Problem 1

(a). the flow of vehicle on level section

q (vph) = total number of vehicles / time (hr)

= number of trucks * speed of trucks + number of cars * speed of cars

= 10% * 50 * 40mi/hr + 90% * 50 * 50 mi/hr

= 200 + 2250

= 2450 vehicle / hr

(b). The density of vehicles on the grade

From the given information we know that the flow on level section equals to the flow on the grade.

k_{grade}^{tot} = $k_{grade}^{truck} + k_{grade}^{car}$

$= \frac{truck\ flow\ on\ the\ grade}{truck\ speed\ on\ the\ grade} + \frac{car\ flow\ on\ the\ grade}{car\ speed\ on\ the\ grade}$

$= \frac{truck\ flow\ on\ level\ section}{truck\ speed\ on\ the\ grade} + \frac{car\ flow\ on\ level\ section}{car\ speed\ on\ the\ grade}$

$= \frac{200}{20} + \frac{2250}{40}$

= 10 + 56.25

= 66.25 vehicle / mile

(c). the percent of trucks on the grade seen on the photo

$\text{fraction}_{\text{truck}} = \frac{density\ of\ trucks\ on\ the\ grade}{total\ density\ of\ vehicles\ on\ the\ grade} = \frac{10}{66.25}$ = 15.1%

(d). the percent of trucks on the grade seen by a stationary observer:

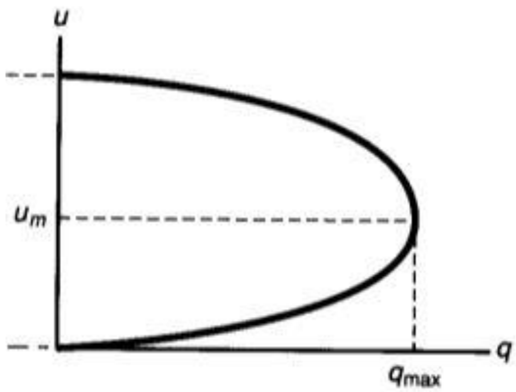
$\text{fraction}_{\text{truck}} = \frac{flow\ of\ trucks\ on\ the\ grade}{total\ flow\ of\ vehicles\ on\ the\ grade} = \frac{200}{2450}$ = 8.2%

Problem 2

Greenshields' Model

(a). according to Greenshields' Model, velocity is negatively related to density. Hence when v drops, the density will increase.

(b). the relation between velocity and flow is bit more complicated. According to Greenshields' Model, the relationship can be depicted as follows:



If the initial velocity $v < U_m$, then $q < q_{max}$. Hence if v decreases, q can only decrease.

If $v > U_m$, and $v' > U_m$, then decreasing v will drive q approaching q_{max} which leads to an increasing q.

If $v > U_m$ and $v' < U_m$:

If $| v - U_m| > |v' - U_m|$ then q will increase;

If $| v - U_m| = |v' - U_m|$ then q will remain the same;

If $| v - U_m| < |v' - U_m|$ then q will decrease.

Problem 3

(a). Since $v = \frac{q}{k}$, if the given formula of k and q are true, dividing the q formula by k we have $v = q_0/k_0$ which is a constant that never changes with time. However, since q and k are all functions of time and displacement x, they must change with time, which leads to a changing velocity. Hence the formula are physically impossible.

(b). the formula of q tells us that with time going by q will continuously decrease, indicating that the speed of the traffic increases. The speed in turn suggest a decreasing density, which can be deduced from the second formula, since as displacement x grows the density decrease. Hence this set of formula is physically possible.

(c).

Problem 4

(a). delay and time in queue

Velocity of free flow V_f = 1 mile/min = 60 mile/hr

Velocity of vehicles moving in the queue V_μ = 50 veh/min = 300veh/hr

Length of the queue dQ = 0.5mile or dQ = 100 vehicles

So the time taken for a vehicle moving from the end to the head of the queue is t = 100/300=0.33hr

Then the velocity of a vehicle in the queue in mile/hr is: V_μ = 0.5mile/0.33hr = 1.52 miles/hr

Hence the delay is:

$$W = \left(\frac{1}{V\mu} - \frac{1}{Vf}\right) * dQ = \left(\frac{1}{1.52} - \frac{1}{60}\right) * 0.5 = 0.32 \text{ hr}$$

And the time for a vehicle spend in the queue is:

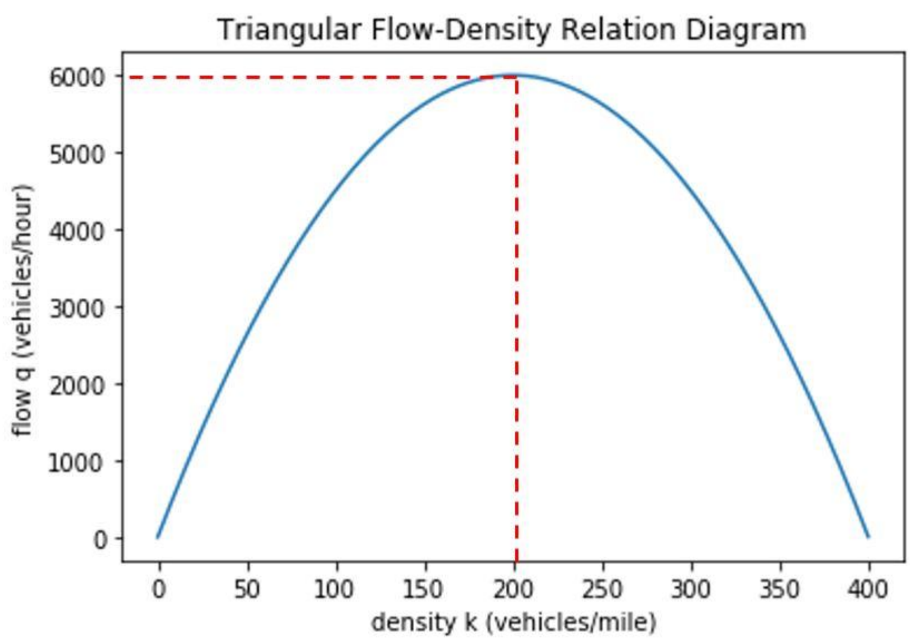
$$tQ = \frac{W}{1-\frac{V\mu}{Vf}} = \frac{0.32}{1-\frac{1.52}{60}} = 0.33 \text{ hr}$$

(b). the (average) density of vehicles in the queue

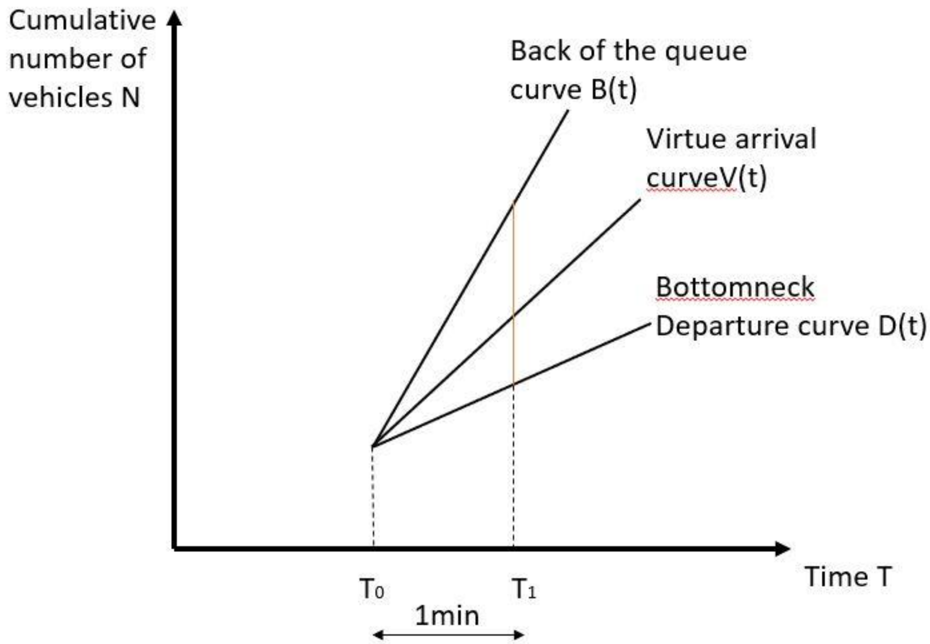
K = (number of vehicles in the queue) / (length of the queue in miles) = 100 / 0.5 = 200 veh/hr

(c). triangular flow-density relation diagram

We use the Greenshields' model to interpret the relationships between the key variables. We assume that the velocity in free flow is the maximum velocity the traffic can reach when the density equals zero, which is V_f = 1 mile/min = 60 mile/hr. the maximum flow is given as q_m = 100 veh/min = 6000 veh/hr. Hence, according to the formula $q_m = \frac{kj*Vf}{4}$, where k_j is the maximum density when velocity equals zero, we can get k_j = 400 veh/mile. Finally ew can get a quadratic formula that describes the relationship between flow q and density k: $q = V_f * k - \frac{Vf}{kj} * k^2 = 60k - 0.15k^2$. the triangular plot is shown as follow:



(d).



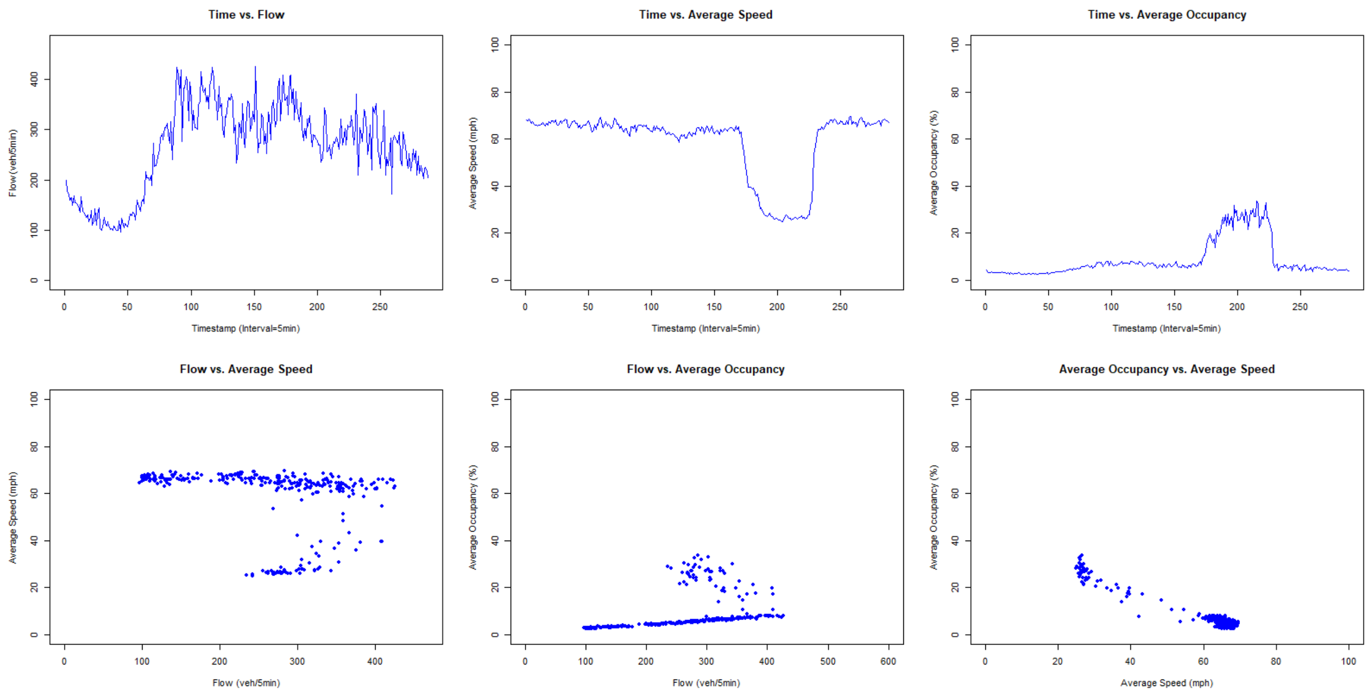
Since the vehicles in the queue move at a rate of 50 veh/min, and there are 100 vehicles when A join the queue, then after 1 min the A would be at 50 vehicles from the bottleneck. Since the upstream flow is 80 vehicles per minute, so in this I minute there will be 80 vehicles coming from upstream staying behind A in the queue. Hence there will be 50 + 80 = 130 vehicles in the queue at 1 minute. When there are 100 vehicles in the queue the length of the queue is 0.5 miles, so a queue of 130 vehicles will be 0.5 * (130/100) = 0.65 mile. Hence vehicle B will be at 0.65 mile upstream from the bottleneck.

Computational Questions

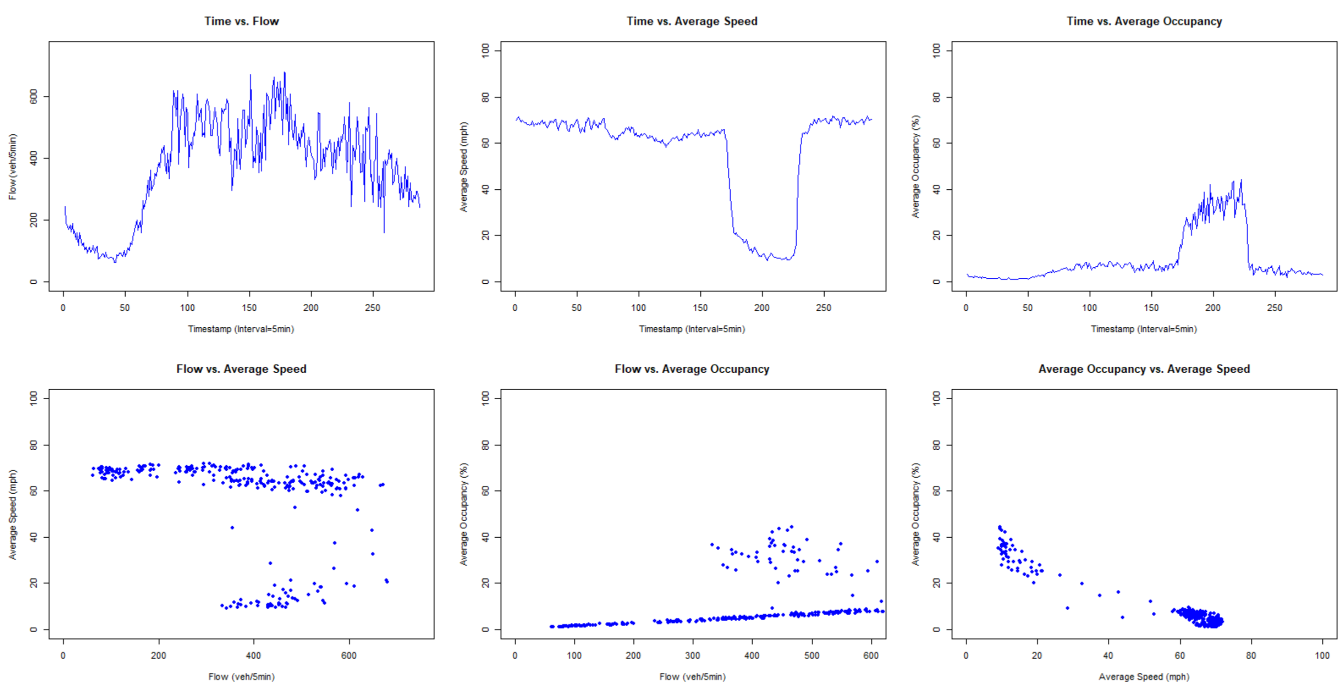
Problem 1

District 4 is selected for this question. Two stations are selected: 400000 and 400001

Plots for station 400000:

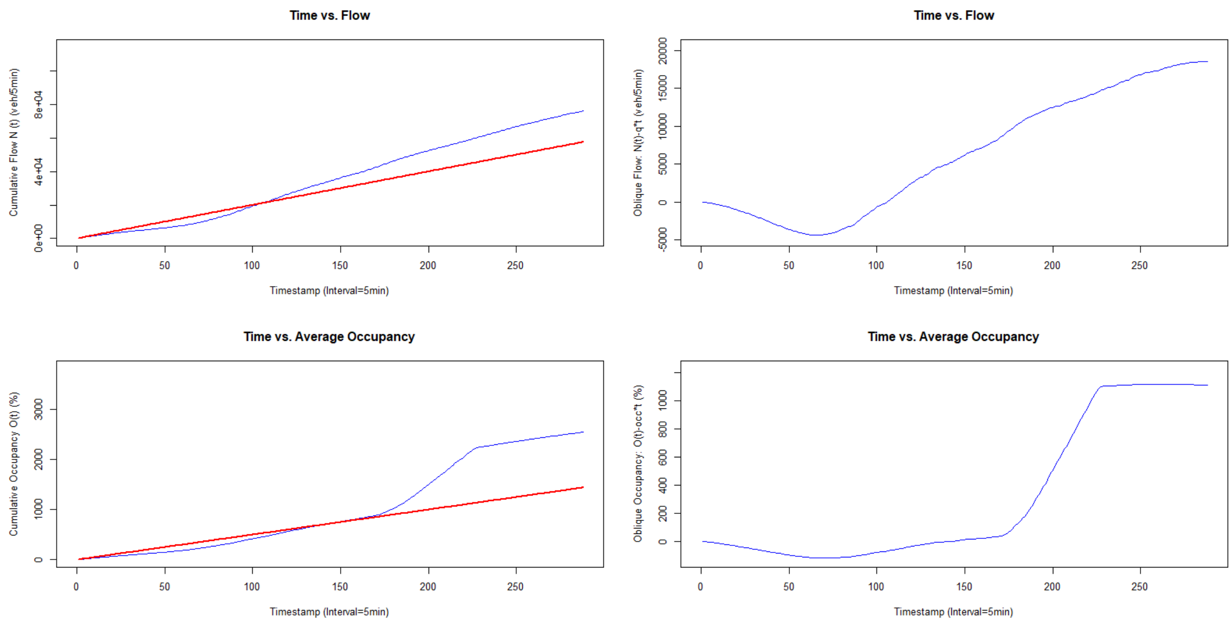


Plots for station 400001:

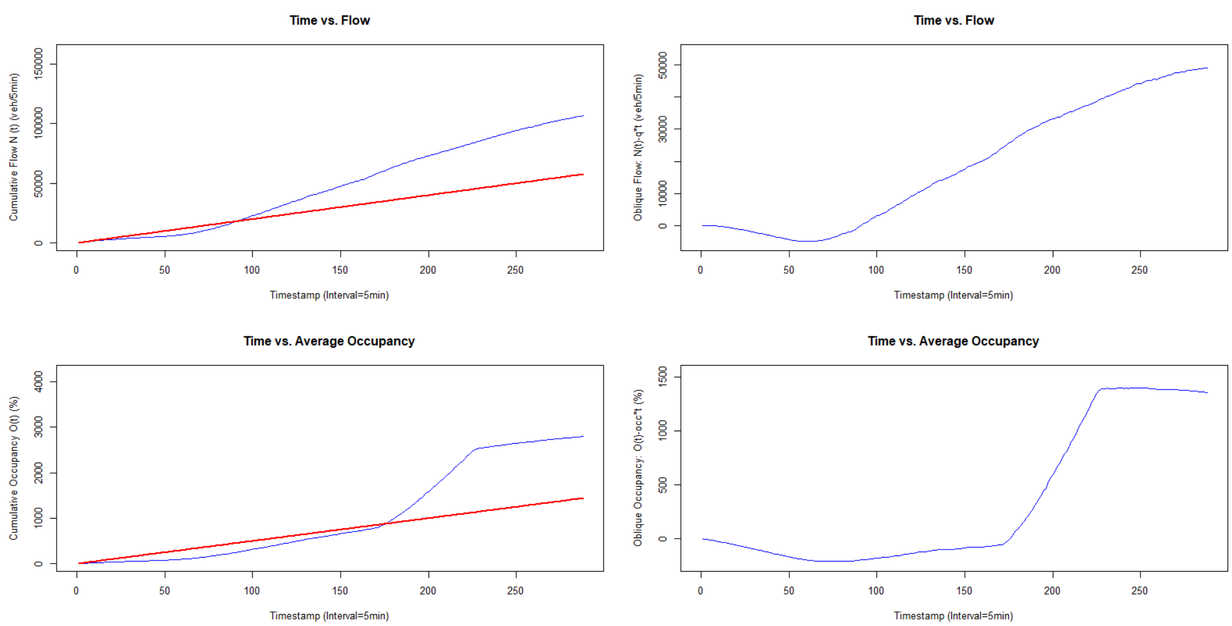


Problem 2

Result plots for station 403430:



Result plots for station 401513:



Traffic flow description:

From the figures above, we can infer that the observed cumulative occupancy curve and cumulative flow curve eventually surpass their respective reference curves, indicating that there may be queuing in the traffic.