A Type is a Set of Values

Consider the statement:

let mut n: i32;

Here we declare n to be a variable of <u>type</u> i32; what we mean, n can take on any value from the <u>set of all 32 bit integer values</u>.

Def: A type is a set of values.

Def: A primitive type is a type a programmer can use but not define.

Def: A <u>constructed type</u> is a user defined type.

Example: Java, primitive type

float q;

type float ⇒ set of all possible floating point values

q is of type float, only a value that is a member of the set of all floating point values can be assigned to q.

Example: Java, constructed type

```
class Rectangle { int xdim; int ydim; };
```

Rectangle r = new Rectangle();

Now the variable c only accepts values that are members of type Rectangle; <u>object instantiations</u> of class Rectangle.

Example: Rust, constructed type

```
1 struct Rectangle { xdim: i32, ydim: i32}
2 
3 fn main() {
4  let r = Rectangle{xdim:3, ydim:4};
5 }
an element of type Rectangle.
```

Example: C, constructed type

the variable a will accept values which are arrays of 3 integers.

Example: Rust, constructed type

the variable a will accept values which are arrays of 3 i64 integers.

Subtypes

Def: a <u>subtype</u> is a <u>subset</u> of the elements of a type.

Example: Java

Short is a subtype of int: $short \subset int$

Observations:

- (1) converting a value of a subtype to a values of the super-type is called <u>widening</u> type conversion. (safe)
- (2) converting a value of a supertype to a value of a subtype is called <u>narrowing</u> type conversion. (not safe)

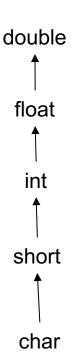
Example: Java

float ⊂ double

Subtypes give rise to type hierarchies and type hierarchies allow for automatic type coercion – widening conversions!

Subtypes & Type Hierarchies

- Here the arrow means "subtype of", e.g., int is a subtype of float.
- In type hierarchies it is always safe to move from subtype to supertype – widening conversion (coercion)
 - short -> int ✓
- Never the other way around
 - float <- int X</p>



Subtypes

- Rust does NOT have subtypes
 - Therefore, it does NOT support automatic type coercion
 - It is an error in Rust to do:

```
let x : i64 = 3;
let i : f64 = x;
```

 You would have to explicitly cast the integer variable as a floating point:

```
let x : i64 = 3;
let i : f64 = x as f64;
```

Function Types

Rust and Haskell (as we will see) treat functions as just another data type that can be manipulated in a very elegant way.

- Functions can be passed as values; just as values that belong to other data types
- Functions belong to function types

Example: in Rust consider the function type

$$fn(f64) \rightarrow f64$$

This type represents the <u>set of all functions from f64 to f64</u>.

Looking at the Rust standard library std::f64 we can find some examples from this set of functions

floor: $fn(f64) \rightarrow f64$

ceil: $fn(f64) \rightarrow f64$

round: $fn(f64) \rightarrow f64$

Function Types

Example: Functions as values

```
fn myround (x:f64) -> i64 {
    x.round() as i64
}

fn main () {
    let foo = myround;
    let v : i64 = foo(3.4);
    println!("{}", v);
}
```

A function is just an element of a particular function set.

Function Types

Example: Functions as function arguments

```
fn myfloor (x:f64) -> i64 {
      x.floor() as i64
 3
   fn myceil (x:f64) -> i64 {
      x.ceil() as i64
    fn myfun (x: fn(f64)->i64, v:f64) -> j.4 {
         \mathbf{x}(\mathbf{v})
10
11
12
    fn main () {
13
         println!("{}", myfun(myfloor, 3.4));
14
         println!("{}", myfun(myceil, 3.4));
15
16
```

A function is just an element of a particular function set.

Why do we use types?

- Types allow the computer/language system to assist the developer write <u>better programs</u>.
 Type <u>mismatches</u> in a program usually indicate some sort of <u>programming error</u>.
 - Static type checking check the types of all statements and expressions at compile time.
 - <u>Dynamic type checking</u> check the types at runtime.

Type Equivalence

I. Name (nominal) Equivalence – two objects are of the same type if and only if they share the same type name.

Example: Rust

```
1  struct Type1 {x:i64, y:i64}
2  struct Type2 {x:i64, y:i64}
3
4  fn main () {
5    let x: Type1 = Type1{x:1,y:2};
6    let y: Type2 = x;
7    println!("{:?}",y);
8  }
Error; even though the same, their name therefore Duct will.
```

Error; even though the types look the same, their names are different, therefore, Rust will not compile.

Rust uses name equivalence

Type Equivalence

II. <u>Structural Equivalence</u> – two objects are of the same type if and only if they share the same <u>type structure</u>.

Example: Haskell

```
type Type1 = (Integer, Integer)
type Type2 = (Integer, Integer)

x :: Type1
y :: Type2

x = (1,2)
y = x
```

Even though the type names are different, Haskell correctly recognizes this statement.

Haskell uses <u>structural equivalence</u>.

- Type inference refers to the automatic detection of the data type of an expression in a programming language.
- To see how this might work let's work through an example.

 Assume we have the following statement in a programming language like Rust:

```
let x : i64 = 3;
let y : i64 = 2 * x;
```

- We want to make sure that all the assignments are legal.
- We will use the type notation '3.{i64}' indicating that this syntactic unit has the type i64.

 We start at the primitives on the right side of the assignments starting with the first statement and then stepping through all the remaining statements

```
let x : i64 = 3; ✓
let x : i64 = 3.\{i64\};
                                         let y : i64 = 2.\{i64\} * x;
let y : i64 = 2 * x;
                                         let x : i64 = 3; \checkmark
                                         let y : i64 = 2.\{i64\} * x.\{i64\};
let x : i64 = 3.\{i64\};
let y : i64 = 2 * x;
                                         let x : i64 = 3;
                                         let y: i64 = 2 *.\{fn(i64,i64)->i64\} x;
                                         let x : i64 = 3; ✓
                                         let y: \frac{164}{2} = 2 \cdot .\{fn(i64,i64)->i64\} x; \checkmark
                                        let x : i64 = 3; ✓
                                        let y : i64 = 2 * x; ✓
```

Let's try a program with a bug in it:

```
let x : i64 = 3;
let y : f64 = 2 * x;
```

```
let x : i64 = 3; ✓
let x : i64 = 3.\{i64\};
                                       let y: f64 = 2.\{i64\} * x;
let y : f64 = 2 * x;
                                      let x : i64 = 3; \checkmark
                                      let y: f64 = 2.\{i64\} * x.\{i64\};
let x : i64 = 3.\{i64\};
let y : f64 = 2 * x;
                                       let x : i64 = 3;
                                       let y: f64 = 2 *.\{fn(i64,i64)->i64\} x;
                                       let x : i64 = 3; ✓
                                       let y: \frac{164}{1} = 2 *.{fn(i64,i64)->i64} x; *
                                       let x : i64 = 3; 	✓
                                       let y : f64 = 2 * x; 🗶
```

Exercises

- Describe the type associated with the set of values {-1,-2,-3,-4,...}, call this type Q.
- Describe the type associate with the set of values {-2,-4,-6,-8,...}, call this type P.
- Is there a subtype-supertype relationship between this types? If so, what is it?
- Let x be a variable of type Q and y be a variable of type P, then is the assignment x := y a safe assignment? Why? Why not?
- ullet Describe the type associated with set $Q \to P$.

Take Away

- Types are sets of values, typically with a common representation and common set of operations.
- Types in programming languages allows compilers and interpreters to check for consistency in your programs.
- Inconsistencies usually show up a type mismatches.
- Type equivalence between constructed types can be established in one of two ways, name equivalence or structural equivalence.