1. MC Question

a). True Integrated Value

True integral value when a=0, b=1: 1.6

True integral value when a=0, b=2: 2.801626

b). With random seed:

if n=100

MC integral value when a=0, b=1: 1.566659

MC integral value when a=0, b=2: 2.754586

if n=1000

MC integral value when a=0, b=1: 1.586322

MC integral value when a=0, b=2: 2.793915

If n=10000

MC integral value when a=0, b=1: 1.604865

MC integral value when a=0, b=2: 2.817034

Without random seed:

if n=100

MC integral value when a=0, b=1: 1.593295

MC integral value when a=0, b=2: 2.837323

if n=1000

MC integral value when a=0, b=1: 1.591592

MC integral value when a=0, b=2: 2.813356

If n=10000

MC integral value when a=0, b=1: 1.59407

MC integral value when a=0, b=2: 2.818247

c). Comments:

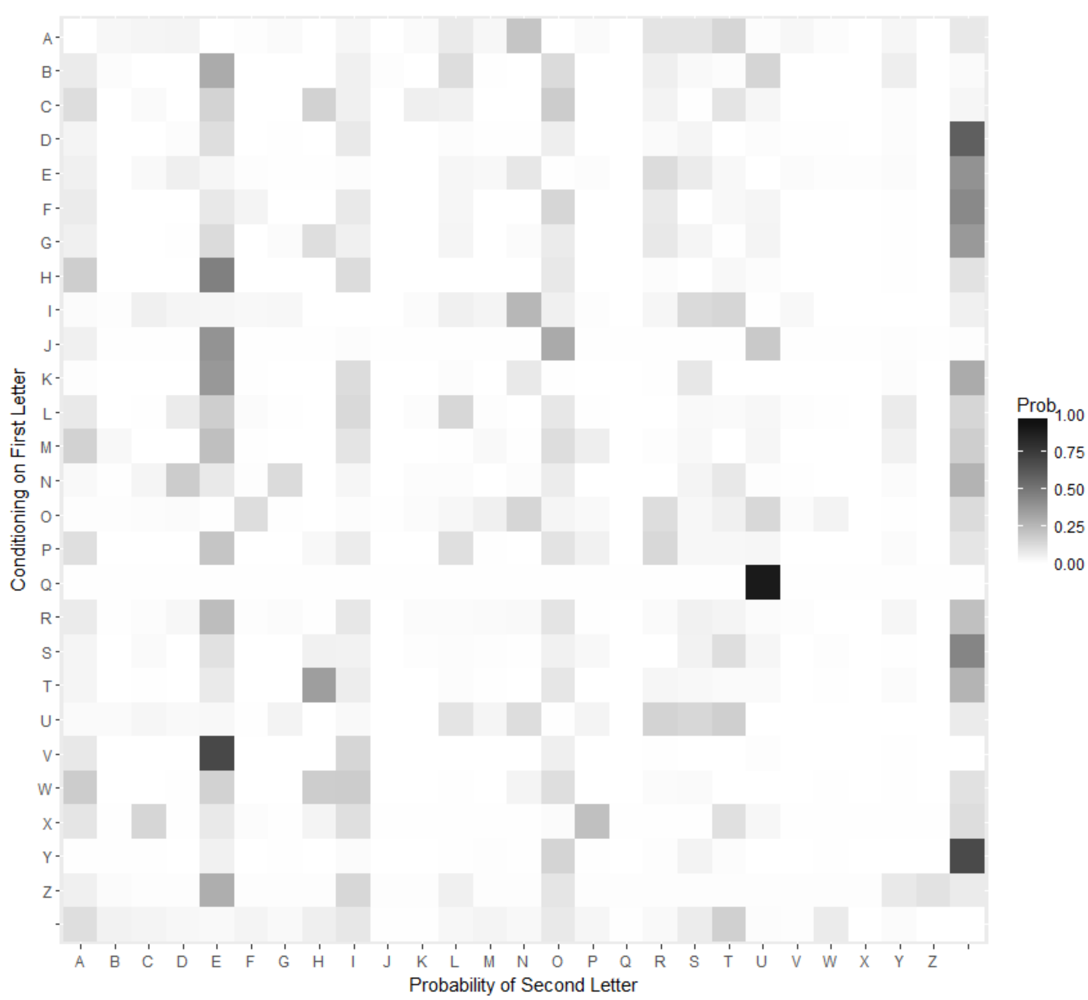
When a random seed is given, the largest n always corresponds to the largest results or, the best simulation to the true results.

If no random seed is specified, the result of MC integral values are unstable, i.e., sometimes the largest results (or the best simulation to the true results) are not generated by the largest n. However, if you run the code several times, you can find that in most cases the optimal results tend to be generated under largest n.

2. MCMC Question

a). the name of book downloaded: Windfalls.txt

transition matrix plot:



b).

codedQuote="RQG LWY C AGG CR CF YOP WXG BOCJB RO EPCDM W RCVG VWNQCJG CJRO W NWX LQY JOR MO CR LCRQ AOVG ARYDG"

the final decoded version of the quote:

" THE WAY I SEE IT IF YOU ARE GOING TO BUILD A TIME MACHINE INTO A CAR WHY NOT DO IT WITH SOME STYLE"

c). results after the jump probability is doubled:

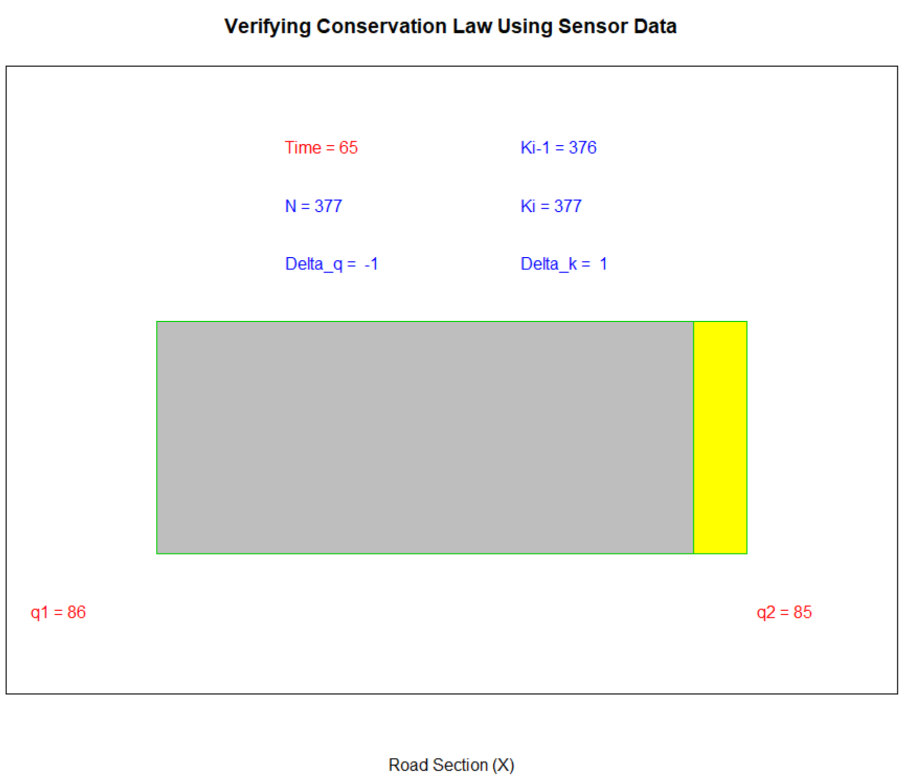
"THE GAR I CEE IT IK ROM ALE DOIND TO UMISP A TIVE VAWHINE INTO A WAL GHR NOT PO IT GITH COVE CTRSE"

The results seems to be much worse than that before doubling the jump probability, since a lot of decoded words are not valid English words. Since the jump probability is two times greater, it is much easier to accept next steps in the Markov Chain Monte Carlo process. Therefore, the letter for each position changes much more drastically and it is much harder for the whole process to converge to the correct answer.

3. Conservation Law

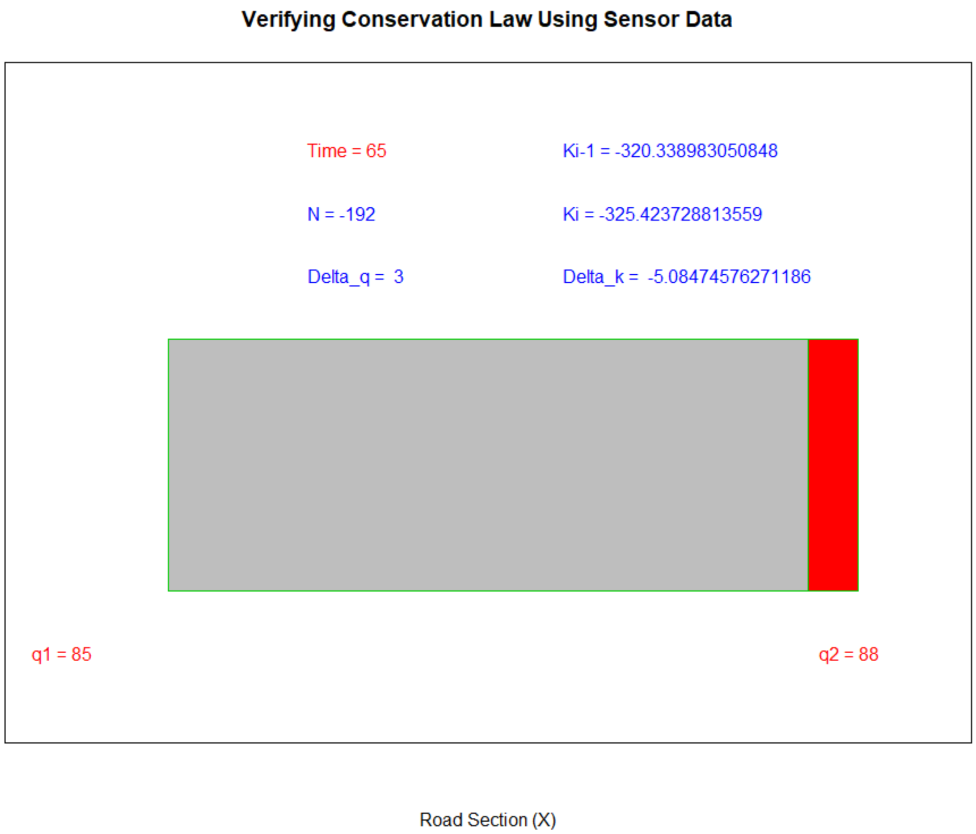
a). The simulation results shows that the conservation results still holds at time step 65.

Plot at time step = 65



b). The simulation results shows that the conservation results does not hold at time step 65.

Plot at time step 65:



c). Comments:

expression for the conservation law:

$$\frac{dq}{dx} + \frac{dk}{dt} = 0$$

In the code the density is calculated as k = $\frac{N+q1-q2}{dX}$, and the number of cars N_0 is updated in each iteration by adding q1 and subtracting q2. In scenario b, the number of car N becomes below zero which indicates that after going through the preceding iterations, the rate that cars leave the road section is greater than the rate of entering the section. This can also explains why the change in density k is a negative value because as more cars leave the road section the traffic become less denser. The unbalanced inflow and outflow may be the most prominent contributor to the failure of conservation law in scenario b. In contrast, in scenario a the change in density k is larger than zero and the magnitudes of delta q and delta k are much smaller than those in scenario b, indicating that the inflow and outflow are much more balanced in scenario a, which in turn supports the conservation law.