Defining Language TWO

- Extend Language ONE with:
  - Variables
  - An ML-style **`let`** expression for defining them

TWO: Syntax

TWO:

*<exp>** ::= *<exp>* **+** *<mulexp>* | *<mulexp>*
<mulexp> ::= *<mulexp>* ***** *<rootexp>* | *<rootexp>*
<rootexp> ::= **let val** *<variable>* **=** *<exp>* **in** *<exp>* **end**
| **(***<exp>***)** | *<variable>* | *<constant>*

- A subset of ML expressions
- This grammar is unambiguous
- A sample Language TWO expression:
  ```
  let val y = 3 in y*y end
  ```
- What does the parse tree for the above expression look like?

TWO: Abstract Syntax

Additional abstract syntax nodes for language TWO:

(1) var(X) dereferences a variable X

(2) let(X,E1,E2) binds the variable X to expression E1 in the context of expression E2.

Example: the TWO program

```
let val y = 3 in y*y end
```

will result in the AST

```
let(y,const(3),times(var(y),var(y)))
```

From Parse Tree to Prolog AST

- Consider: 2 * let x = 5 in 1+x end
  - Parse tree?
  - AST?
  - Prolog AST?

TWO: Semantics

In order to provide semantics we need to remember the values assigned to variables -- binding environments, contexts.

In our case, for the Prolog based semantics, we let the terms bind(X,K) represent the binding of variable X to value K. A context is simply a list of these binding terms:

```
        [bind(y,3),bind(q,20),bind(z,5)]
```

Given this binding structure, we can write a predicate, lookup/3, that returns a variable binding for a particular Var

```
        lookup(Var,[bind(Var,Value)| _ ],Value).
        lookup(Var,[ _ |Rest],Value) :- lookup(Var,Rest,Value).
```

Finds the most recent binding of variable `Var` if there is one.

TWO: Prolog Interpreter

```
val2(plus(X,Y),C,Value) :-
        val2(X,C,XValue),
        val2(Y,C,YValue),
        Value is XValue + YValue.

val2(times(X,Y),C,Value) :-
        val2(X,C,XValue),
        val2(Y,C,YValue),
        Value is XValue * YValue.

val2(const(X),_,X).

val2(var(X),C,Value) :-
        lookup(X,C,Value).

val2(let(X,Exp1,Exp2),C,Value) :-
        val2(Exp1,C,XValue),
        val2(Exp2,[bind(X,XValue)|C],Value).
```

val2 / 3 - interpretation predicate, first argument: AST; second argument: context; third argument: semantic value.

Examples

```
let val y = 3 in y*y end
```

```
?- val2(let(y,const(3),times(var(y),var(y))),[ ],X).

X = 9

Yes
```

```
let val y = 3 in
  let val x = y*y in
    x*x
  end
end
```

```
let val y = 1 in
  let val y = 2 in
    y
  end
end
```

Exercises

- Use the semantics of TWO to show the following:
  - Assume that the context C = [bind(y,3)] then the semantic value of ‘2*y’ is 6
  - The semantic value of ‘2 * let x = 3 in x * x end’ is 18
  - The semantic value of
    ‘let x = 1 in let y = x + 1 in y end end’
    is 2

Exercises

- Use thesemantics to compute the meaning of the following expressions in TWO (use the rules given in the notes, the book has many typos):

```
1) let val y = 3 in 2*y end

2) let val y = 1 in
    let val y = 2 in
      y
    end
   end
```

Note: first construct an abstract syntax tree, then give the representation in Prolog notation, and then show the computation in oursemantics.