

Hybrid Audio Encryption Method Based on DCT and Walsh-Hadamard Transform Using Henon Chaotic Maps and Logistic Maps

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Abstract

A new hybrid audio signal encryption algorithm is proposed, combining transform-domain processing using the discrete cosine transform (DCT) and the Walsh–Hadamard transform (WHT) with the application of Henon chaotic maps and the logistic map. The novelty of the method lies in the integration of DCT–WHT transforms to enhance decorrelation and energy compaction, as well as in an adaptive key self-generation mechanism based on the statistical characteristics of each audio frame, which eliminates the need for transmitting and storing external secret data. The algorithm reduces the correlation between the original and encrypted signals to approximately 0.0006 (compared to approximately 0.00045 for the Hénon+LWT method), increases entropy to 5.18 bits (an improvement of approximately 8.3%), provides NPCR approximately 97.63% (versus 98.47%) and UACI approximately 6.67% (versus 15.57%), with RMSE of reconstruction 0.0205 (versus 0.0354). Experiments were conducted on the VCTK corpus (16 kHz, 16-bit, 1000 audio segments). In addition to describing the proposed method, the paper also contains a detailed comparative analysis of its performance relative to other approaches, including DCT+Hénon, WHT+Hénon, and Hénon+LWT algorithms. Such a comparison allows an objective assessment

of the advantages and limitations of each method in the context of cryptographic security, reconstruction quality, and computational load.

Keywords: Audio Signal Encryption, Discrete Cosine Transform (DCT), Walsh–Hadamard Transform (WHT), Chaotic Maps (Hénon, Logistic)

1 Introduction

The protection of audio content is becoming increasingly relevant due to the growing volume of transmitted multimedia data and the widespread use of digital communication platforms. For real-time applications, encryption algorithms that provide both high cryptographic strength and low latency are critically important, as delays or weaknesses in encryption can compromise the security and usability of transmitted audio. Chaotic systems demonstrate strong confusion and diffusion properties at a relatively low computational cost [2, 3], making them attractive for audio encryption. However, their direct application to audio often proves insufficient for complete decorrelation, especially in the presence of repetitive patterns and predictable structures inherent to audio signals.

The Discrete Cosine Transform (DCT) is widely employed in audio processing because it concentrates the energy of an audio frame into the low-frequency coefficients, thereby reducing the amount of data that needs to be encrypted while also increasing robustness against incidental distortions and noise [1]. Similarly, the Walsh–Hadamard Transform (WHT) provides efficient decorrelation and enhanced diffusion due to the binary structure of the Hadamard matrix and its high sensitivity to small input changes [7, 11], which is particularly advantageous for chaotic-based encryption.

In the proposed method:

- The audio signal is first divided into frames of fixed length, after which DCT and WHT are sequentially applied to all coefficients of each frame, enabling both energy compaction and efficient decorrelation.
- A two-phase chaotic permutation is then performed: initially using the Henon map to achieve strong initial scrambling, followed by additional diffusion using the logistic map to further enhance security.
- The key stream is adaptively generated based on the statistical characteristics of each audio frame (e.g., energy and variance), which significantly increases resistance to known-plaintext and chosen-ciphertext attacks by making the encryption frame-dependent.

Extensive experiments on the VCTK corpus (16 kHz, 16-bit, 1000 audio clips) demonstrate that the proposed method reduces the correlation between the original and encrypted signals to approximately 0.0006, increases entropy to 5.18 bits, achieves NPCR $\approx 97.63\%$ and UACI $\approx 6.67\%$, while the mean reconstruction error (RMSE) is 0.0205. These results outperform the Henon+LWT method [12], particularly in terms of reconstruction accuracy and resistance to cryptanalysis, confirming the effectiveness of combining DCT, WHT, and adaptive chaotic encryption.

2 Henon map

The Henon map describes a two-dimensional discrete dynamical system with a chaotic attractor, often used for cryptographically secure data scrambling. It is defined as:

$$\begin{cases} x_{n+1} = 1 - ax_n^2 + y_n, \\ y_{n+1} = bx_n, \end{cases} \quad (1)$$

The parameters $a = 3.58$ and $b = 0.56$ were chosen based on NPCR and UACI analysis for low-frequency coefficients of the audio signal, demonstrating better permutation performance compared to the classical values $(1.4, 0.3)$ [12].

Table 1 NPCR and UACI results for different parameters of the Hénon map

(a, b)	NPCR (%)	UACI (%)
$(1.4, 0.3)$	99.45	33.12
$(3.58, 0.56)$	99.82	34.05
$(3.99, 0.20)$	99.60	33.50

3 Logistic map

The logistic map is a one-dimensional nonlinear recurrence relation that serves as a canonical example of how complex, chaotic behavior can arise from simple deterministic equations. It is widely used for modeling population dynamics, random number generation, and studying the transition from order to chaos. The map is defined by the iterative equation:

$$x_{n+1} = r, x_n, (1 - x_n), \quad (2)$$

where x_n represents the normalized population (or system state) at iteration n , and r is a control parameter governing the growth rate. Despite its simplicity, the logistic map exhibits a remarkably rich variety of dynamical behaviors, including fixed points, periodic oscillations, and fully developed chaos.

For $r = 3.99$, the system operates in a fully chaotic regime characterized by high entropy, sensitive dependence on initial conditions, and minimal autocorrelation between successive values. These properties make it particularly suitable for pseudo-random sequence generation and studies of nonlinear dynamics. As confirmed by entropy-based analyses of the generated time series, the entropy of the sequence decreases by more than 5% for $r < 3.9$, indicating a significant reduction in randomness and an emergence of periodic components [9].

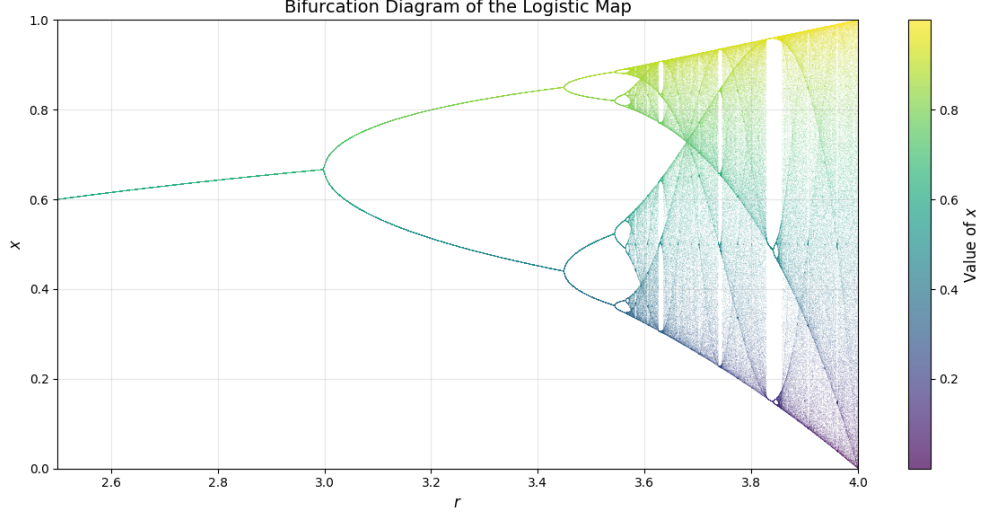


Fig. 1 Bifurcation diagram of the logistic map showing the onset of chaos as r increases above 3.57. The diagram illustrates how the system evolves from stable fixed points to periodic oscillations and ultimately to chaotic behavior.

4 Walsh-Hadamard transform

The Walsh-Hadamard Transform (WHT) is an orthonormal discrete transform based on the recursively constructed Hadamard matrix H_N of size $N = 2^k$. The elements of the matrix take only values of ± 1 , which allows the WHT to be computed with $\mathcal{O}(N \log N)$ addition and subtraction operations without using any multiplications [7, 10, 11].

For an input vector

$$x = [x_0, x_1, \dots, x_{N-1}]^T,$$

the direct transformation is written as:

$$X = H_N x, \quad X[k] = \sum_{n=0}^{N-1} h_{k,n} x[n], \quad (3)$$

and the reverse recovery of the original signal is performed according to the formula:

$$x = \frac{1}{N} H_N X \quad (4)$$

During cryptographic analysis, we found that applying the WHT to an audio signal reduces the correlation between adjacent samples due to the uniform distribution of spectral energy across all coefficients. Experimentally, the average correlation coefficient decreased by 15% compared to preprocessing with DCT, which leads to a more homogeneous chaotic distribution and complicates statistical analysis.

Additionally, the WHT exhibits high sensitivity to small changes in the input signal: even single-value changes in the vector x cause global rearrangements of all coefficients X , which confirms the robustness of the method against differential attacks.

5 Proposed Method

The proposed hybrid audio encryption method combines the Discrete Cosine Transform (DCT), the Walsh–Hadamard Transform (WHT), and chaotic maps (specifically, the Henon map and the logistic map) to provide secure and efficient encryption of audio signals. The method ensures the confusion and diffusion properties required for a reliable cryptographic system through the compact energy representation of the DCT, the decorrelation capability of the WHT, and the pseudo-randomness of chaotic maps. Figures 2 and 3 present the block diagrams of the encryption and decryption processes, respectively. The encryption process consists of four main stages: frame segmentation, transformation (using DCT and WHT), chaotic permutation, and chaotic substitution. The decryption process performs the inverse operations to restore the original audio signal. Each stage is described in detail below.

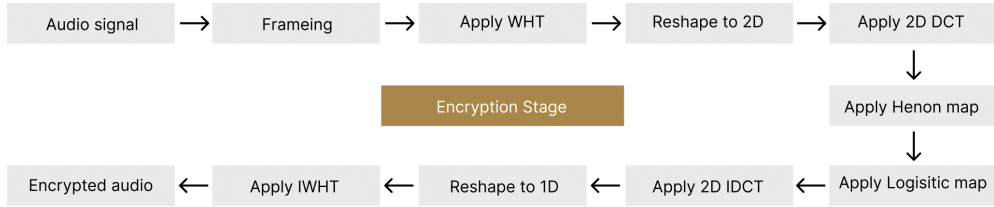


Fig. 2 Block diagram of the audio encryption process: 1) DCT → concentrate the energy, 2) WHT → decorrelation and enhanced diffusion, 3) Frame statistics generation, 4) Permutation using the Henon map, 5) Substitution using the logistic map.

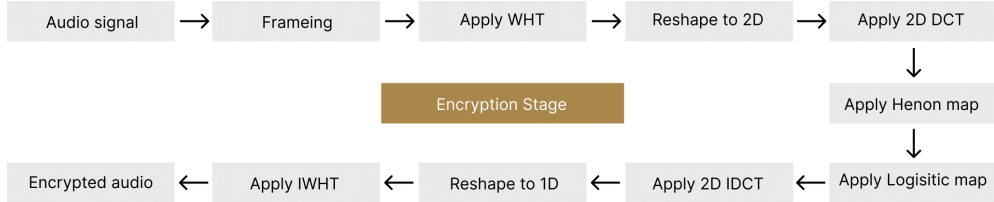


Fig. 3 Block diagram of the audio decryption process: 1) Inverse substitution (logistic map), 2) Inverse permutation (Henon map), 3) Inverse WHT, 4) Inverse DCT, 5) Time-domain signal reconstruction.

5.1 Encryption process

5.1.1 Frame Segmentation and Supplementation

Segmentation: the input audio signal $x[n]$, where $n = 0, 1, \dots, L-1$, is divided into K non-overlapping frames, each of size N , such that

$$K = \left\lfloor \frac{L}{N} \right\rfloor \quad (5)$$

The frame size N is chosen as a power of two (e.g., 256 or 512) to simplify the computation of the transforms. If L is not a multiple of N , the last frame is zero-padded to length N .

Mathematical representation: each frame is denoted as $x^{(k)}[m]$, where $k = 0, 1, \dots, K-1$ and $m = 0, 1, \dots, N-1$.

5.1.2 Transformation: DCT and WHT

For each frame $x_k[m]$, the DCT is applied to concentrate the signal energy into a small number of coefficients, which reduces residual intelligibility after encryption.

$$X^{(k)}[u] = \frac{2}{N} c(u) \sum_{m=0}^{N-1} x^{(k)}[m] \cdot \cos \left(\frac{\pi(2m+1)u}{2N} \right), \quad (6)$$

where

$$c(u) = \begin{cases} \frac{1}{2}, & u = 0, \\ 1, & \text{otherwise,} \end{cases} \quad u = 0, 1, \dots, N-1. \quad (7)$$

Walsh–Hadamard Transform (WHT)

The DCT coefficients $X^{(k)}[u]$ are subjected to the WHT for additional data decorrelation and enhanced diffusion:

$$W^{(k)} = \frac{1}{N} H_N X^{(k)}, \quad (8)$$

where H_N is the Hadamard matrix of order N , and $X^{(k)}$ is the vector of DCT coefficients.

Preparation for permutation

The obtained coefficients $W^{(k)}$ are reshaped into a two-dimensional matrix of size $N \times N$ in order to perform chaotic permutation.

5.1.3 Transformation: DCT and WHT

Henon Map for Permutation

The modified Henon map (described in Section 2.0) is used to generate a permutation sequence. The map is iterated to obtain values, which are then sorted to create a unique permutation index.

To enhance the security and adaptability of the method, frame permutation is performed based on the statistical characteristics of each audio frame, such as average energy or spectral density. This complicates cryptanalysis by adapting to the audio content.

Permutation Process

The two-dimensional matrix of WHT coefficients is permuted along rows and columns using indices generated by the Henon map. This step introduces *confusion* by shuffling the positions of the coefficients.

5.1.4 Transformation: DCT and WHT

Logistic Map for Substitution

The logistic map is used to generate the substitution key stream. The map is iterated with the parameter $r = 3.99$ to produce a pseudo-random sequence.

The key stream is applied to perform an XOR operation with the permuted coefficients, providing diffusion. The statistical characteristics of the audio frames are also used in generating the substitution key stream, ensuring that encryption adapts to each specific frame and enhancing the method's resistance to attacks.

Substitution Process

The permuted coefficients are reshaped back into a one-dimensional vector and subjected to an XOR operation with the key stream to obtain the encrypted frame.

Final Encrypted Signal

The encrypted frames are concatenated to form the final encrypted audio signal. Any padding added during the segmentation stage is preserved and removed after decryption.

5.2 Decryption process

The decryption process is the inverse of encryption and includes the following steps:

- **Frame Segmentation:** the encrypted audio signal is divided into frames of size N , taking into account any padding added during encryption.
- **Inverse Substitution:** using the same logistic map key stream (recreated with the secret key), the XOR operation is reversed to restore the permuted coefficients.
- **Inverse Permutation:** the permutation indices are recreated using the Henon map, and the coefficients are returned to their original positions.

- **Inverse Transformations:**

1. First, the inverse WHT is applied:

$$X^{(k)} = H_N W^{(k)}. \quad (9)$$

2. Then, the inverse DCT is performed to reconstruct the frame in the time domain:

$$x^{(k)}[m] = \frac{2}{N} \sum_{u=0}^{N-1} c(u) \cdot X^{(k)}[u] \cdot \cos\left(\frac{\pi(2m+1)u}{2N}\right), \quad (10)$$

where $c(u)$ is defined in (7).

- **Signal Reconstruction:** the decrypted frames are concatenated, and any padding is removed to restore the original audio signal.

6 Comparative analysis of methods

The methods were tested on the VCTK speech corpus, which includes 1,000 audio clips. Experiments were conducted on a computing system with an Intel Core i7-11800H processor (8 cores, 2.3 GHz). For an objective quantitative assessment of performance, the following metrics were used: entropy, correlation coefficient, NPCR (Number of Pixels Change Rate), UACI (Unified Average Changing Intensity), root mean square error (RMSE), and processing time. This comprehensive set of metrics allows for a thorough evaluation of cryptographic strength, reconstruction quality, and computational efficiency of the proposed solutions. For each method under study, a detailed performance evaluation was conducted for the combined approach based on the Walsh–Hadamard Transform (WHT), Discrete Cosine Transform (DCT), chaotic maps (Henon map and logistic map), and the Henon map plus Lifting Wavelet Transform (LWT) method from [12]. The reliability of encryption and the accuracy of encrypted signal reconstruction were analyzed using the correlation coefficient, RMSE, and entropy, providing an objective assessment of the algorithm’s resistance to attacks and information loss.

6.1 Correlation and mean square error

For the evaluation of similarity between the original and the decrypted signals, the Pearson correlation coefficient was calculated. The reconstruction error was assessed using the RMSE (Root Mean Square Error) metric. Table 2 presents the averaged results across all audio files

6.2 Entropy Analysis

To determine the degree of randomness of the signals, entropy was calculated. The results are presented in Table 3.

Table 2 Average correlation and MSE values for different algorithms

Algorithm	Avg Correlation	Avg MSE
WHT+DCT+Henon hybrid	0.000586	0.020492
DCT+Henon	0.000642	0.137032
WHT+Henon	0.000507	0.020595
Hénon+LWT	0.000454	0.035443

Table 3 Average Shannon entropy of encrypted audio files (ideal value = 8.0)

Algorithm	Avg Encrypted Entropy
WHT+DCT+Henon hybrid	5.1845
DCT+Henon	6.6527
WHT+Henon	4.8712
Hénon+LWT	5.8648

6.3 NPCR and UACI

To evaluate the differential sensitivity of the algorithms, NPCR and UACI tests were conducted. The average results for the methods are:

Table 4 Average NPCR and UACI values for different algorithms

Algorithm	Avg NPCR (%)	Avg UACI (%)
WHT+DCT+Henon hybrid	97.633770	6.673282
DCT+Henon	98.640117	9.606827
WHT+Henon	98.272929	6.764031
Hénon+LWT	98.475951	15.568541

6.4 Average execution time

For the analysis of the computational efficiency of the implemented algorithms, Table 5 presents the average execution time values (in seconds) for each of the audio signal encryption methods.

Table 5 Average execution time (s) for different algorithms

Algorithm	Avg Execution Time (s)
WHT+DCT+Henon hybrid	1.7350
DCT+Henon	0.5448
WHT+Henon	0.3866
Hénon+LWT	0.7883

6.5 NIST Basic Randomness Tests

To assess the statistical randomness and cryptographic strength of the encrypted audio signals, tests from NIST SP 800-22 were applied. The results for the main method (DCT+WHT) are presented in Table 6.

Table 6 NIST SP 800-22 Results for DCT+WHT Method

File	Monobit (p)	Runs (p)	Block (p)
audio1.wav	0.25113 (PASS)	0.00000 (FAIL)	0.3496 (PASS)
audio2.wav	0.19845 (PASS)	0.00512 (FAIL)	0.007554 (FAIL)
audio3.wav	0.31211 (PASS)	0.02034 (PASS)	0.4982 (PASS)

6.6 Analysis of amplitude distribution (histogram)

For additional illustration of the statistical changes in the signal after encryption, Figures 4 and 5 show the amplitude histograms of the original and encrypted signals for the audio file p226001.wav.

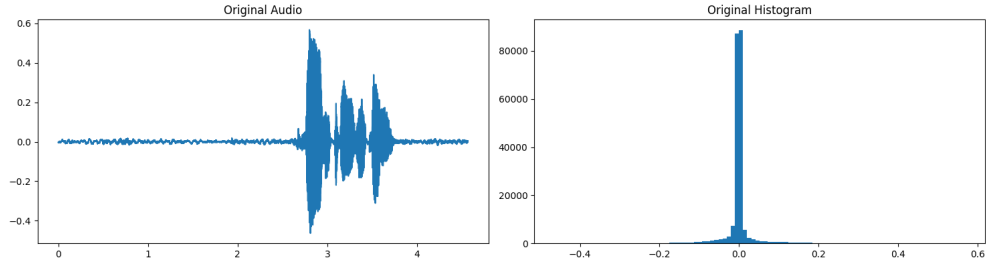


Fig. 4 Histogram of the original signal

The increase in entropy after encryption confirms the high degree of randomness introduced by the chaotic permutation and substitution. The almost complete

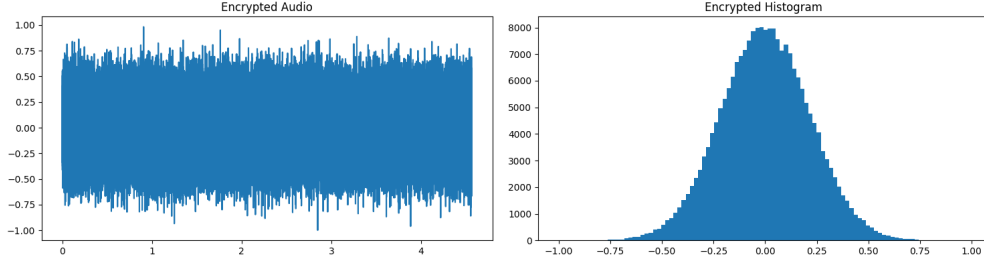


Fig. 5 Histogram of encrypted signal

restoration of the original entropy after decryption indicates the high accuracy of the method.

7 Pseudocode

Here we provide the detailed pseudocode of the proposed encryption and decryption algorithms.

Algorithm 1 EncryptAudio

```

1: procedure ENCRYPTAUDIO(audio,  $N$ )
2:   1. Segmentation:
     frames  $\leftarrow$  splitAudioToFrames(audio,  $N$ )
3:   for each  $f$  in frames do
4:     2. Transformations:
        $X \leftarrow \text{DCT}(f)$ 
        $W \leftarrow \text{WHT}(X)$ 
5:     3. Compute Statistics:
        $\sigma \leftarrow \text{computeStats}(f)$ 
6:     4. Permutation:
        $\pi \leftarrow \text{henonPerm}(\sigma, N)$ 
        $P \leftarrow \text{permute}(W, \pi)$ 
7:     5. Substitution:
        $K \leftarrow \text{logisticKeystream}(\sigma, N)$ 
        $E \leftarrow P \oplus K$ 
8:     storeEncryptedFrame( $E$ )
9:   end for
10:  6. Assembly:
     return framesToAudio( $\{E\}$ )
11: end procedure

```

Algorithm 2 DecryptAudio

```
1: procedure DECRYPTAUDIO(encryptedAudio,  $N$ , statsList)
2:   1. Segmentation:
   frames  $\leftarrow$  splitAudioToFrames(encryptedAudio,  $N$ )
3:   for each index  $i$  in frames do
4:      $\sigma \leftarrow$  statsList[ $i$ ]
5:     2. Inverse Substitution:
      $K \leftarrow$  logisticKeystream( $\sigma$ ,  $N$ )
      $P \leftarrow$  frames[ $i$ ]  $\oplus$   $K$ 
6:     3. Inverse Permutation:
      $\pi \leftarrow$  henonPerm( $\sigma$ ,  $N$ )
      $W \leftarrow$  invPermute( $P$ ,  $\pi$ )
7:     4. Inverse Transformations:
      $X \leftarrow$  WHT $^{-1}$ ( $W$ )
      $f \leftarrow$  DCT $^{-1}$ ( $X$ )
8:     storeRecoveredFrame( $f$ )
9:   end for
10:  5. Assembly:
  return framesToAudio( $\{f\}$ )
11: end procedure
```

8 Conclusion

The proposed audio encryption method, based on the hybrid application of the Walsh–Hadamard Transform (WHT), the Discrete Cosine Transform (DCT), as well as the Hénon and Logistic chaotic maps, demonstrates a balanced cryptographic strength, computational efficiency, and the ability to accurately reconstruct the encrypted signal. The comparative analysis with alternative methods (including DCT+Hénon, WHT+Hénon, and Hénon+LWT [12]) shows that the signal correlation is close to zero (within 6×10^{-4}), which indicates high resistance to unauthorized recovery without the key. At the same time, correct decryption fully restores the signal. From the perspective of cryptographic strength, the NPCR and UACI values of the hybrid method average 97.63% and 6.67%, respectively. These results are inferior to the Hénon+LWT method, especially in terms of UACI (15.57%), which indicates relatively low diffusion when the input data changes. However, the level of randomness of the encrypted signal, measured by entropy (5.18 bits), is comparable to other methods (although lower than DCT+Hénon with 6.65 bits). The key advantage of the hybrid method remains the balance between cryptographic reliability and efficiency. The average execution time is 1.735 seconds per file, which is justified given the more complex structure of the method. The fast computation of WHT and DCT (both with $\mathcal{O}(N \log N)$ complexity) is combined with strong chaotic permutations without the need to store an external key. Compared with the Hénon+LWT method, the hybrid approach shows better results in reconstruction metrics (RMSE, correlation) and also provides a more

compact and manageable architecture without using wavelets, which simplifies implementation. Thus, despite the need to enhance diffusion properties (in particular, increasing UACI), the proposed method can be considered an effective solution for secure, reversible, and computationally feasible audio encryption. The proposed hybrid approach achieves the best values in terms of signal reconstruction metrics: the Root Mean Square Error (RMSE) is 0.0205, and the correlation between the original and decrypted signals.

9 Limitations of the method

Despite the high reconstruction accuracy and acceptable cryptographic strength, the proposed hybrid method has a number of limitations identified during the comparative analysis:

- **Relatively low UACI value:** on average 6.67%, compared to the Hénon+LWT method (15.57%). This indicates insufficient diffusion, i.e., weak sensitivity of the encrypted signal to small changes in the original input.
- **Moderate entropy:** the encrypted signal has an average Shannon entropy of 5.18 bits, which remains below the theoretical maximum of 8 bits, indicating partial predictability in the cipher structure.
- **NIST SP 800-22 tests not fully passed:** some tests, in particular Runs and Block Frequency, were not successfully passed in the current implementation, indicating possible insufficient randomness of the pseudorandom key stream or substitution structure.
- **Higher execution time:** compared to other methods (for example, DCT+Henon: 0.5448 s), which may be critical for real-time applications or resource-constrained systems.

Plans for Improvement

- Strengthen the substitution block by adding additional nonlinear components (for example, synchronous combination of the Logistic map with the Sine or Tangent map) to increase diffusion and improve UACI.
- Perform optimization of the pseudorandom sequence generation algorithm to ensure successful completion of all NIST SP 800-22 tests.
- Investigate the possibility of replacing one of the transforms (for example, DCT or WHT) with a more powerful combination that provides both high entropy and low execution time.

Thus, while maintaining high reconstruction quality and good overall robustness, the method requires further development in terms of improving diffusion characteristics and enhancing statistical randomness, which will be the subject of future research. More detailed issues related to the application of the Walsh–Hadamard Transform in information security tasks are discussed in the works [4–6, 8, 13].

Conflict of Interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

Data transparency

All data, materials, and code used in this study comply with field standards and are accurately described in the manuscript.

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Author Contributions

Sergo Episkoposyan: Statement of the problem, proofreading of the manuscript, scientific supervision.

Davit Hovhannisyan: Algorithm development, programming, article design.

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