Cup y = new CoffeeCup();
- 3. TeaCup z = new Cup();
- TeaCup t = new TeaCup();
 Cup c = t;

Note: Type coercion in type hierarchies gives rise to polymorphic programming in OO - objects can appear in different type contexts.

Cup

CoffeeCup

TeaCup

Notation:

A < B means A is subtype of B

Exercises

- Describe the type associated with the set of values {-1,-2,-3,-4,...}, call this type Q.
- Describe the type associated with the set of values {-2,-4,-6,-8,...}, call this type P.
- Is there a subtype-supertype relationship between these types? If so, what is it?
- Let x be a variable of type Q and y be a variable of type P, then is the assignment x := y a safe assignment? Why? Why not?

Hint: A type is a set of values!

Take Away

- Types are sets of values, typically with a common representation and common set of operations.
- Types in programming languages allows compilers and interpreters to check for consistency in your programs.
- Inconsistencies usually show up a type mismatches.
- Type equivalence between constructed types can be established in one of two ways, name equivalence or structural equivalence.
- Class hierarchies in OO languages give rise to subtype-supertype relationships due to inheritance.

Assignments

Assignment #2 – See BrightSpace