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1 clear
2 %generate data for a state space model
3 %Y=Beta[t]*X+e1
4 %Beta[t]=mu+F*Beta[t-1]+e2
5 %var(e1)=R
6 %var(e2)=Q


$$Y_t = X_t \beta_t + v_t$$


$$\beta_t = \mu + F \beta_{t-1} + e_t$$


$$\text{VAR}(v_t) = R$$


$$\text{VAR}(e_t) = Q$$


7 t=500;
8 Q=0.001;
9 R=0.01;
10 F=1; %these are fixed
11 mu=0; %these are fixed
12 e1=randn(t,1)*sqrt(R);
13 e2=randn(t,1)*sqrt(Q);
14 Beta=zeros(t,1);
15 Y=zeros(t,1);
16 X=randn(t,1);
17 for j=2:t
18     Beta(j,:)=Beta(j-1,:)+e2(j,:);
19     Y(j)=X(j,:)*Beta(j,:)'+e1(j);
20 end
Start of the Kalman filter
21 %%Step 1 Set up matrices for the Kalman Filter

22 beta0=zeros(1,1); %state variable b[0/0]  $\beta_{0|0}$ 
23 p00=1; %variance of state variable p[0/0]  $p_{0|0}$ 
24 beta_tt=[]; %will hold the filtered state variable
25 ptt=zeros(t,1,1); % will hold its variance
26 %initialise the state variable

27 beta1l=beta0;  $\beta_{t-1|t-1}$ 
28 p1l=p00;  $p_{t-1|t-1}$ 
29 for i=1:t Loop from period 1 to end of sample
30     x=X(i);
31     %Prediction

32 beta10=mu+beta1l*F';  $\beta_{t|t-1} = \mu + F \beta_{t-1|t-1}$ 
33 p10=F*p1l*F'+Q;  $p_{t|t-1} = F p_{t-1|t-1} F' + Q$ 
34 yhat=(x*(beta10)')';  $X_t \beta_{t|t-1}$ 
35 eta=Y(i,:)-yhat;  $\eta_{t|t-1} = Y_t - X_t \beta_{t|t-1}$ 
36 feta=(x*p10*x')+R;  $f_{t|t-1} = X_t p_{t|t-1} X_t' + R$ 
37 %updating
38 K=(p10*x')*inv(feta);  $K_t = p_{t|t-1} X_t' f_{t|t-1}^{-1}$  Kalman gain
39 beta1l=(beta10'+K*eta)';  $\beta_{t|t} = \beta_{t|t-1} + K_t \eta_{t|t-1}$ 

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40 p11=p10-K*(x*p10);
41 ptt(i,:,:)=p11;
42 beta_tt=[beta_tt;beta11];
43 end
44 %%%%%%%%%end of Kalman
Filter%%%%%%%%
45 plot([beta_tt Beta])
46 axis tight
47 legend('estimated \beta_{t}','true \beta_{t}');

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$$p_{t|t} = p_{t|t-1} - K_t X_t p_{t|t-1}$$