Universitá degli studi di Milano

Time Series Econometrics
B-74-3-B
M.Sc. Data Science & Economics

Inflation forecast for the Euro area

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Introduction

The European Central Bank (ECB) is the institution in charge of monetary policy in the Euro area. Their aim is to keep prices stable: *price stability is the best contribution that monetary policy can make to economic growth.* Their objective is to keep inflation at 2%, holding the value of the euro and bringing economic stability to the area.

In this text, I am applying the techniques learned in the Time Series Econometrics course to forecast the quarterly inflation in the Euro area for the period 2015Q1 to 2020Q1, using as sample the observed inflation from 1999Q4 to 2014Q4. The forecast will be then compared against three different series:

1. Forecast from the professional forecasters
2. Naive forecast (forecast equal to the last observed realisation)
3. The 2% target inflation set by the European Central Bank (ECB)

Inflation in the Euro area 1999 - 2014

The sample series Y_t of the inflation in the Euro area from 1999Q4 to 2014Q4 can be observed in Figure 1. It is composed by 61 observations, characterised by a mean of 1.98, a maximum of 4.0 in 2008Q2 and a minimum of -0.3 in 2009Q3 and a standard deviation of 0.869.

$$\text{T=61} \quad \overline{Y} = 1.98,\ Y_{max} = 4.0,\ Y_{min} = -0.3,\ \sigma_Y = 0.869$$

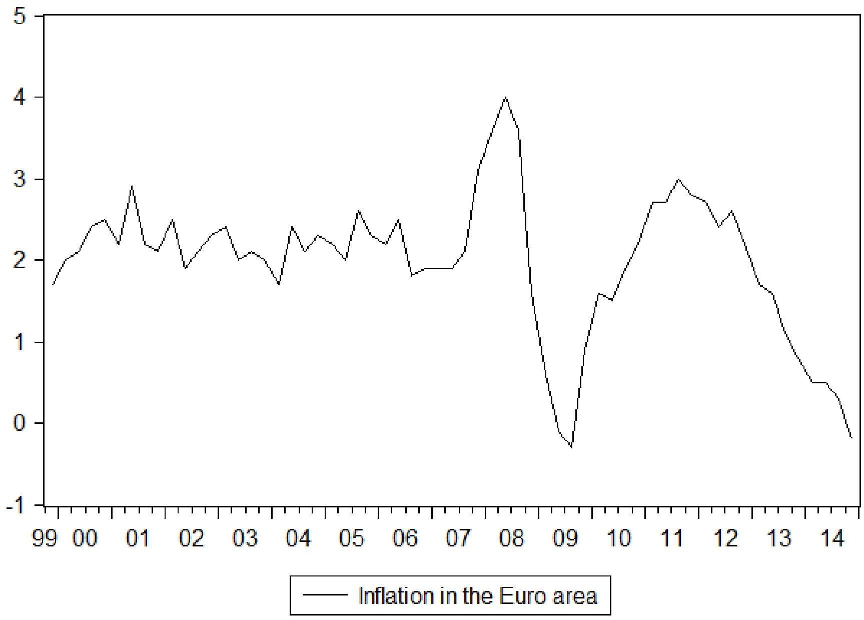


Figure 1: Inflation in the Euro area - 1999Q4 - 2014Q4

The discussion of whether inflation is a I(0) or I(1) process is constantly being studied by different researchers, and although the objective of this text is not to enter in those details, it must be said that inflation does not show all the characteristics of a I(1) process: 1) it does not have infinite variance and 2) it is not infinitely diverging away from the mean. In other words, inflation is not a random walk per se, but the series usually behaves as an I(1) process.

By this reason, the first step to produce a forecast is to know if the sample series is stationary. This can be done by testing if the series has a unit root. If the sample series has a unit root, then it is not stationary, and a first difference model should be used on the data. On the other hand, if the series does not have a unit root, then a model at levels can be estimated.

Writing Y_t as the sample series, in order to determine if the sample series has a unit root, an Augmented Dickey Fuller test is run over the following equation:

$$\Delta Y_t = \alpha + (\rho - 1)Y_{t-1} + \mu_t, \text{where } \mu_t = \sum_{j=1}^{p-1} \xi_j \mu_{t-j} + \varepsilon_t \text{ and } \varepsilon_t \; w.n.(0, \sigma^2)$$

The equation above corresponds to the Case 2 for unit root testing. The case is chosen based on the fact that there is not a strong reason to think that inflation has mean 0 (Case 1 must be applied in that scenario), and the sample series is not showing a trend over the whole period, so Case 3 and Case 4 are excluded as well.

In this regression, our null hypothesis is $H_0 : \{\rho - 1 = 0\}$ (and $\alpha = 0$) and our alternative hypothesis is $H_A : \{|\rho| < 1\}$ (and $\alpha \neq 0$). In Figure 2, the test results are displayed, where we do not reject the null hypothesis, as the corresponding t-statistic (-2.72) is higher than the critical value (-2.86), therefore the inflation series has a unit root.

Null Hypothesis: REALISED has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.726986	0.0739
Test critical values:	1% level	-3.514426	
	5% level	-2.898145	
	10% level	-2.586351	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(REALISED)
Method: Least Squares
Date: 12/04/21 Time: 13:30
Sample (adjusted): 2000Q2 2020Q1
Included observations: 80 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
REALISED(-1)	-0.150769	0.055288	-2.726986	0.0079
D(REALISED(-1))	0.260683	0.111316	2.341823	0.0218
C	0.245885	0.108183	2.272860	0.0258

R-squared	0.118671	Mean dependent var	-0.016250
Adjusted R-squared	0.095779	S.D. dependent var	0.472924
S.E. of regression	0.449705	Akaike info criterion	1.276331
Sum squared resid	15.57210	Schwarz criterion	1.365657
Log likelihood	-48.05324	Hannan-Quinn criter.	1.312144
F-statistic	5.184018	Durbin-Watson stat	2.045274
Prob(F-statistic)	0.007724		

Figure 2: Augmented Dickey-Fuller test for the inflation sample

It is clear from the test output that the value of our estimated $\hat{\rho}$ is $\hat{\rho} = 0.849$ and, as mentioned above, that we are dealing with a I(1) process. As our series is not stationary, we must use a unit root model in order to compute forecasts. Our model is then:

$$\Delta Y_t = \mu_t, \text{where } \mu_t = \sum_{j=1}^{p-1} \xi_j \mu_{t-j} + \varepsilon_t \text{ and } \varepsilon_t \; w.n.(0, \sigma^2)$$

Nevertheless, just to be sure that one difference is enough to solve the unit root problem, we run the ADF test again on the first difference equation. Figure 3 shows the results of the test, where the null hypothesis - ΔY_t has a unit root- is rejected.

Null Hypothesis: D(REALISED) has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.274128	0.0000
Test critical values:	1% level	-3.514426	
	5% level	-2.898145	
	10% level	-2.586351	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(REALISED,2)
Method: Least Squares
Date: 12/04/21 Time: 13:41
Sample (adjusted): 2000Q2 2020Q1
Included observations: 80 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(REALISED(-1))	-0.815507	0.112111	-7.274128	0.0000
C	-0.015328	0.052315	-0.292986	0.7703

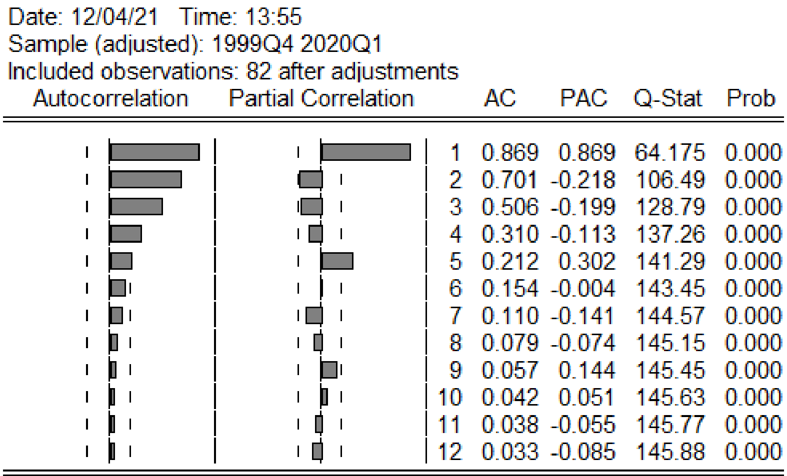
R-squared	0.404184	Mean dependent var	-0.011250
Adjusted R-squared	0.396546	S.D. dependent var	0.602315
S.E. of regression	0.467892	Akaike info criterion	1.343525
Sum squared resid	17.07601	Schwarz criterion	1.403075
Log likelihood	-51.74099	Hannan-Quinn criter.	1.367400
F-statistic	52.91294	Durbin-Watson stat	1.992820
Prob(F-statistic)	0.000000		

Figure 3: Augmented Dickey-Fuller test for the inflation (first difference) sample

Model Selection

Estimation from the correlogram

Before estimating a series of models on the sample, it is often useful to look at the correlogram of the series to extract further information about its behavior. In Figure 4 is displayed the correlogram of the sample series at 12 lags. Here, it is clear from the Autocorrelation Function (AF) that dependence is still strong up to the 5^{th} lag. From the Partial Autocorrelation Function (PACF) we can see a sort of cyclical behaviour (seasonality), suggesting a model with complex roots may fit better the series.



Date: 12/04/21 Time: 13:55
Sample (adjusted): 1999Q4 2020Q1
Included observations: 82 after adjustments

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.869	0.869	64.175	0.000
		2	0.701	-0.218	106.49	0.000
		3	0.506	-0.199	128.79	0.000
		4	0.310	-0.113	137.26	0.000
		5	0.212	0.302	141.29	0.000
		6	0.154	-0.004	143.45	0.000
		7	0.110	-0.141	144.57	0.000
		8	0.079	-0.074	145.15	0.000
		9	0.057	0.144	145.45	0.000
		10	0.042	0.051	145.63	0.000
		11	0.038	-0.055	145.77	0.000
		12	0.033	-0.085	145.88	0.000

Figure 4: Correlogram of the sample series

To correctly identify the model that best fit the data, ARMA models from order 1 to 4 are tested on the sample. The criterion used to select the best fitting model is the Bayes Information Criterion (Schwarz Information Criterion), as the Akaike Information Criterion (AIC) may indicate some preference over models with more parameters, hence, higher variance on the estimate. It is worth noticing that an ARMA (p,q) model with equal predictive ability is always preferred over an ARMA (p+1,q) or ARMA (p,q+1) (Parsimonious modelling).

In Table 1 are displayed the estimation results of the models tested on the data. In the table is shown the Schwarz Information Criterion and whether the MA roots of the models are invertible or not. Models with no invertible MA components are not taking into account, as they are not suitable for forecasting.

Table 1: Models tested on the sample Inflation series

Model order	BIC	Invertible MA
MA(1)	1.590657	Yes
AR(1)	1.584259	-
MA(2)	1.560168	No
AR(2)	1.649977	-
ARMA(1,1)	1.650995	Yes
MA(3)	1.458424	No
AR(3)	1.716967	-
ARMA(2,1)	1.621699	No
ARMA(1,2)	1.593146	No
MA(4)	1.382195	No
AR(4)	1.551164	-
ARMA(3,1)	1.637150	No
ARMA(1,3)	1.455521	No
ARMA(2,2)	1.506537	No

After excluding the models with no invertible MA roots, the model with the lower BIC is the AR(4). Nevertheless, as the difference in the BIC is not very high compared to the AR(1) model, the AR(1) model will be used to estimate the forecast (taking into account that a model with less parameters will result in less variance and an easier interpretation).

In Figure 5 is shown the estimation output for the estimated AR(1) model.

Dependent Variable: D(REALISED)
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/07/21 Time: 09:13
Sample: 2000Q1 2014Q4
Included observations: 60
Convergence achieved after 5 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.032381	0.085070	-0.380641	0.7049
AR(1)	0.240728	0.133988	1.796642	0.0777
SIGMASQ	0.232369	0.029003	8.011939	0.0000

R-squared	0.058590	Mean dependent var	-0.031667
Adjusted R-squared	0.025558	S.D. dependent var	0.501013
S.E. of regression	0.494569	Akaike info criterion	1.479442
Sum squared resid	13.94212	Schwarz criterion	1.584159
Log likelihood	-41.38326	Hannan-Quinn criter.	1.520403
F-statistic	1.773751	Durbin-Watson stat	1.998739
Prob(F-statistic)	0.178933		

Inverted AR Roots	.24

Figure 5: Estimation output AR(1) model

It can be observed that the coefficient for ϕ_1 is significant and equal to 0.24. With this in mind, the equation that best fit the data is:

$$\Delta Y_t = \alpha + \phi_1 \Delta Y_{t-1} + \varepsilon_t,$$

$$\text{where } \alpha = -0.03 \text{ and } \phi_1 = 0.24$$

$$\Delta Y_t = -0.03 + 0.24 \Delta Y_{t-1} + \varepsilon_t$$

To be sure that the AR(1) model fitted to the data is capturing the series behaviour, a Portmanteau test is run on the residuals. In Figure 6 are shown the results of the test.

Date: 12/07/21 Time: 10:20
Sample (adjusted): 2000Q1 2014Q4
Q-statistic probabilities adjusted for 1 ARMA term

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.010	-0.010	0.0059	
		2	0.056	0.056	0.2056	0.650
		3	0.085	0.087	0.6825	0.711
		4	-0.442	-0.448	13.679	0.003
		5	-0.079	-0.106	14.101	0.007
		6	-0.040	0.019	14.209	0.014
		7	-0.015	0.096	14.225	0.027
		8	-0.031	-0.269	14.293	0.046
		9	0.042	-0.063	14.421	0.071
		10	-0.037	-0.046	14.525	0.105
		11	0.018	0.106	14.551	0.149
		12	0.125	0.009	15.756	0.150

Figure 6: Portmanteau test on AR(1) residuals

Note that the 4^{th} lag is statistically significant, that is, there's still some information on the series that our model did not capture. This makes sense, as in the Model selection stage the model with the lowest BIC was the AR(4).

As mentioned above, the problem with the AR(4) model is the variance imposed on the model by estimating 4 parameters, even though this parameters are statistically equal to 0 (see Appendix for the estimation output). In this case, what can be done is to estimate an AR(4) model setting $\phi_2 = \phi_3 = 0$. This way,the 1^{st} lag dependence is captured by ϕ_1 and the seasonality is captured by ϕ_4 (the series is composed by quarterly data). In Figure 7 are shown the results of this estimation.

Dependent Variable: D(REALISED)
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/07/21 Time: 10:49
Sample: 2000Q1 2014Q4
Included observations: 60
Convergence achieved after 17 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.032461	0.051039	-0.636006	0.5274
AR(1)	0.232997	0.140772	1.655139	0.1035
AR(4)	-0.423998	0.095923	-4.420201	0.0000
SIGMASQ	0.183747	0.031267	5.876749	0.0000

R-squared	0.255574	Mean dependent var	-0.031667
Adjusted R-squared	0.215694	S.D. dependent var	0.501013
S.E. of regression	0.443702	Akaike info criterion	1.292344
Sum squared resid	11.02483	Schwarz criterion	1.431967
Log likelihood	-34.77033	Hannan-Quinn criter.	1.346958
F-statistic	6.408581	Durbin-Watson stat	1.996976
Prob(F-statistic)	0.000826		

Inverted AR Roots	.63-.57i	.63+.57i	-.52+.57i	-.52-.57i

Figure 7: Reduced AR(4) estimation output

It is worth noticing that ϕ_4 has a negative sign, supporting the fact of a cyclical component in the series. The model equation is then[1]:

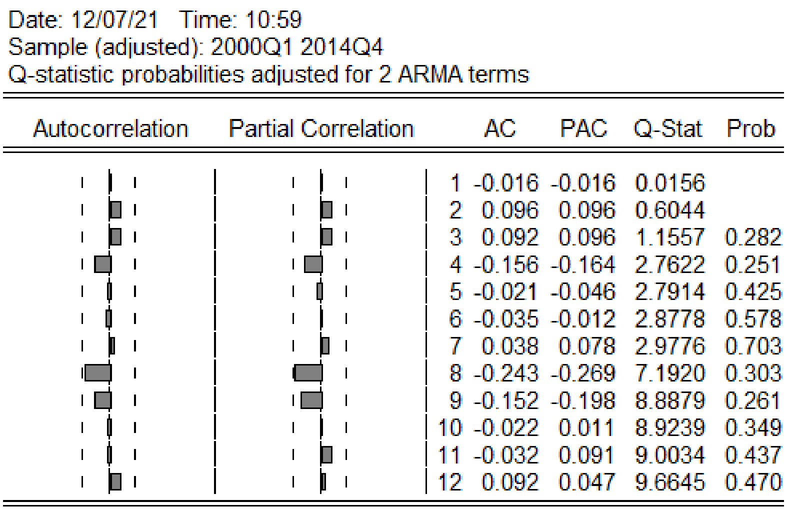
$$\Delta Y_t = \alpha + \phi_1 \Delta Y_{t-1} + \phi_2 \Delta Y_{t-2} + \phi_3 \Delta Y_{t-3} + \phi_4 \Delta Y_{t-4} + \varepsilon_t,$$

$$\text{where } \alpha = -0.03,\ \phi_1 = 0.23,\ \phi_2 = \phi_3 = 0, \text{and } \phi_4 = -0.42$$

$$\Delta Y_t = -0.03 + 0.23 \Delta Y_{t-1} - 0.42 \Delta Y_{t-4} + \varepsilon_t$$

If we take a look at the correlogram of the residuals in Figure 8, we observe that they are all statistically equal to 0, meaning there is no further dependence on past values that has not been captured by the model fitted to the data.

[1] We will cal this the Reduced AR(4) model



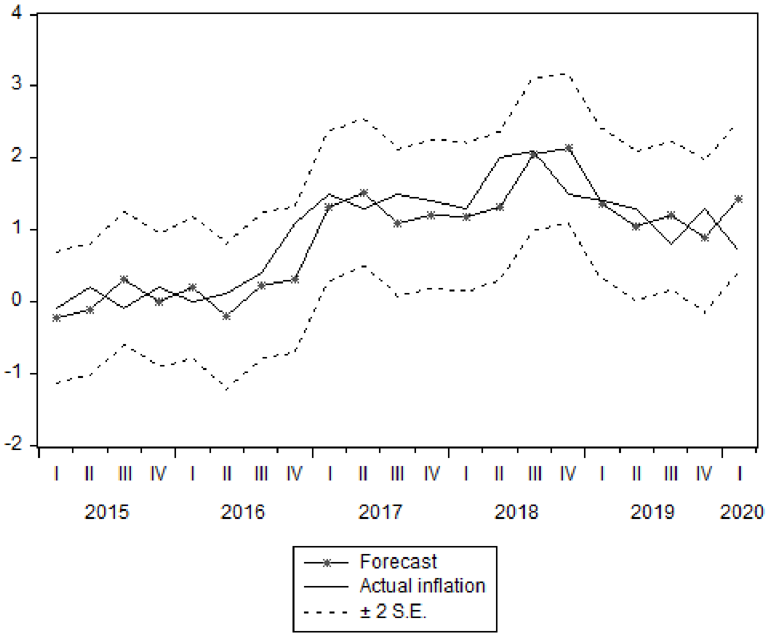
Date: 12/07/21 Time: 10:59
Sample (adjusted): 2000Q1 2014Q4
Q-statistic probabilities adjusted for 2 ARMA terms

Lag	AC	PAC	Q-Stat	Prob
1	-0.016	-0.016	0.0156	
2	0.096	0.096	0.6044	
3	0.092	0.096	1.1557	0.282
4	-0.156	-0.164	2.7622	0.251
5	-0.021	-0.046	2.7914	0.425
6	-0.035	-0.012	2.8778	0.578
7	0.038	0.078	2.9776	0.703
8	-0.243	-0.269	7.1920	0.303
9	-0.152	-0.198	8.8879	0.261
10	-0.022	0.011	8.9239	0.349
11	-0.032	0.091	9.0034	0.437
12	0.092	0.047	9.6645	0.470

Figure 8: Portmanteau test on AR(4) reduced model residuals

Forecast

Now that a model to fit the data has been selected, we can proceed to forecast the 2015 - 2020 period. In Figure 9 is presented the forecast and the actual series with the corresponding confidence intervals. On the next section, we test the predictive ability of the forecast.



Forecast: F1	
Actual: REALISED	
Forecast sample: 2015Q1 2020Q1	
Included observations: 21	
Root Mean Squared Error	0.389982
Mean Absolute Error	0.326340
Mean Abs. Percent Error	NA
Theil Inequality Coef.	0.171244
Bias Proportion	0.044794
Variance Proportion	0.004132
Covariance Proportion	0.951074
Theil U2 Coefficient	NA
Symmetric MAPE	75.18267

Figure 9: Estimated Forecast and actual values 2015Q1 - 2020Q1

Predictive ability of the forecast

To test if the forecast made has actual predictive ability, we are going to use a Diebold and Mariano test. This test measures if the difference between the squared errors of two different forecasts is equal to 0.

If we consider Y as the sample series for inflation, we want to forecast Y_{t+m}, and we have two forecasts $\tilde{Y}^{(1)}_{t+m|t...}$ and $\tilde{Y}^{(2)}_{t+m|t...}$, then, we can denote the forecast errors as

$$\epsilon_t = Y_{t+m} - \tilde{Y}^{(1)}_{t+m|t...} \text{and } \mu_t = Y_{t+m} - \tilde{Y}^{(2)}_{t+m|t...}$$

Where ϵ_t is the forecast error of the first forecast and μ_t is the forecast error of the second forecast. Then, we estimate $d = \epsilon_t^2 - \mu_t^2$ and test under the null hypothesis $H_0 : \{d = 0\}$ using a standard normal distribution.

This test will be run between the reduced AR(4) forecast (e_1) made in the last section and the following series:

1. Forecast from the professional forecasters (e_2)
2. Naive forecast (forecast equal to the last observed realisation) (e_3)
3. The 2% target inflation set by the ECB (e_4)

In Figure 10 is presented the Diebold and Mariano test between e_1 and e_2. As the p value of the constant is lower than 0.05, we reject the null hypothesis, so the two forecast do not have equal predictive ability. To determine which one has a better predictive ability, we must look at the sign of the estimated value. In this case, the constant is equal to -0.32, this is, $e_2 > e_1$. The error of the forecast from the professional forecasters is larger than the error of our estimated model[2].

Dependent Variable: SQE1-SQE2
Method: Least Squares
Date: 12/08/21 Time: 11:29
Sample: 2015Q1 2020Q1
Included observations: 21

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.315533	0.127073	-2.483097	0.0220

R-squared	0.000000	Mean dependent var	-0.315533
Adjusted R-squared	0.000000	S.D. dependent var	0.582319
S.E. of regression	0.582319	Akaike info criterion	1.802852
Sum squared resid	6.781917	Schwarz criterion	1.852592
Log likelihood	-17.92995	Hannan-Quinn criter.	1.813647
Durbin-Watson stat	0.928709		

Figure 10: DM test between the reduced AR(4) model and the professional forecast

[2] The forecast from the professional forecasters is three periods ahead; for the sake of the exercise, we pretended that the forecast had been made just in the period before

Now, we test $d_2 = e_1 - e_3$ and $d_3 = e_1 - e_4$ (see Appendix for estimation outputs) and display the results in Table 2.

Table 2: Diebold and Mariano tests outputs

Forecasts tested	Coefficient value	p-value	Comments
$d_1 = e_1 - e_2$	-0.32	0.02	Reduced AR(4) forecast has better predictive ability
$d_2 = e_1 - e_3$	0.01	0.43	Both forecasts have equal predictive ability
$d_3 = e_1 - e_4$	-1.41	0.00	Reduced AR(4) forecast has better predictive ability

By this stage we can now conclude that the estimated AR(4) model fitted onto the data has equal predictive ability as the naive forecast, and better predictive ability than the professional forecasters forecast and the 2% target set by the ECB. In Figure 11 are shown the three evaluated forecasts compared to the actual inflation series for the 2015Q1 - 2020Q1 period:

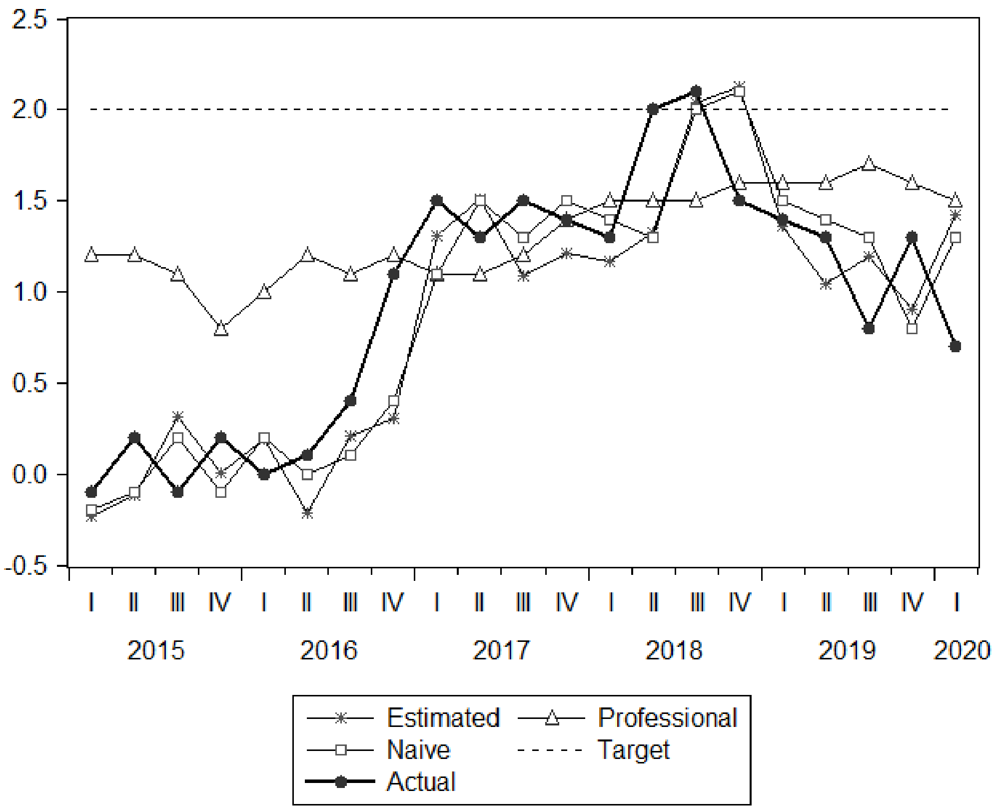


Figure 11: Estimated inflation forecasts 2015Q1 - 2020Q1

Appendix

Regression output, AR(4) estimation on inflation series 1999Q4 - 2014Q4

Dependent Variable: D(REALISED)
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/07/21 Time: 10:00
Sample: 2000Q1 2014Q4
Included observations: 60
Convergence achieved after 28 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.032521	0.057882	-0.561857	0.5765
AR(1)	0.209466	0.143817	1.456473	0.1511
AR(2)	0.081531	0.167417	0.486995	0.6282
AR(3)	0.067814	0.177598	0.381842	0.7041
AR(4)	-0.448739	0.107557	-4.172111	0.0001
SIGMASQ	0.180466	0.031479	5.732954	0.0000

R-squared	0.268865	Mean dependent var	-0.031667
Adjusted R-squared	0.201168	S.D. dependent var	0.501013
S.E. of regression	0.447793	Akaike info criterion	1.341729
Sum squared resid	10.82798	Schwarz criterion	1.551164
Log likelihood	-34.25188	Hannan-Quinn criter.	1.423651
F-statistic	3.971559	Durbin-Watson stat	1.957047
Prob(F-statistic)	0.003860		

Inverted AR Roots	.65-.53i	.65+.53i	-.55+.58i	-.55-.58i

Figure 12: Estimation output, AR(4) model

Diebold and Mariano tests outputs:

1. Reduced AR(4) model against Naive forecast ($d = e_1 - e_3$)

Dependent Variable: SQE1-SQE3
Method: Least Squares
Date: 12/08/21 Time: 12:07
Sample: 2015Q1 2020Q1
Included observations: 21

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.013514	0.016722	0.808158	0.4285

R-squared	0.000000	Mean dependent var	0.013514
Adjusted R-squared	0.000000	S.D. dependent var	0.076631
S.E. of regression	0.076631	Akaike info criterion	-2.253171
Sum squared resid	0.117448	Schwarz criterion	-2.203431
Log likelihood	24.65829	Hannan-Quinn criter.	-2.242376
Durbin-Watson stat	2.575310		

Figure 13: DM test between the reduced AR(4) forecast and the naive forecast

2. Reduced AR(4) forecast against the 2% target set by the ECB ($d = e_1 - e_4$)

Dependent Variable: SQE1-SQE4
Method: Least Squares
Date: 12/08/21 Time: 12:37
Sample: 2015Q1 2020Q1
Included observations: 21

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.412200	0.359195	-3.931569	0.0008

R-squared	0.000000	Mean dependent var	-1.412200
Adjusted R-squared	0.000000	S.D. dependent var	1.646038
S.E. of regression	1.646038	Akaike info criterion	3.881068
Sum squared resid	54.18884	Schwarz criterion	3.930807
Log likelihood	-39.75121	Hannan-Quinn criter.	3.891862
Durbin-Watson stat	0.278869		

Figure 14: DM test between the reduced AR(4) model and the 2% target set by the ECB