- Definition: In higher-order programming, one can pass functions as arguments to other functions and functions can be the return value of other functions.
- This, just like patterns matching, is a concept that was developed for functional programming languages.
- Actually, the idea of functions accepting functions as parameters and/or returning functions is even older than that...
 - it dates back to the 1930's when Alan Turing and Alfonso Church explored what computation actually is. Turing did this with Turing machines and Church did this with lambdacalculus – a computational model which had functions as its primitive notion rather than a state machine as in the Turing machines.
 - The lambda calculus can be considered a precursor to our modern notion of functional programming languages.

- We can use higher-order programming to customize the behavior of functions
- Consider a function that computes multiples
 - We can customize the behavior of this function by passing in functions with specific behavior

```
fn unit (x: i64) -> i64 {
                                         ⊞
                                                                             Q
                                                                                \equiv
                                                             Terminal
                                                                                    ×
                                         ubuntu$ ./multiples
    fn double (x: i64) -> i64 {
                                        ubuntu$
         2*x
    fn triple(x: i64) -> i64 {
         3*x
10
11
12
13
    fn multiples(f:fn(i64)->i64, q:i64) -> i64 {
         f(q)
14
15
16
    fn main() {
17
18
         println!("{}", multiples(unit, 3));
19
         println!("{}", multiples(double, 3));
         println!("{}", multiples(triple, 3));
20
21
```

- One consequence of higher-order programming is you will wind up with the definition of a lot of small functions
- Rather than defining a full fledge functions for these simple computation use closures (also called lambda functions)
- In Rust closures are fully fledged functions that can be used to compute values:

```
1 fn main() {
2    let result = (|i:i64|->i64{2*i})(3);
3    println!("{}", result)
4 }
```

```
1 fn multiples(f:fn(i64)->i64, q:i64) -> i64 {
2    f(q)
3 }
4 
5 fn main() {
6    println!("{}", multiples(|i:i64|->i64 {i}, 3));
7    println!("{}", multiples(|i|2*i, 3));
8    println!("{}", multiples(|i|3*i, 3));
9 }
```

Abbreviated form of a closure

- As the paradigm definition suggests we can also return functions from functions
- This again allows us to write general code that can be specialized using higher-order functions
- Consider a function that returns a specific computation based on some input parameters.

```
enum Multipliers {
         Unit,
         Double,
    fn multiples(what: Multipliers) -> fn(i64)->i64 {
         use Multipliers::*;
         match what {
                                                  ∄
                                                                      Terminal
                                                                                      Q
                                                                                         =
             Unit => |x:i64|->i64\{x\},
                                                 ubuntu$ ./return func
             <u>Double</u> => |x:i64|->i64\{2*x\},
10
                                                 ubuntu$
11
12
13
    fn main() {
14
         use Multipliers::*;
15
16
         let f = multiples(Unit);
17
         println!("{}",f(3));
18
19
         println!("{}",multiples(Double)(3));
20
```

- In the functional programming language context higher-order programming also gives us access to partially evaluated functions
- Unfortunately this is not available in Rust but we will see this when we discuss Haskell
- Partially evaluated functions are a powerful tool when customizing code from a given library
- An interesting application of higher-order programming are iterators

 Definition: An iterator is an object that enables a programmer to traverse a container, particularly lists.

- Typically we classify iterators into two classes:
 - Internal iterators: Internal iterators are higher order functions implementing the traversal across a container, applying the given function to every element in turn.
 - External iterators: An external iterator may be thought of as a type of pointer that has two primary operations: referencing one particular element in a collection (called *element access*) and modifying itself so it points to the next element (called *element traversal*).

- Here is a basic Rust external iterator:
 - Element access
 - Forward to next element

```
1 fn main() {
2    let a = [1,2,3];
3    let a_iter = a.iter();
4
5    for e in a_iter {
6        println!("{}",e);
7    }
8 }
```

We can also use iterators to modify the container

```
1  fn main() {
2    let mut a = [1,2,3];
3    let a_iter = a.iter_mut();
4
5    for e in a_iter {
6      *e += 1;
7    }
8    println!("{:?}",a);
9  }
```

- In Rust internal iterators sit on top of external iterators
 - This is not true for many other languages
 - We will see true internal iterators in Haskell

- A basic internal iterator program
- Notice, no explicit iteration needed
- Note: we needed to switch from arrays to vectors because 'collect' only supports vectors.

Assignments

Assignment #3