@NTU

Exploring Links Between Lattice-based NIZKs and Various Signature Schemes (in the Standard Model)

Shuichi Katsumata (AIST, PQShield)

Overview of This Talk

1. Non-Interactive Zero-Knowledge (NIZK)

2. Links Between NIZKs and Signatures

3. Result: Group Signatures w/o CRS-NIZK

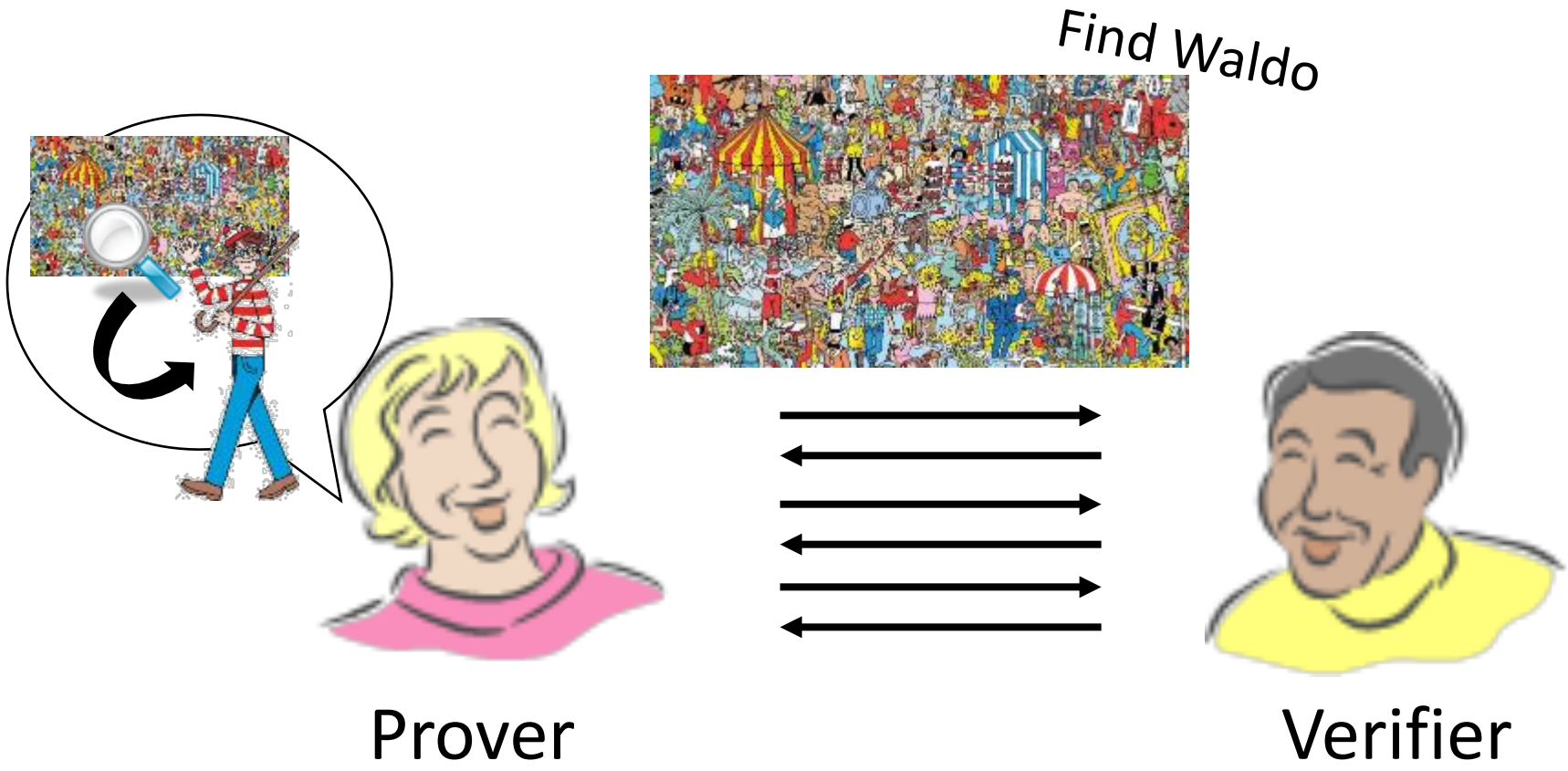
[KY19@EC]

- New Notion:
  Multi-User Designated-Prover NIZKs

1. Introduction:

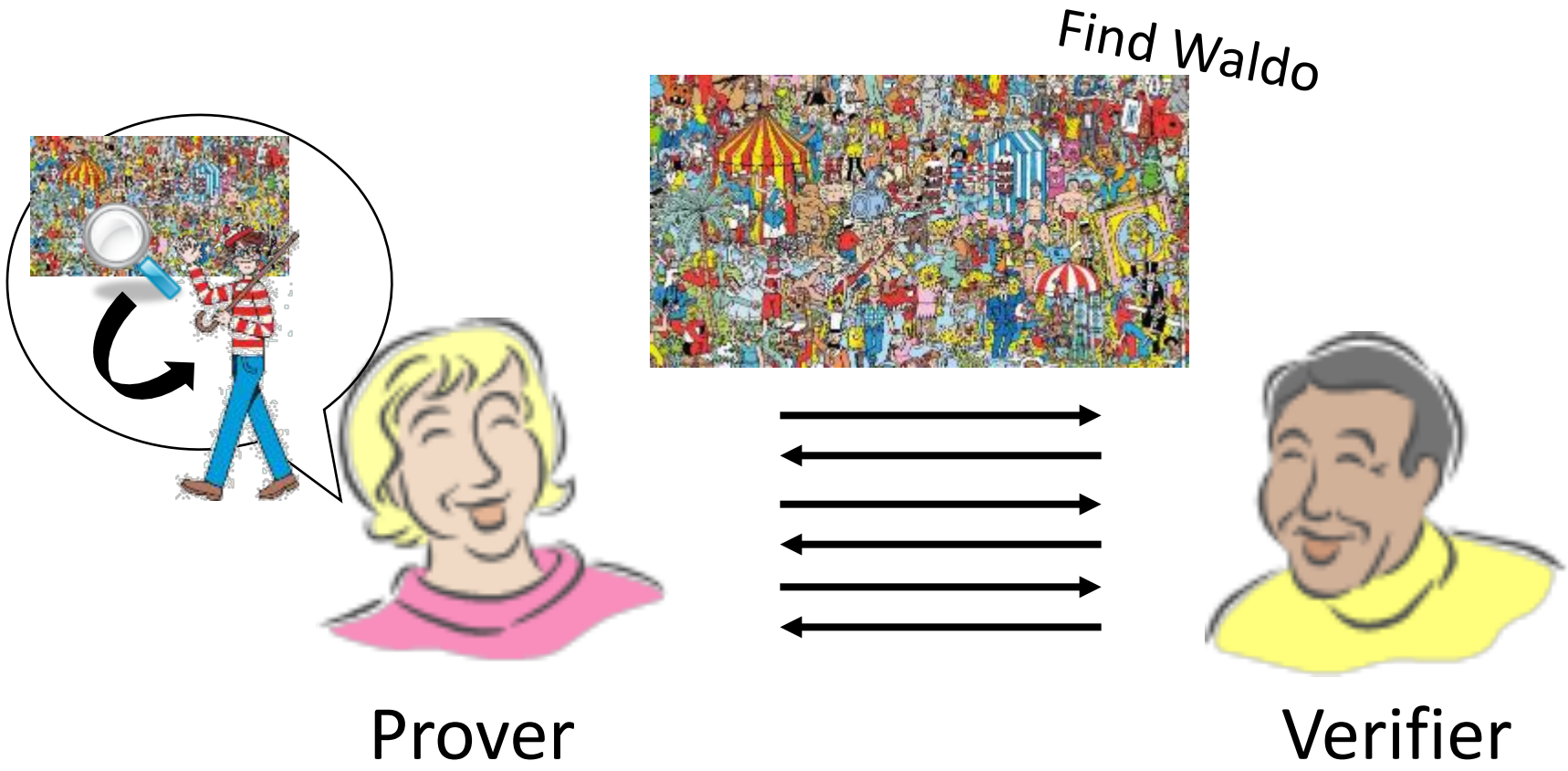
Non-Interactive Zero-Knowledge

Zero-Knowledge Proof Systems



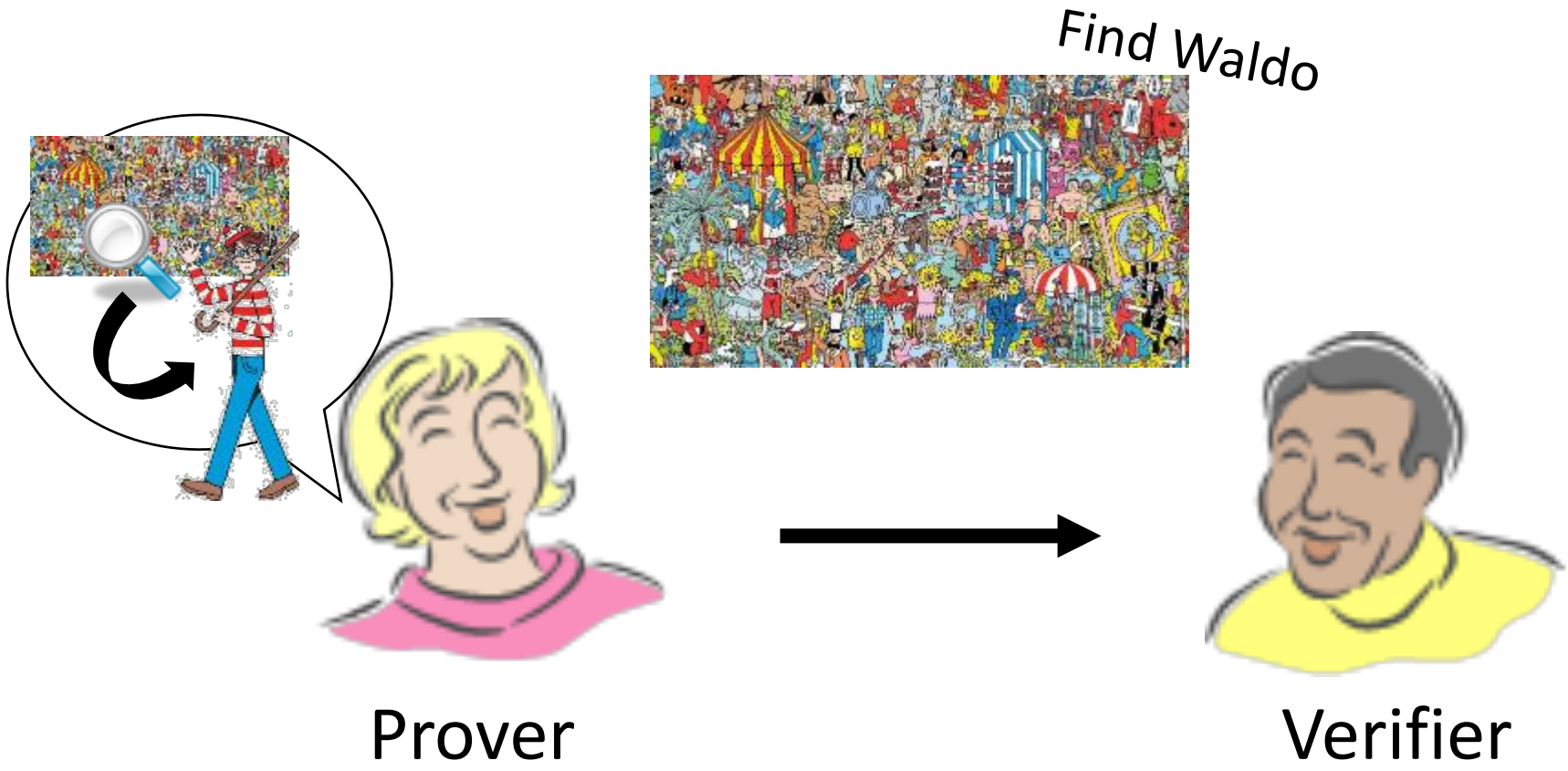
- Verifier is convinced that Prover knows where Waldo is.

Zero-Knowledge Proof Systems



- Verifier is convinced that Prover knows where Waldo is.
- …BUT, Verifier doesn't learn where Waldo is!

Non-Interactive ZK (NIZK)



- Prover **<u>sends only one message</u>** to Verifier.

More Formally: NIZKs

$x \in L$

Prover (x, w) Verifier x

$\xrightarrow{\pi}$

✓ **Completeness**: If $(x, w) \in R_L$, then Verifier is convinced.

More Formally: NIZKs

Cheating Prover x

$x \notin L$

Verifier x

π

➡ Reject π.

- ✓ **Completeness**: If $(x, w) \in R_L$, then Verifier is convinced.
- ✓ **Soundness**: If $x \notin L$, cheating Prover cannot convince Verifier.

More Formally: NIZKs

$x \in L$

Prover (x, w)

π

Verifier x

- ✓ **Completeness**: If $(x, w) \in R_L$, then Verifier is convinced.
- ✓ **Soundness**: If $x \notin L$, cheating Prover cannot convince Verifier.
- ✓ **Zero-Knowledge**: If $x \in L$, Verifier only learns that $x \in L$.

Motivation for NIZK

Many Applications of NIZKs

- OWF + NIZK ⇒ signature scheme [BG89@CRYPTO]
- CPA-PKE + NIZK ⇒ CCA-PKE [NY90@STOC]
- Semi-honest secure MPC + NIZK ⇒ Malicious secure MPC [GMW86@CRYPTO]
- ….

Theoretical Interest

- Connections with complexity theory

Building NIZKs (for all of NP)

Do not exist w/o trusted setup! ☹ [GO94]

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- Random Oracle Model [FS87]
  - Practically appealing solution.
- With Trusted Setup [FLS90]
  - Provable security.
  - Theoretically appealing solution.

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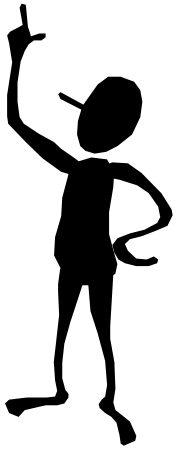
This Talk



Various Types of Trusted Setup

→ **CRS**: (public) common reference string

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CRS, $x \in L$

Prover (x, w)

$\xrightarrow{\pi}$

Verifier x

CRS-NIZK

(Most standard NIZK)

Various Types of Trusted Setup

CRS: (public) common reference string

$\mathbf{k_v}$: (private) verification key

Various Types of Trusted Setup

CRS: (public) common reference string

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Prover (x, w)

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Verifier x

π

Designated Verifier-NIZK (DV-NIZK)

$\Rightarrow$Require private $\mathrm{k_v}$ to verify proof π!

Various Types of Trusted Setup

CRS: (public) common reference string

$\mathbf{k_p}$: (private) proving key

Prover (x, w)

CRS, $x \in L$

Verifier x

$\xrightarrow{\pi}$

Designated Prover-NIZK (DP-NIZK)

$\Rightarrow$Require private $\mathrm{k_p}$ to generate proof π!

*Subtleties in (DV, DP)-NIZKs

Since verifier/prover maintains secret information, definition requires more care than CRS-NIZK.

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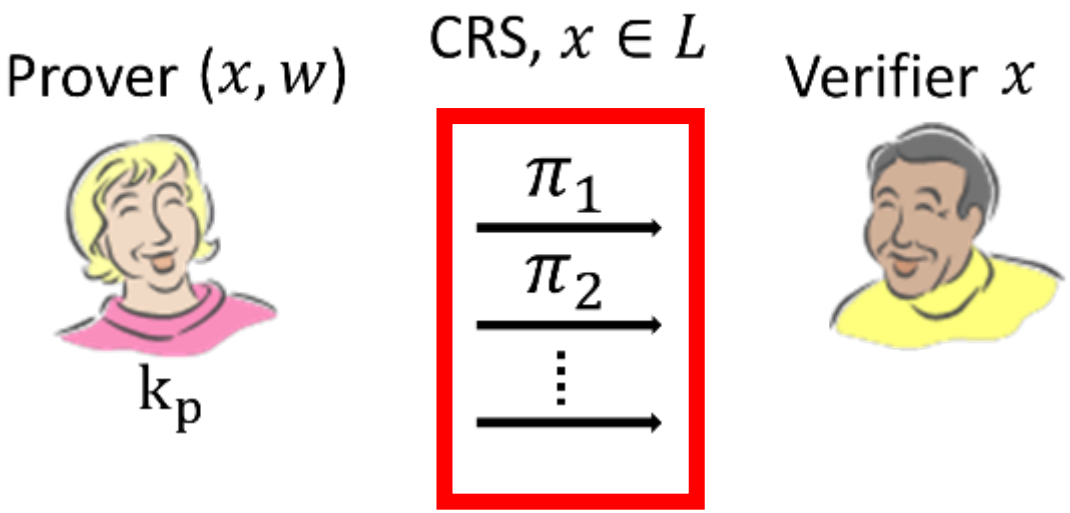

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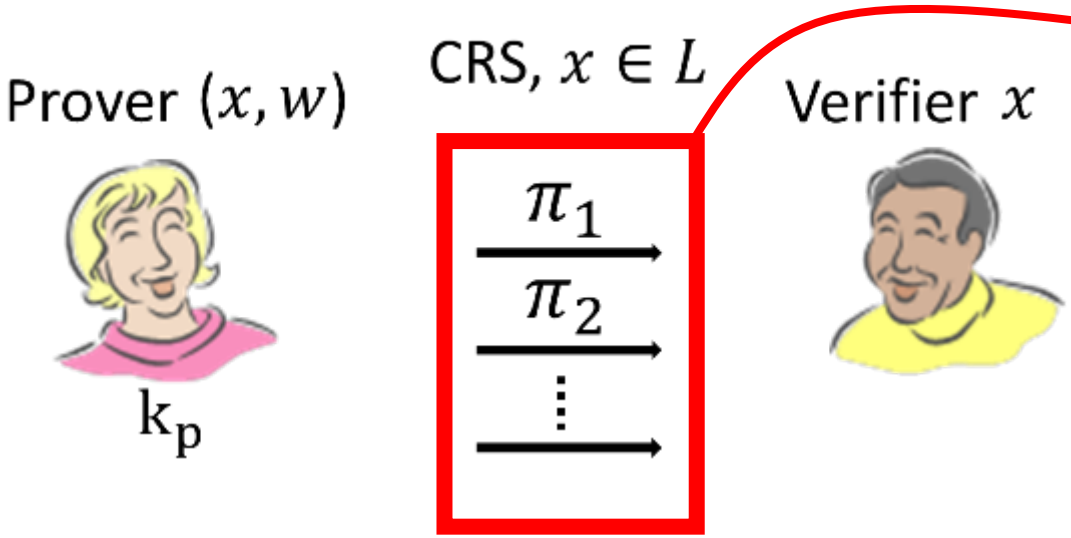

Each proof may leak some information of k_p!

***Consequence**
Parallel repetition may fail.

State-of-Affairs

*Non Exhaustive

1990

CRS-NIZK

Trapdoor Permutations [FLS90, BY96, G04]

DV-NIZK

DP-NIZK

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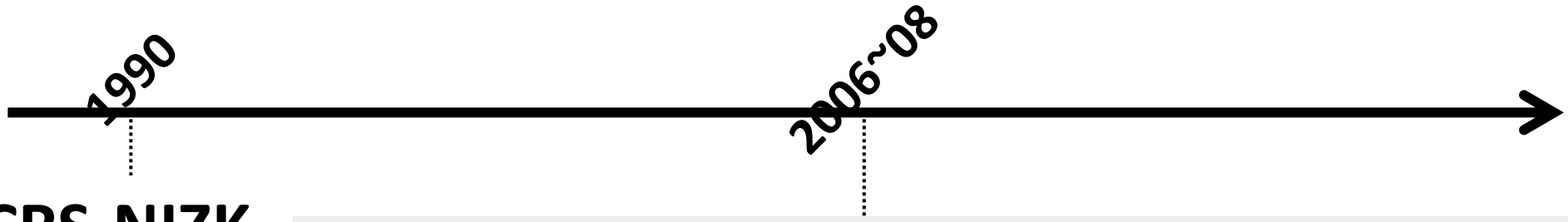

CRS-NIZK

Pairings [GOS06, GS08,…]

Trapdoor Permutations [FLS90, BY96, G04]

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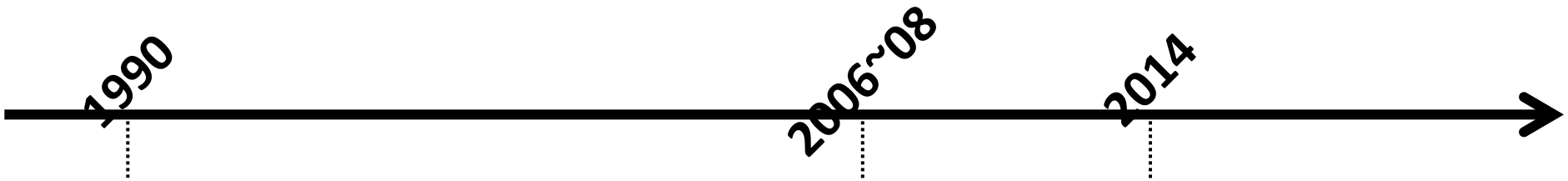

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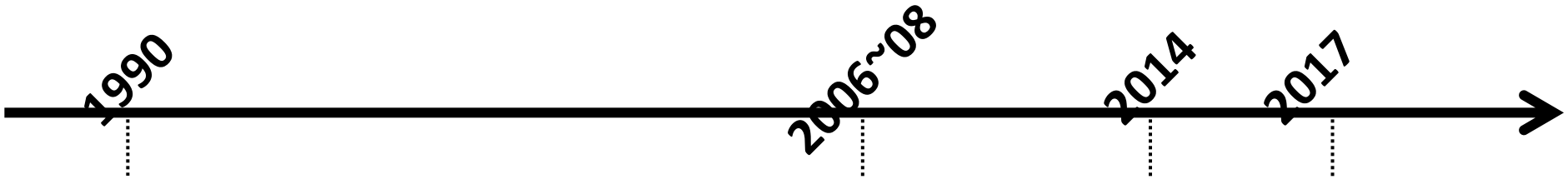

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Correlation Interactable Hash [KRR17,…]

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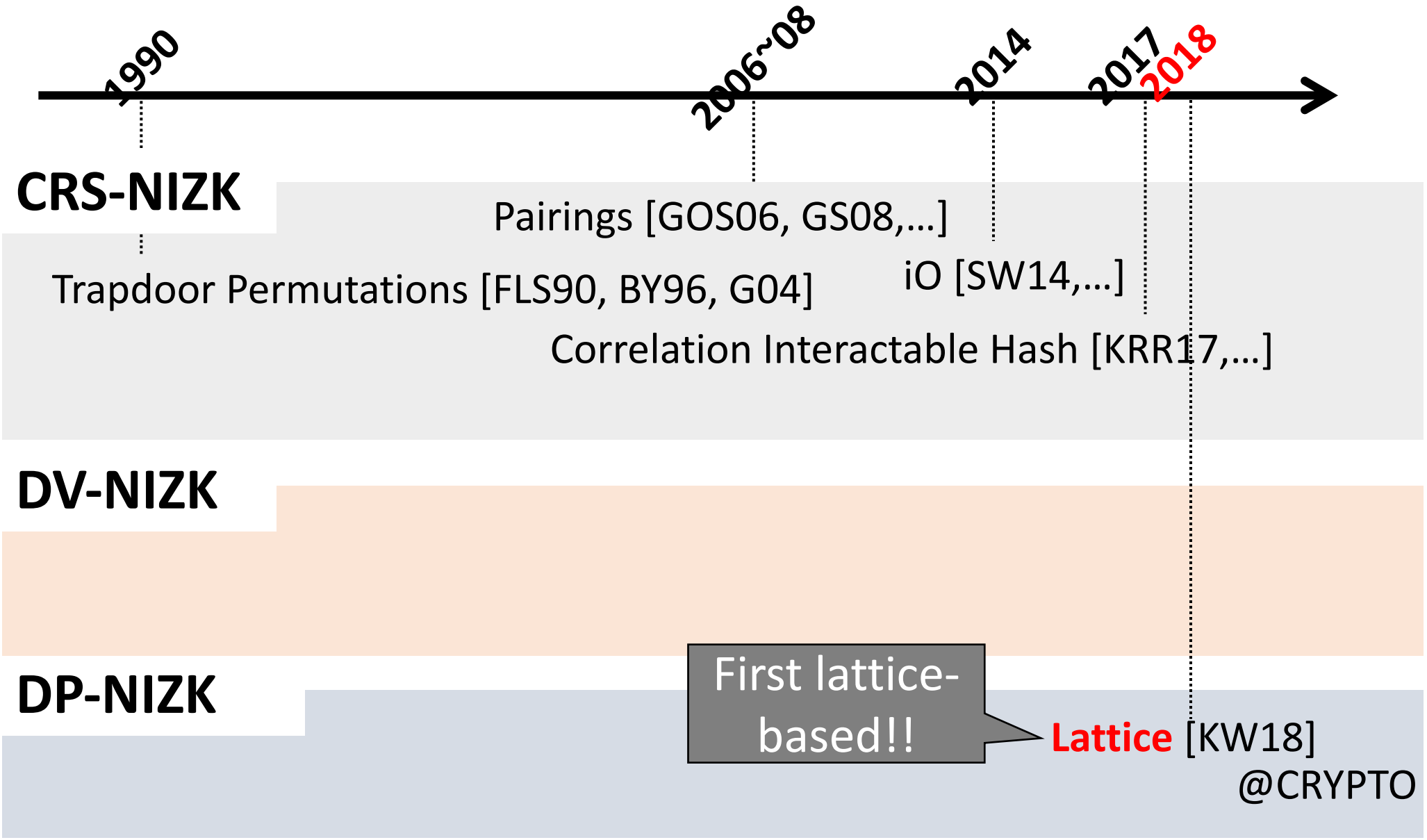

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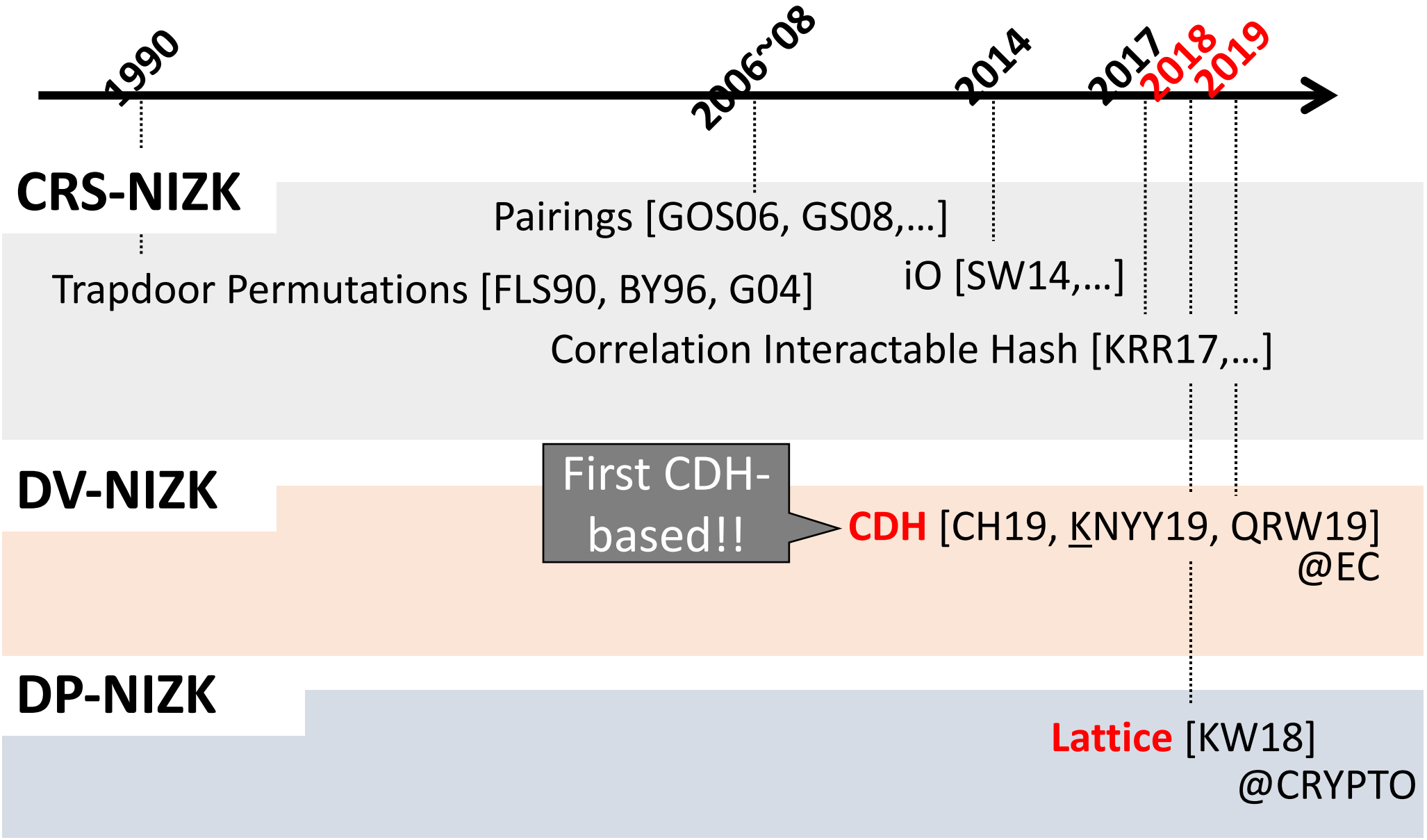

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Correlation Interactable Hash [KRR17,…]

Lattice [PS19] @CRYPTO

Finally!!

DV-NIZK

CDH [CH19, KNYY19, QRW19] @EC

Lattice [LQRWW19] @CRYPTO

DP-NIZK

Lattice [KW18] @CRYPTO

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DP-NIZK

Lattice [KW18] @CRYPTO

Is [KW18] and [LQRWW19] simply a special case of [PS19]??

Closer Look at Lattice-based NIZKs

- CRS-NIZK [PS19]
  - Follows the correlation interactable hash paradigm.
  - Based on **polynomial LWE** **(Learning with Errors)**.

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  - Generic construction from Fully-Homomorphic Signatures.
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  - Generic construction from Fully-Homomorphic Signatures.
  - Based on **polynomial SIS** **(Short Integer Solution)**.

➡ From a theoretical stand point,
DP-NIZK requires the **weakest assumption**!

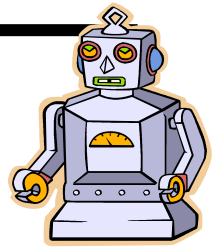
*The state-of-the-art can change anytime! ☺

SIS vs LWE

SIS

- One way function
- Collision resistant hash
- Digital signature scheme
- Attribute-based Signature
- Fully homomorphic signature
- …

LWE



- Public key encryption
- Oblivious transfer
- Attribute-based Encryption
- Fully homomorphic encryption
- …

+

Whatever SIS can.

*In a world of PT quantum algorithms, LWE and SIS are equivalent.

2. Exploration:

Links Between NIZKs and Signature Schemes

Warm Up: Standard Signatures

Well Known Fact…

OWF + **CRS**-NIZK ⇒ Signature Schemes [BG89]

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However,

OWF + **DP**-NIZK ⇒ Signature Schemes

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Well Known Fact…

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However,

OWF + DP-NIZK ⇒ Signature Schemes

Why? At a high level…

In a signature scheme, the **keys are generated honestly** and **secret key is never revealed** to an adversary.

Warm Up: Standard Signatures

Well Known Fact…

OWF + CRS-NIZK ⇒ Signature Schemes [BG89]

→ **LWE**

However,

OWF + DP-NIZK ⇒ Signature Schemes

→ **SIS**

Why? At a high level…

In a signat

honestly a

an adversa

Aligns with prior knowledge that SIS implies signature schemes.

In a Bit More Detail…

*Use fact that OWF implies MAC, COM

Signer

$$\mathrm{vk} = (\mathrm{crs}_{\mathrm{DP}}, \mathrm{com}_{\mathrm{sk}_{\mathrm{MAC}}})$$

$$\mathrm{sk} = (\mathrm{k}_{\mathrm{P}}, \mathrm{sk}_{\mathrm{MAC}})$$

Verifier

In a Bit More Detail…

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$$\mathrm{vk} = (\mathrm{crs}_{\mathrm{DP}}, \mathrm{com}_{\mathrm{sk}_{\mathrm{MAC}}})$$

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$$\mathrm{sk} = (\mathrm{k}_{\mathrm{P}}, \mathrm{sk}_{\mathrm{MAC}})$$

$\mathbf{Sign}(\mathrm{sk}, \mathrm{M})$:

1. $\sigma_{\mathrm{M}} \leftarrow \mathrm{Sign}_{\mathrm{MAC}}(\mathrm{sk}_{\mathrm{MAC}}, \mathrm{M})$

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$\mathbf{Sign}(\mathrm{sk}, \mathrm{M})$:

1. $\sigma_{\mathrm{M}} \leftarrow \mathrm{Sign}_{\mathrm{MAC}}(\mathrm{sk}_{\mathrm{MAC}}, \mathrm{M})$
2. $\pi \leftarrow \mathrm{Prove}(\mathrm{x} = (\mathrm{M}, \sigma_{\mathrm{M}}, \mathrm{com}_{\mathrm{sk}_{\mathrm{MAC}}}), \mathrm{w} = \mathrm{sk}_{\mathrm{MAC}})$

$$\boxed{\begin{aligned}(\mathrm{x}, \mathrm{w}) \in R \Leftrightarrow (&\mathrm{Verify}_{\mathrm{MAC}}(\mathrm{sk}_{\mathrm{MAC}}, \mathrm{M}, \sigma_{\mathrm{M}}) = \top \\ &\wedge \mathrm{com}_{\mathrm{sk}_{\mathrm{MAC}}} = \mathrm{COM}(\mathrm{sk}_{\mathrm{MAC}}))\end{aligned}}$$

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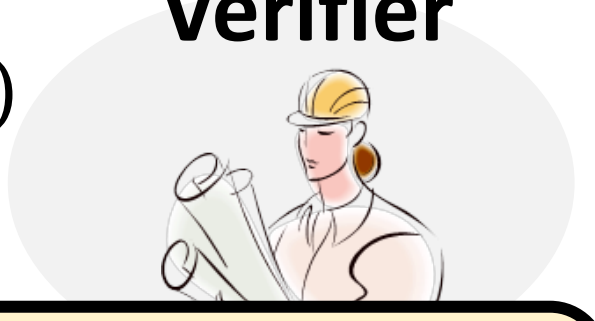
$\mathrm{vk} = (\mathrm{crs}_{\mathrm{DP}}, \mathrm{com}_{\mathrm{sk}_{\mathrm{MAC}}})$

Verifier



$\mathrm{sk} = (\mathrm{k}_{\mathrm{P}}, \mathrm{sk}_{\mathrm{MAC}})$

Prove that

✓Signature is valid

✓Signed using the committed $\mathrm{sk}_{\mathrm{MAC}}$

Sign$(\mathrm{sk}, \mathrm{M})$:

1. $\sigma_{\mathrm{M}} \leftarrow \mathrm{Sign}_{\mathrm{MAC}}(\mathrm{sk}_{\mathrm{MAC}}, \mathrm{M})$
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In a Bit More Detail…

*Use fact that OWF implies MAC, COM

Signer

$\mathrm{sk} = (\mathrm{k_P}, \mathrm{sk_{MAC}})$

$\mathrm{vk} = (\mathrm{crs_{DP}}, \mathrm{com_{sk_{MAC}}})$

$= (\pi, \sigma_\mathrm{M})$

Verifier

Sign$(\mathrm{sk}, \mathrm{M})$:

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Verify$(\mathrm{vk}, \mathrm{M},$ $)$:

Check validity of proof

In a Bit More Detail…

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Verifier

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):

f proof

Take Away

DP-NIZK suffices since the "**signer**" is the "**designated prover**".

Lattice NIZKs and Signatures (Feasibility)

Fully-Hom. Signature

↓ [KW18]

DP-NIZK **SIS**

Correlation Interactable Hash + Fiat-Shamir

↓ [PS 19]

CRS-NIZK **LWE**

*Ignore DV-NIZK since it doesn't seem useful for signatures.

Lattice NIZKs and Signatures (Feasibility)

Fully-Hom. Signature

↓ [KW18]

DP-NIZK **SIS**

↓

Digital Signature

Correlation Interactable Hash + Fiat-Shamir

↓ [PS 19]

CRS-NIZK **LWE**

↓ [BG89]

Digital Signature

*All arrow assumes "+OWF"

Lattice NIZKs and Signatures (Feasibility)

Fully-Hom. Signature

↓ [KW18]

DP-NIZK

↓

Digital Signature

? →

Attribute-based Signature

SIS

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[MPR11] →

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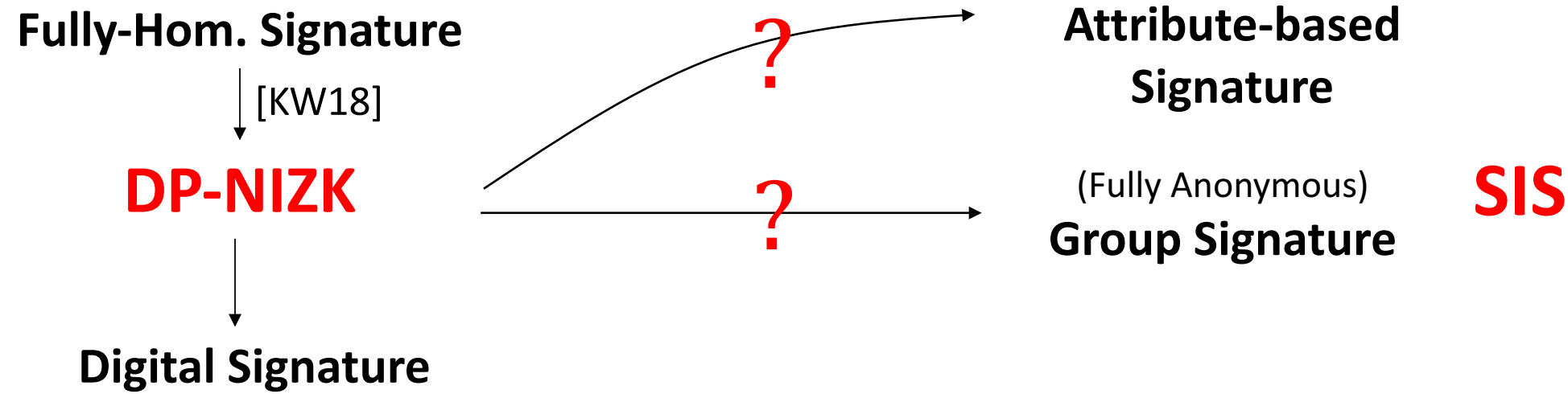

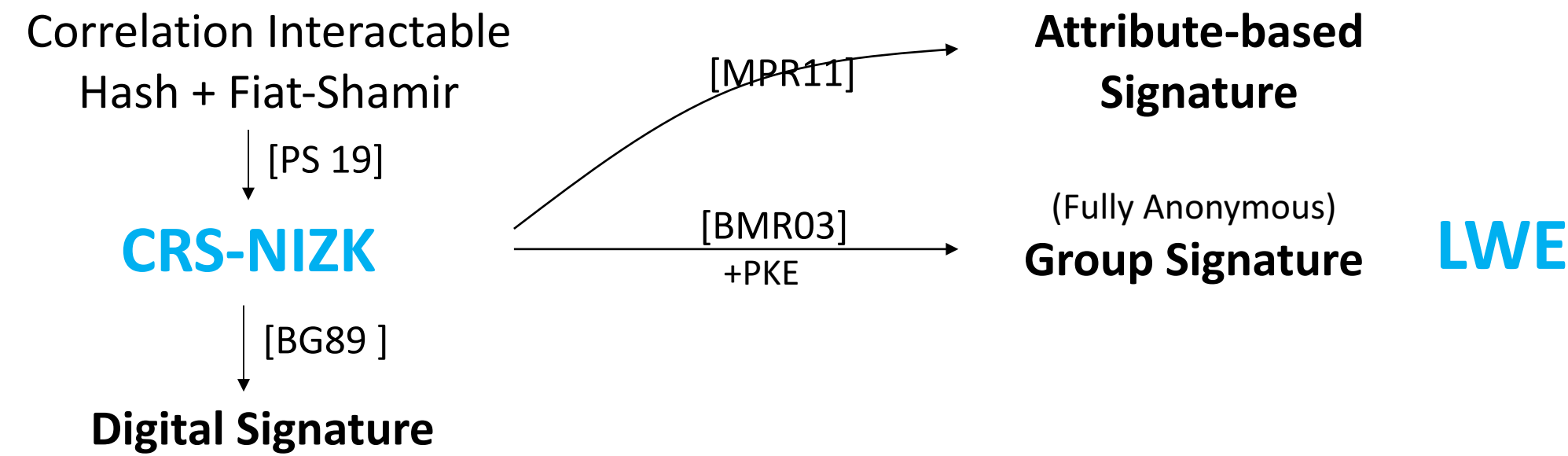

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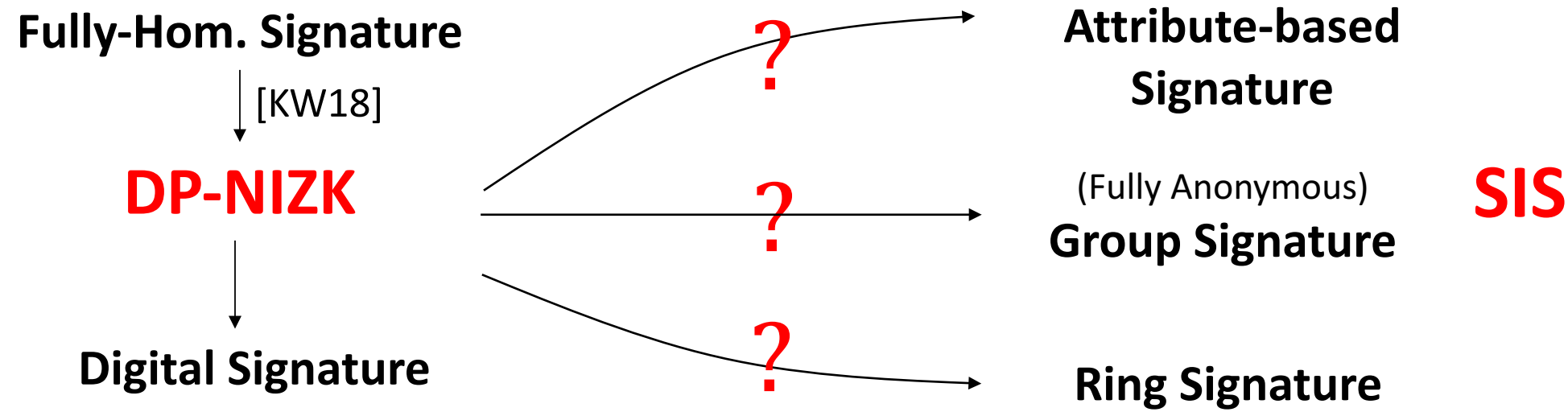

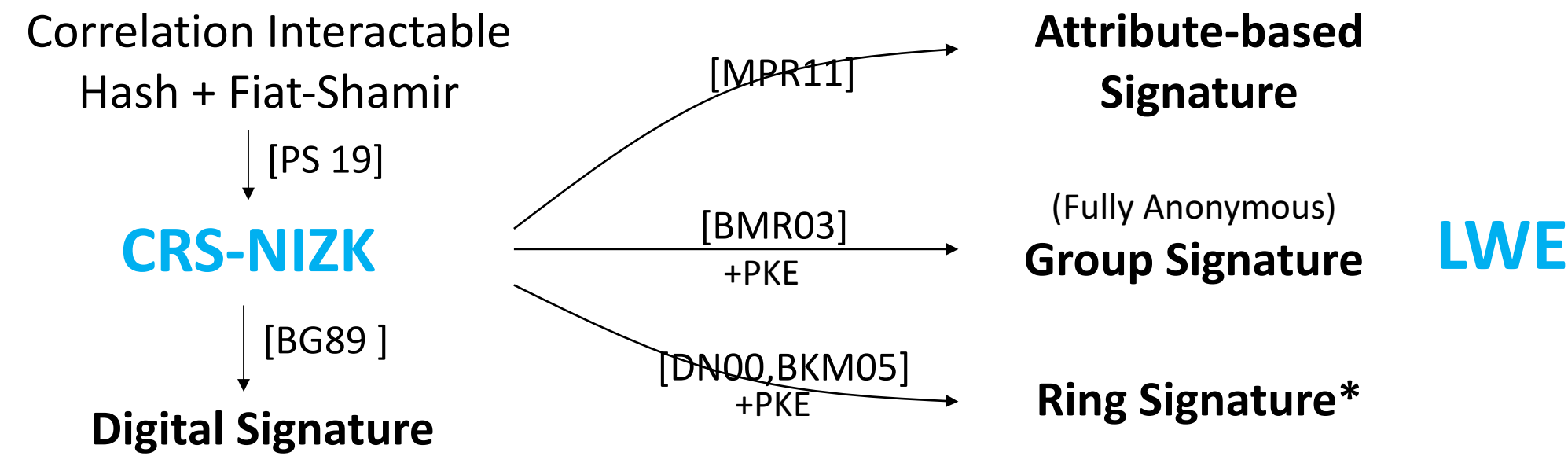

*Unfortunately, this implication does not hold for lattice-based CRS-NIZKs ☹

Lattice NIZKs and Signatures (Feasibility)

Fully-Hom. Signature

[KW18]

DP-NIZK → [KY19] *generalization* → **Multi-User DP-NIZK**

Digital Signature

Attribute-based Signature

(Fully Anonymous) **Group Signature**

Ring Signature

SIS

Correlation Interactable Hash + Fiat-Shamir

[PS 19]

CRS-NIZK

[BG89]

Digital Signature

[MPR11] → **Attribute-based Signature**

[BMR03] +PKE → (Fully Anonymous) **Group Signature**

[DN00,BKM05] +PKE → **Ring Signature***

LWE

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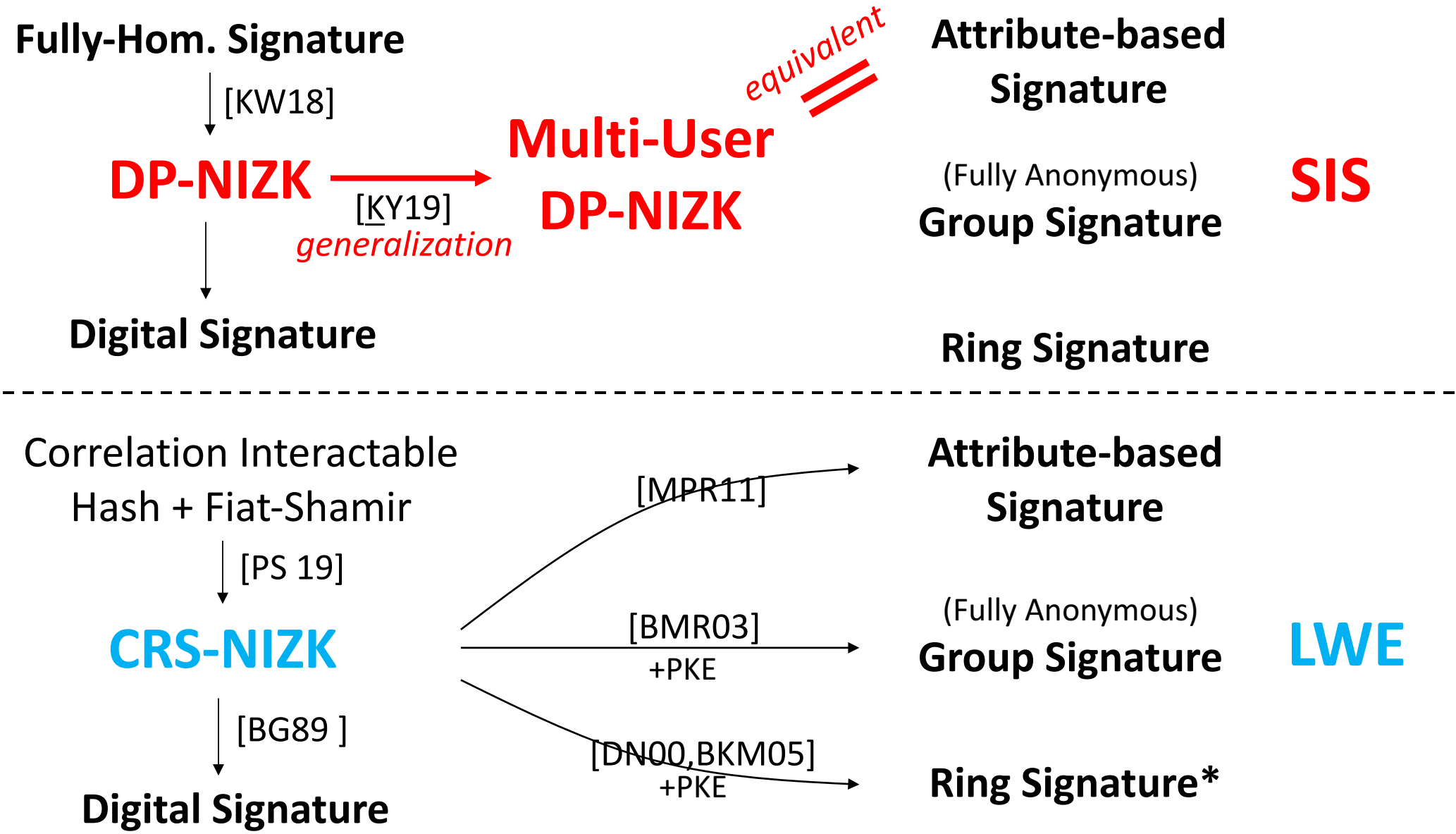
Lattice NIZKs and Signatures (Feasibility)



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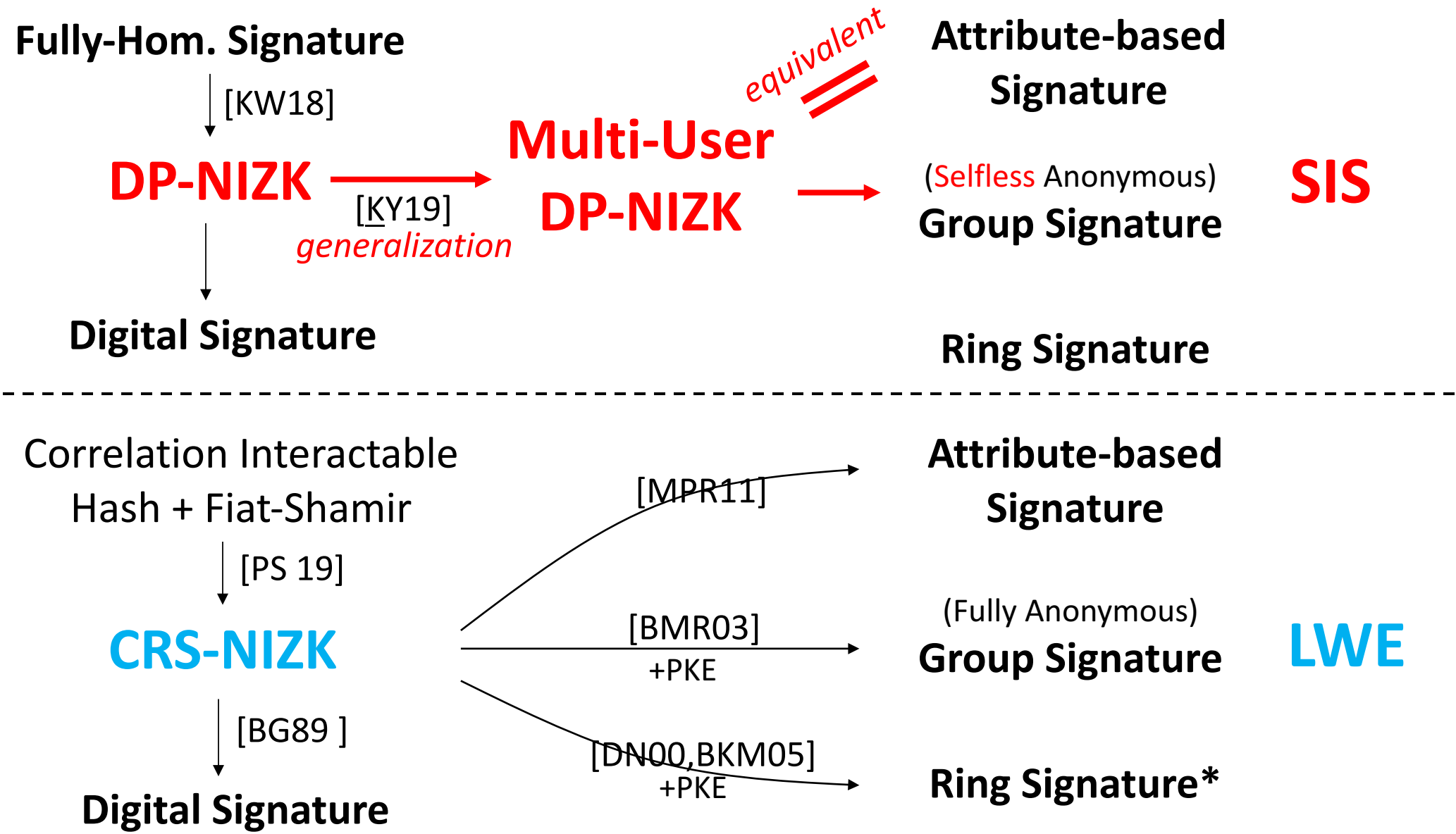

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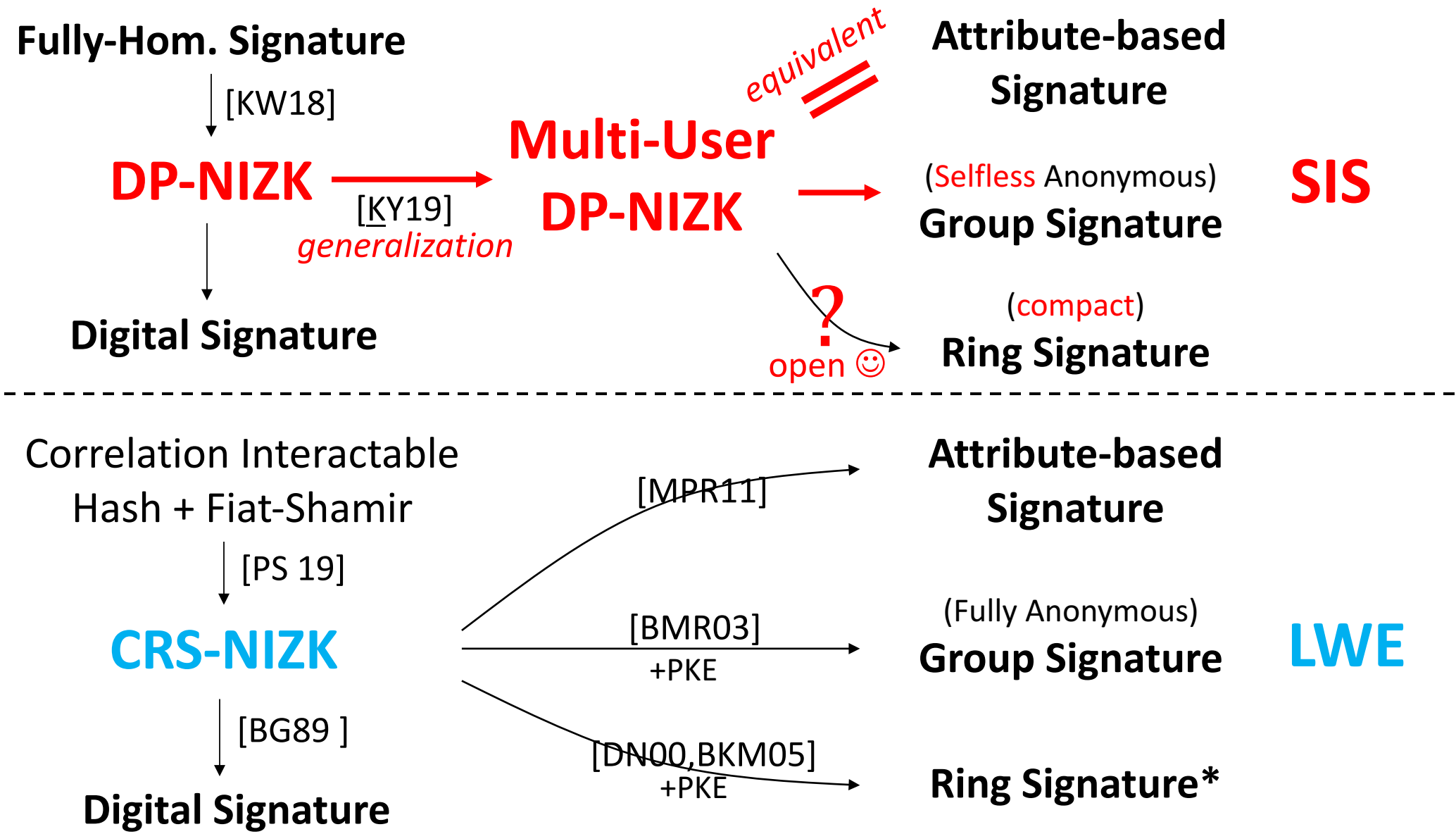

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3. Result:

Lattice-based Group Signature

[KatYam@EC'19]

via "*Multi-User DP-NIZKs*"

*Disclaimers

After our paper was accepted@EC, **CRS**-NIZK from LWE was finally resolved [PS19@CRYPTO].

Accordingly, the following presentation@EC is made under the “old” fact that CRS-NIZK from lattices do not exist yet.

Our Result in Short

① Construct the **first group signatures from lattices in the standard model**.

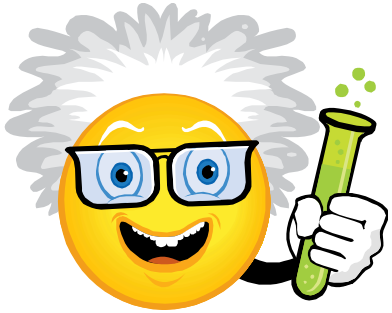
② Achieves full traceability [BMW03] and selfless anonymity [CG04].

③ Constructions from various assumptions.

- ✓ SIS w/ subexp-modulus.
- ✓ LWE w/ poly-modulus.
- ✓ SIS w/ poly-modulus + LPN w/ const. noise rate

Our Techniques in Short

Avoid using CRS-NIZK, a necessary component in existing frameworks [BMW03, CG04,…], but not known from lattices.



A) Extend DP-NIZK to **Multi-User (MU) DP-NIZK**.
B) Show MU-DP-NIZK ⇒ GS.
C) Provide construction of MU-DP-NIZK from SIS.

Outline of “3. Result”

Definition of Group Signatures

Previous Techniques

Our Work

Outline of “3. Result”

Definition of Group Signatures

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Syntax of Group Signature (GS)

$$\left(\mathrm{gpk}, \mathrm{gok}, \{\mathrm{gsk}_i\}_{i\in[N]}\right) \leftarrow \mathrm{GS.KeyGen}(1^k, 1^N)$$

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$$(\mathrm{gpk}, \mathrm{gok}, \{\mathrm{gsk}_i\}_{i\in[N]}) \leftarrow \mathrm{GS.KeyGen}(1^k, 1^N)$$

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*Signature signed by **SOMEBODY** in the group.

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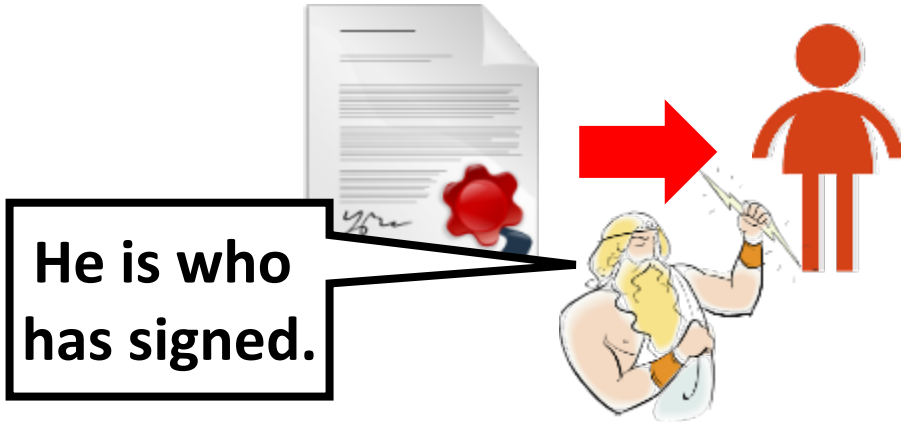

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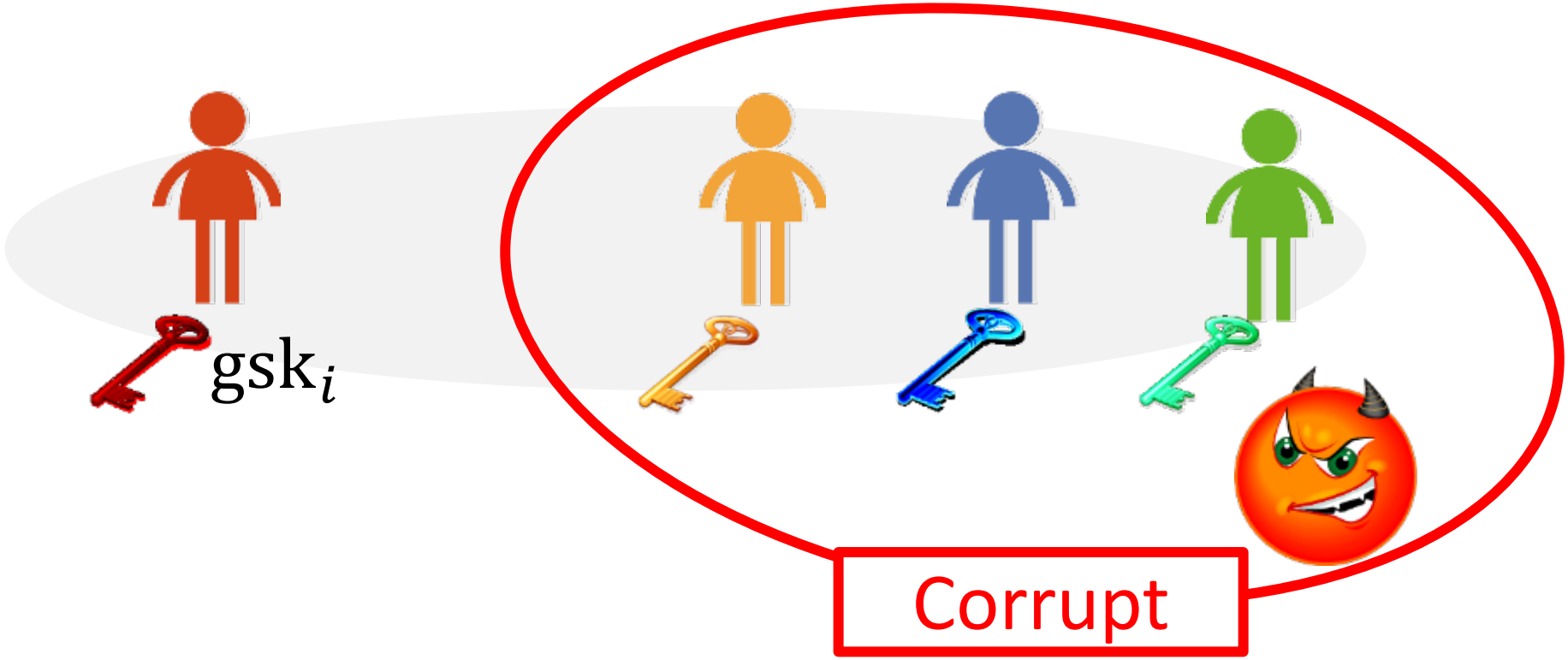
Security: Full Traceability [BMW03]



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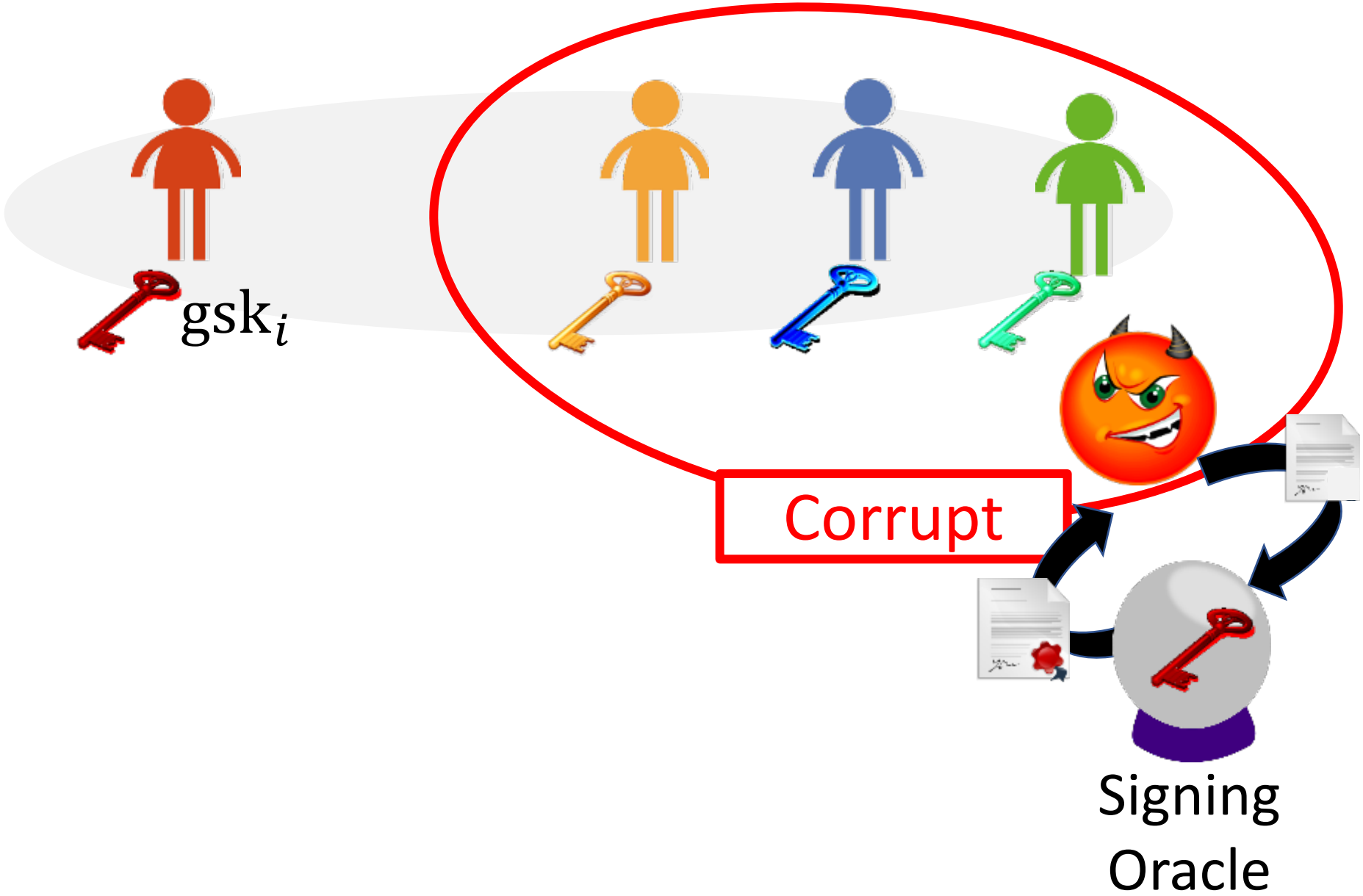

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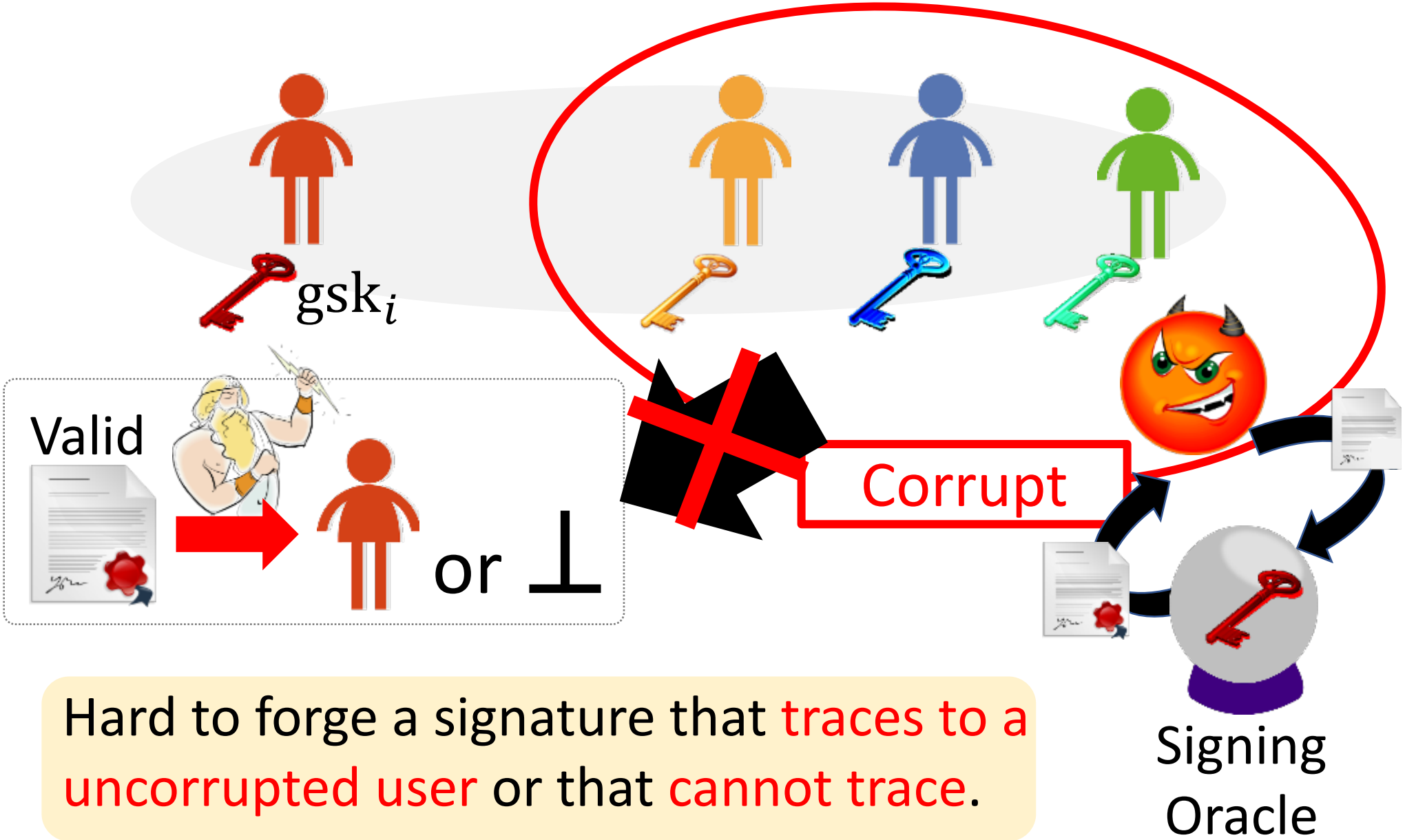
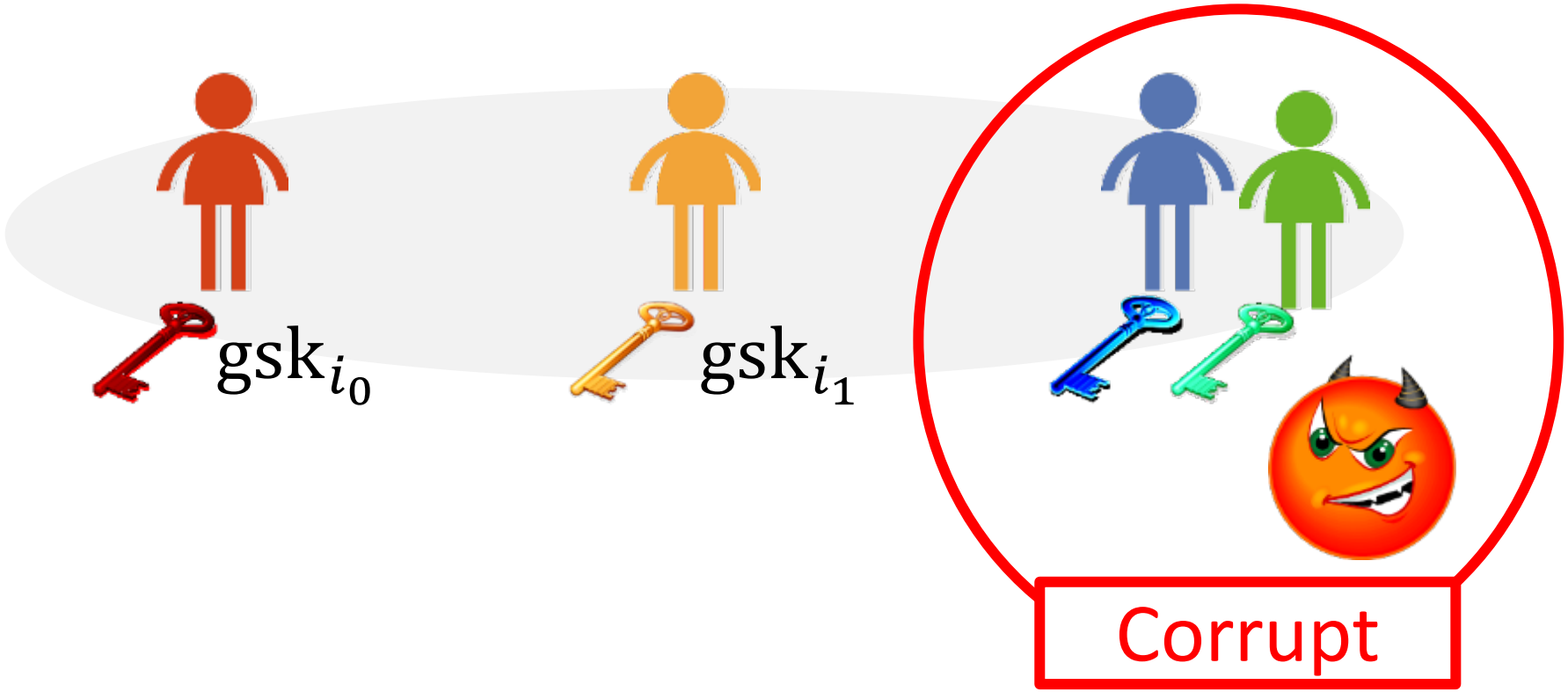


Hard to forge a signature that traces to a uncorrupted user or that cannot trace.

Security: Selfless Anonymity [CG04]



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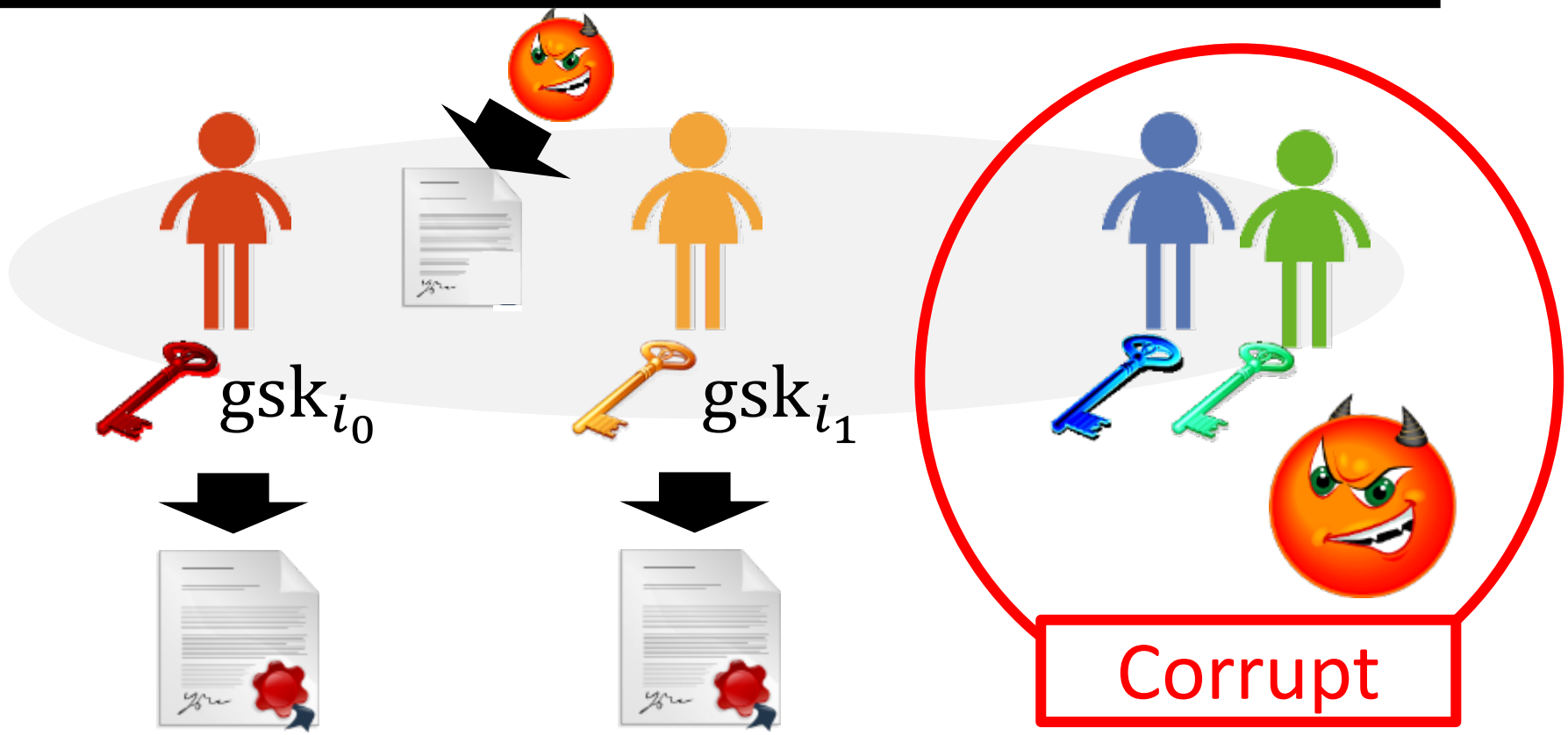

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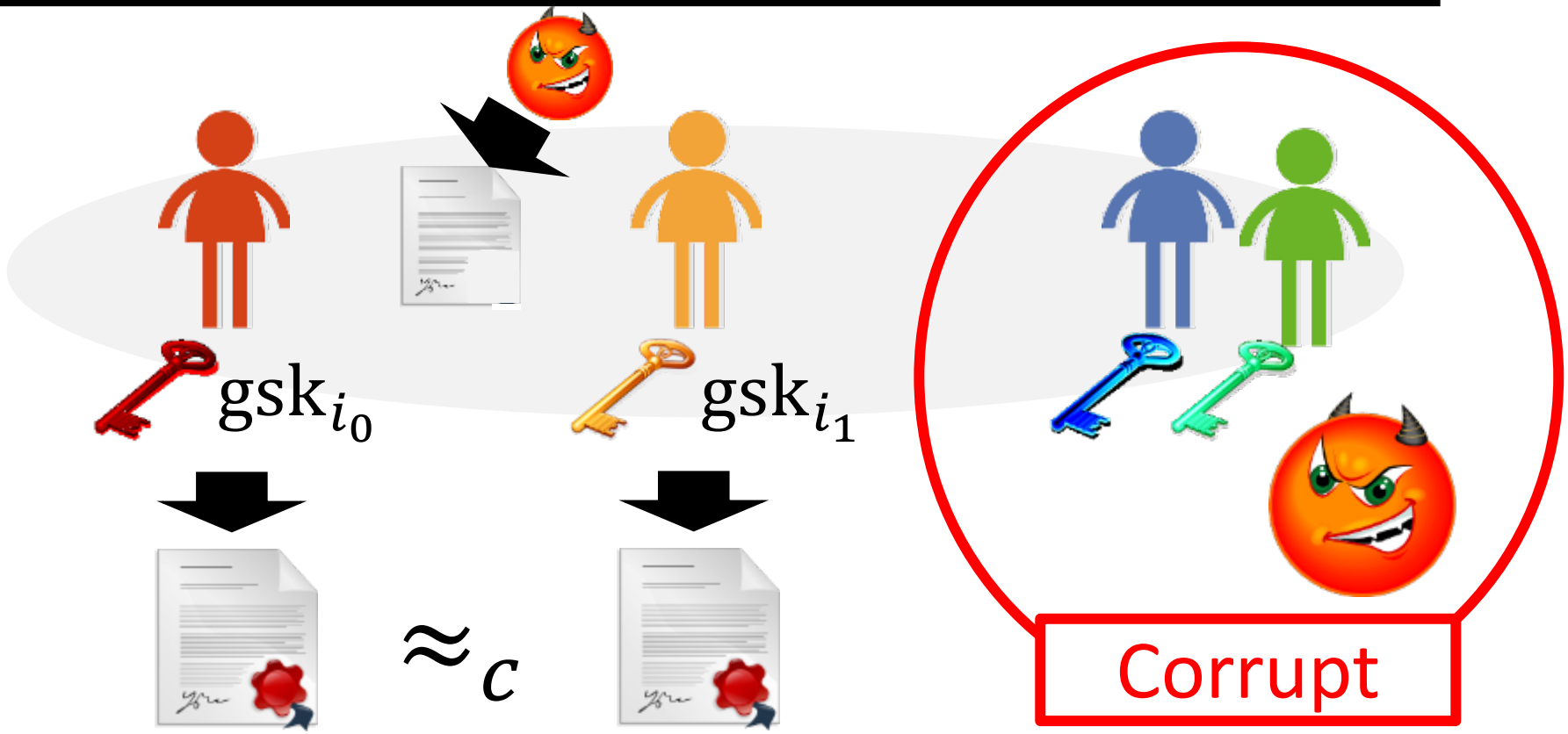

Signatures of two users i_0 and i_1 are ind,
even given gpk, $\{\mathrm{gsk}_j\}_{j \neq i_0, i_1}$ (and open oracle)

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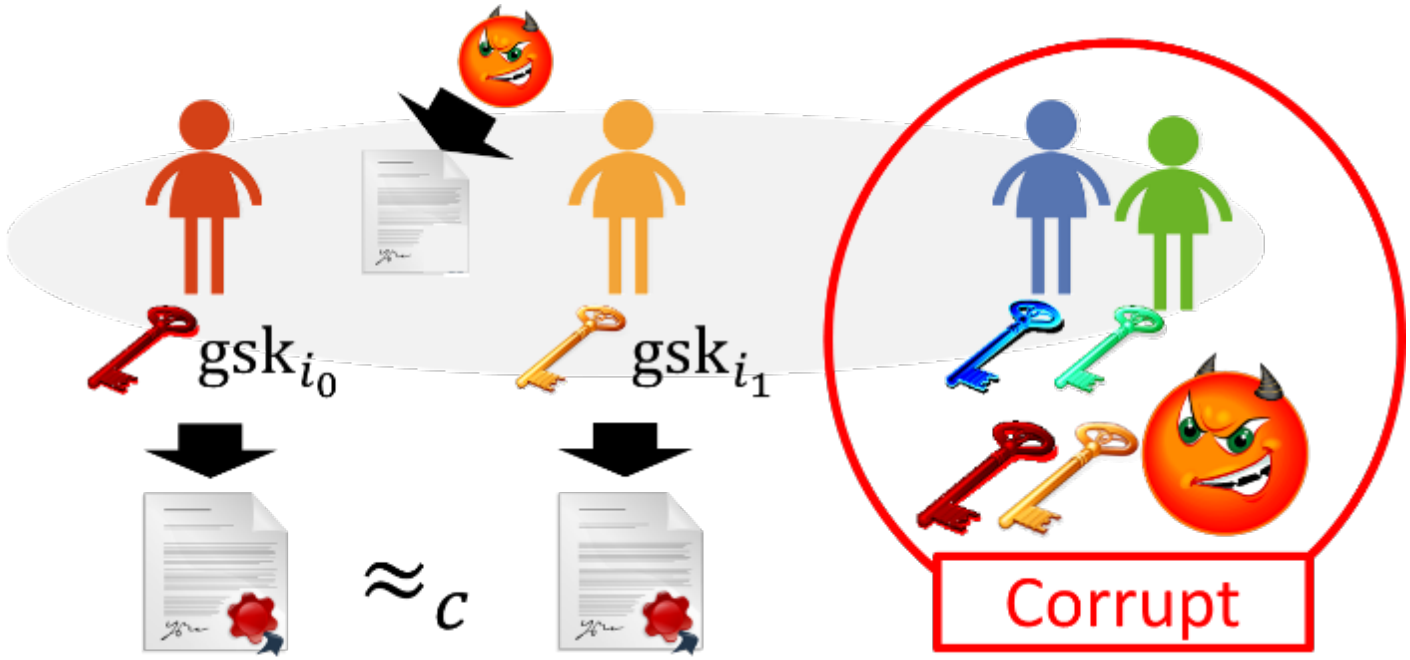
A **stronger** notion than "selfless" anonymity.

The adversary is also given gsk_{i_0} and gsk_{i_1}.



Known to imply public-key encryption. [CG04,AW04]

(Hence, probably non-obtainable from SIS.)

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This Talk

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“Sign then Enc and Prove” Framework

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Ingredients: Signature + PKE + CRS-NIZK

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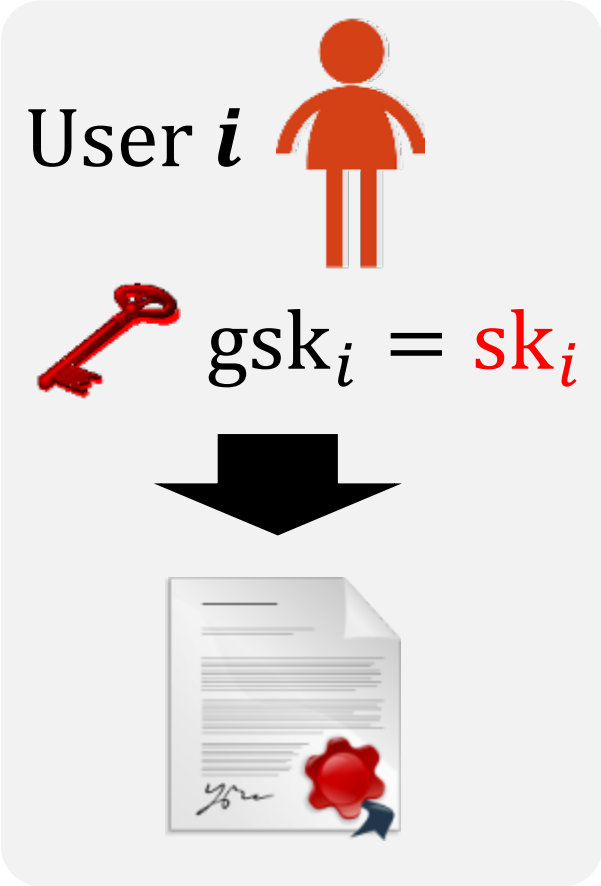

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The information of who signed the message is hidden!

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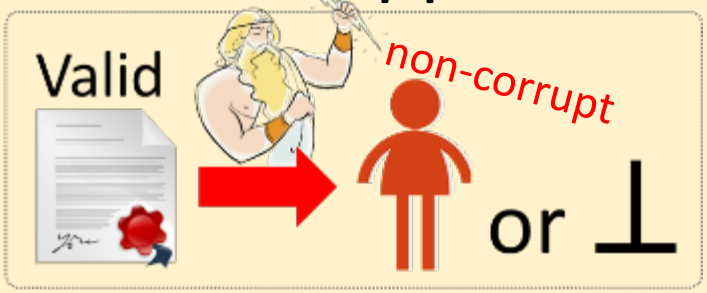

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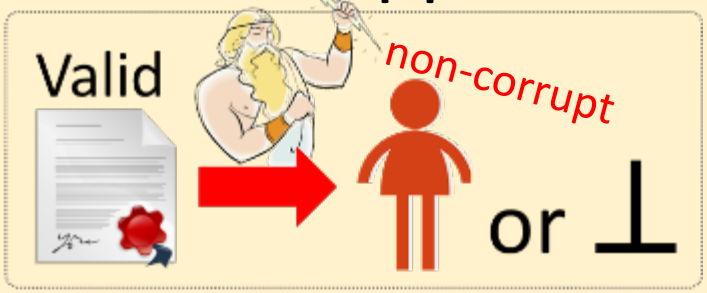

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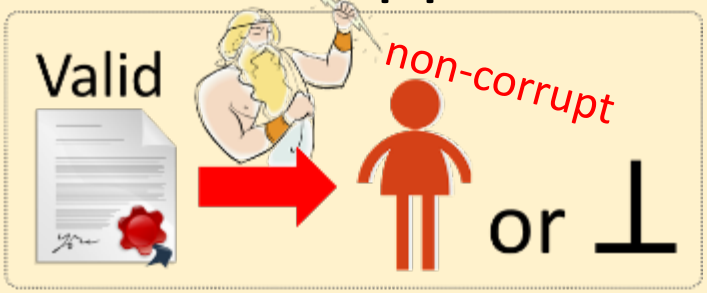

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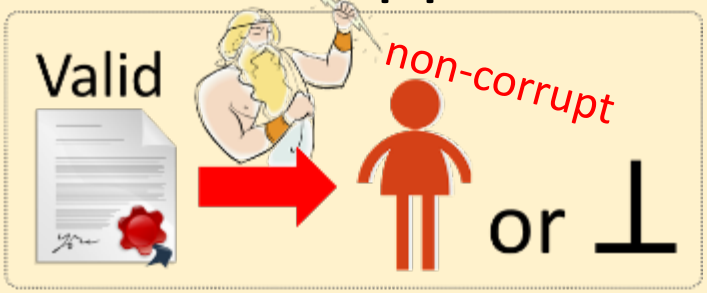

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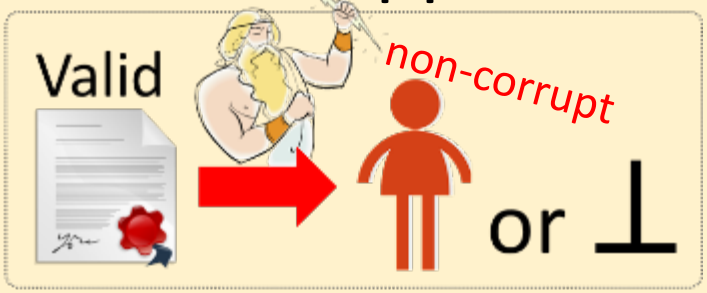

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How to construct lattice-based GS w/o CRS-NIZK??

Ingredients: Signature + PKE + CRS-NIZK

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▶ Will result in the first lattice-based GS! Moreover, from the SIS assumption ☺

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② To avoid problem, we introduce a **Multi-User DP-NIZK** and use it as replacement.

③ We observe that MU-DP-NIZK is implied by attribute-based signatures (**ABS**) and construct a new lattice-based ABS.

Recap: DP-NIZK

CRS: (public) common reference string

$\mathbf{k_p}$: (private) proving key

Prover (x, w)

CRS, $x \in L$

Verifier x

$\mathbf{k_p}$

π

ZK property is satisfied as long as $\mathbf{k_p}$ is kept secret from the Verifier.

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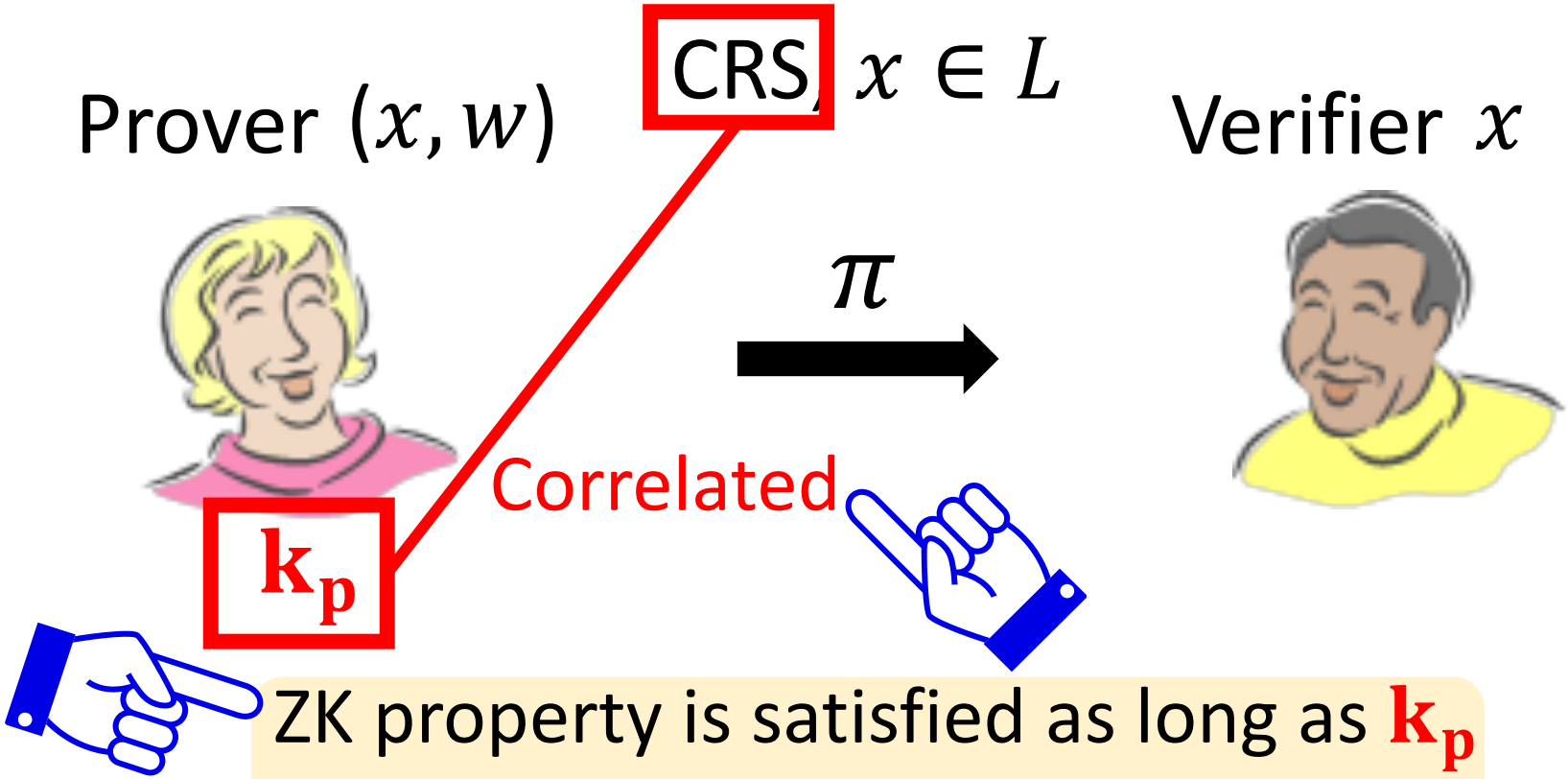

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DP-NIZK proof

Corruption of single user reveals $\mathrm{k_P}$

$\Rightarrow$ Ruins ZK property of DP-NIZK

$\Rightarrow$ Breaks Anonymity of the resulting GS

Second Attempt

Use different $\mathrm{k_p}$ for different users.

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The DP-NIZK proof (GS signature) does not hide the instance i.

$\Rightarrow$ Breaks Anonymity of the resulting GS

Lesson Learned from Failures

- Need multiple prover variant of DP-NIZK.
- Need security against corruption of provers.
- Proof should not leak prover identity.

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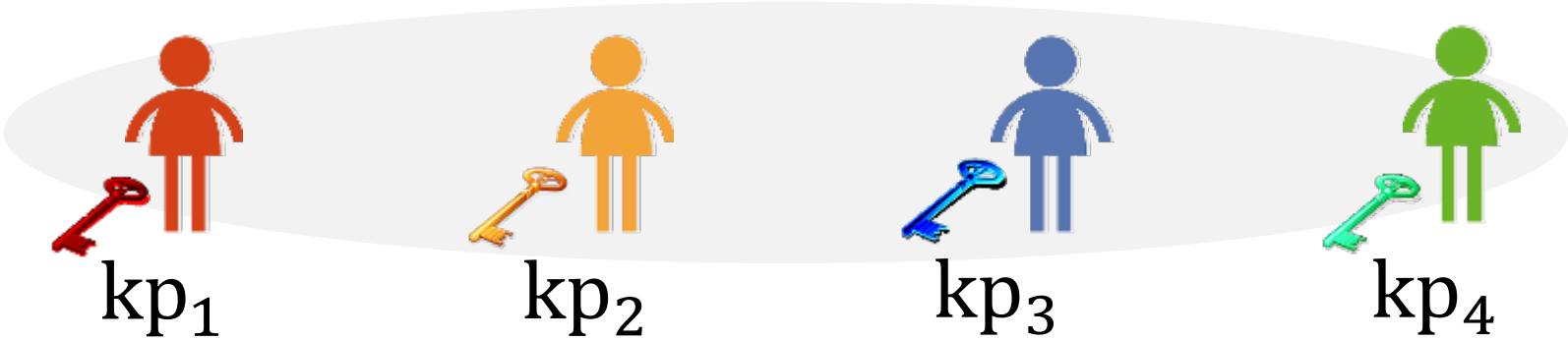
Our Solution

Construct anonymous ***Multi-User DP-NIZK***
(Attribute-based signature + [KW18] technique)

New Notion: Multi-User DP-NIZK

Essentially, a DP-NIZK with multiple users ☺

→ Public: CRS, Private: $(\mathrm{kp}_1, \ldots, \mathrm{kp}_N)$



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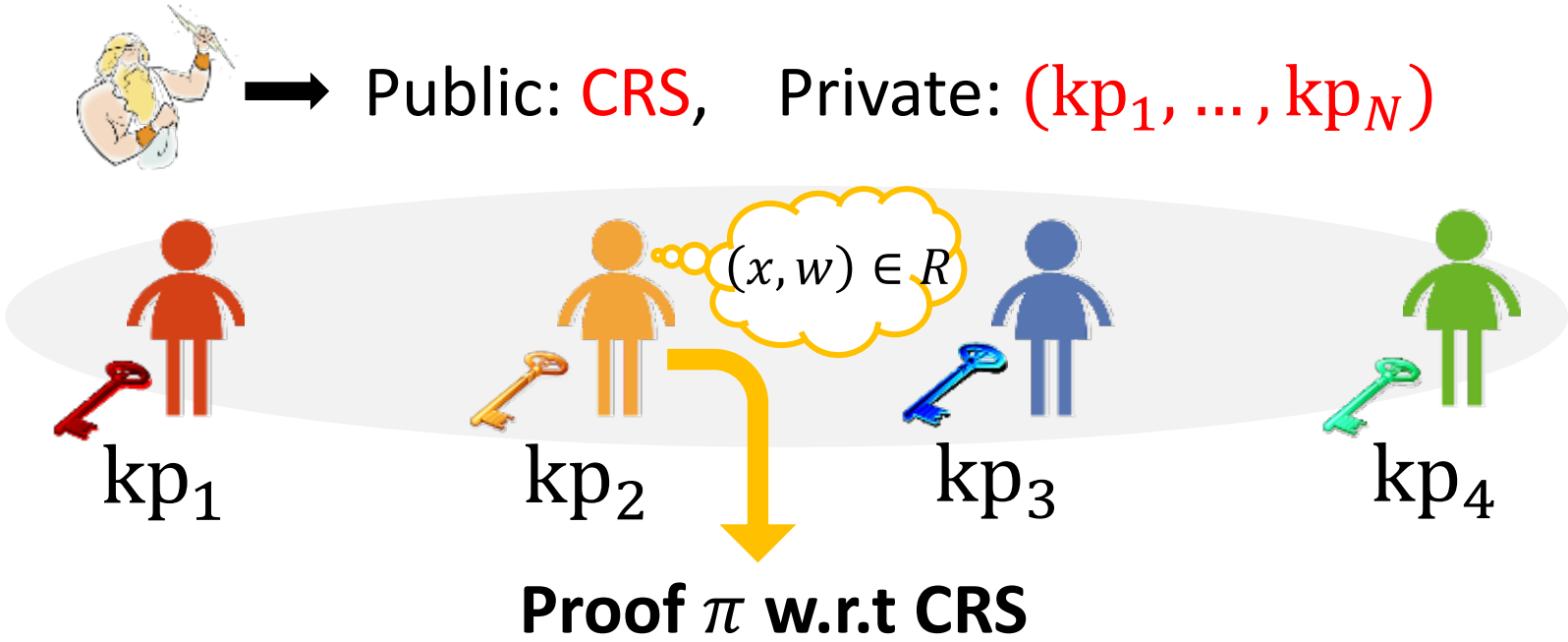

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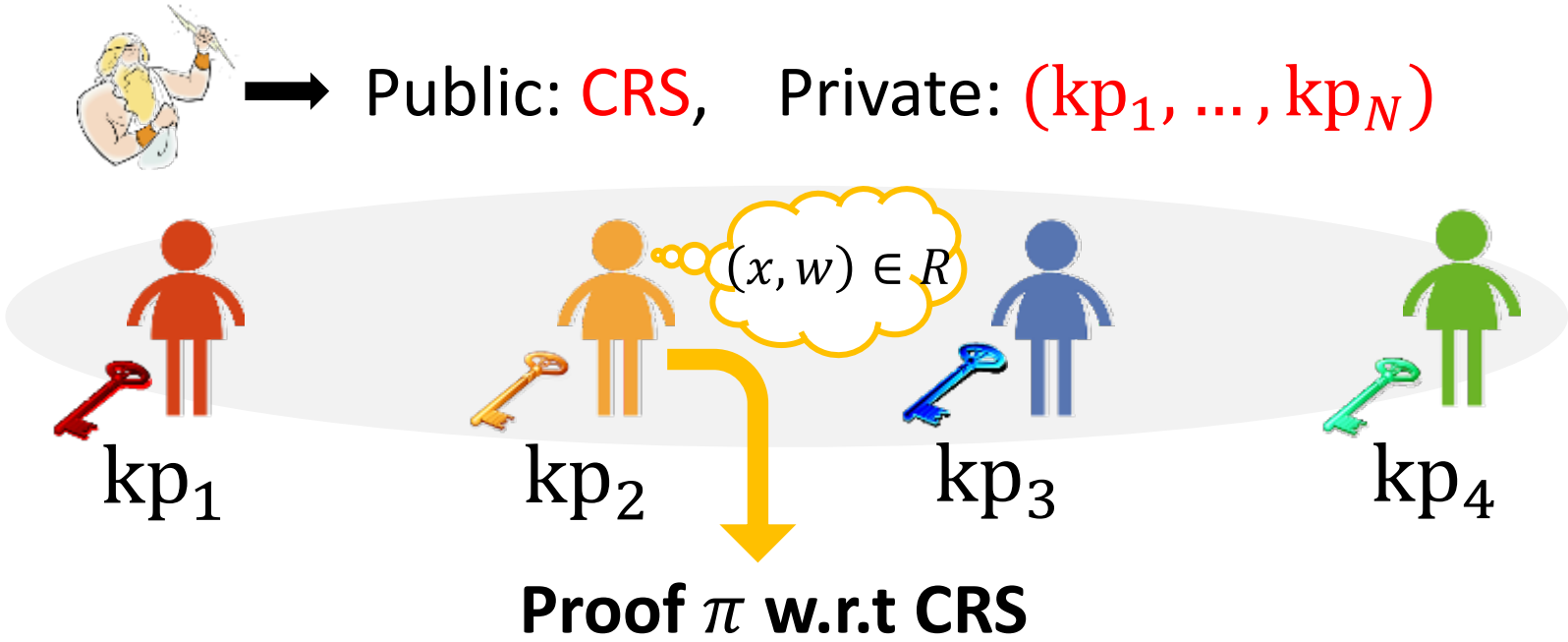

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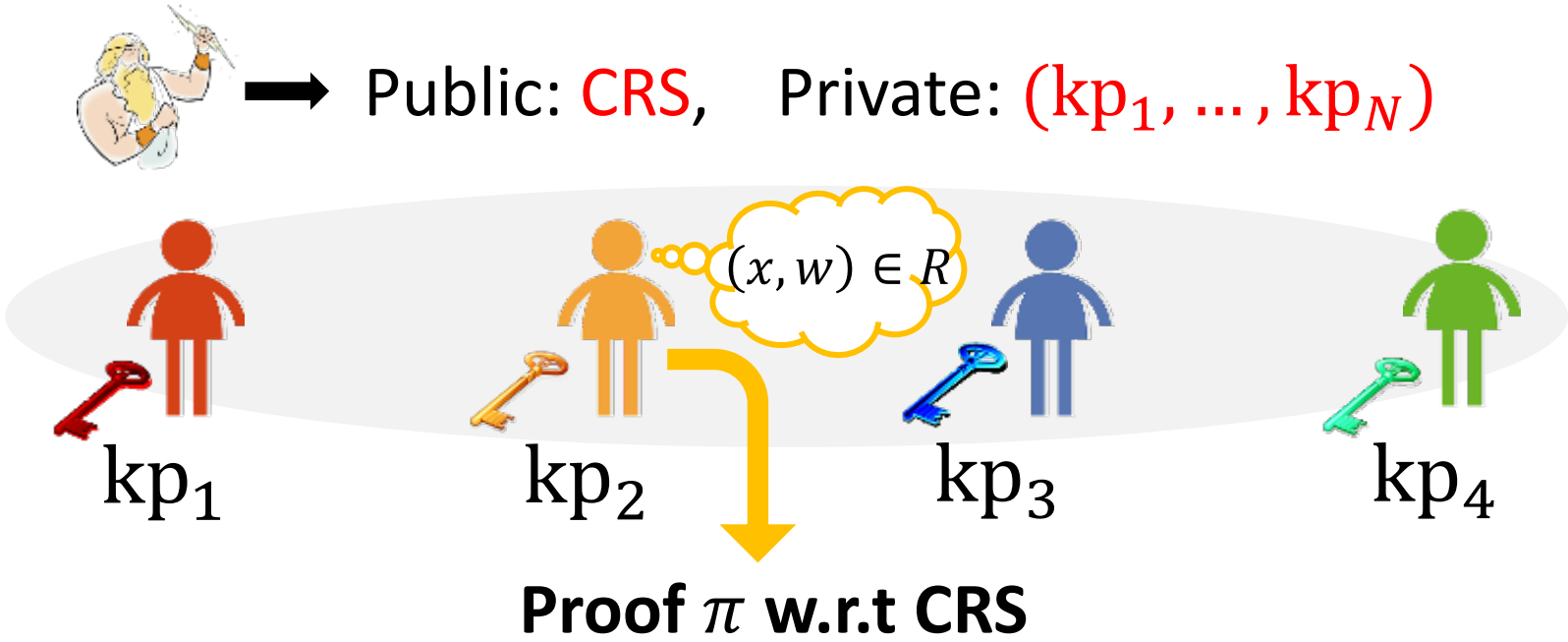

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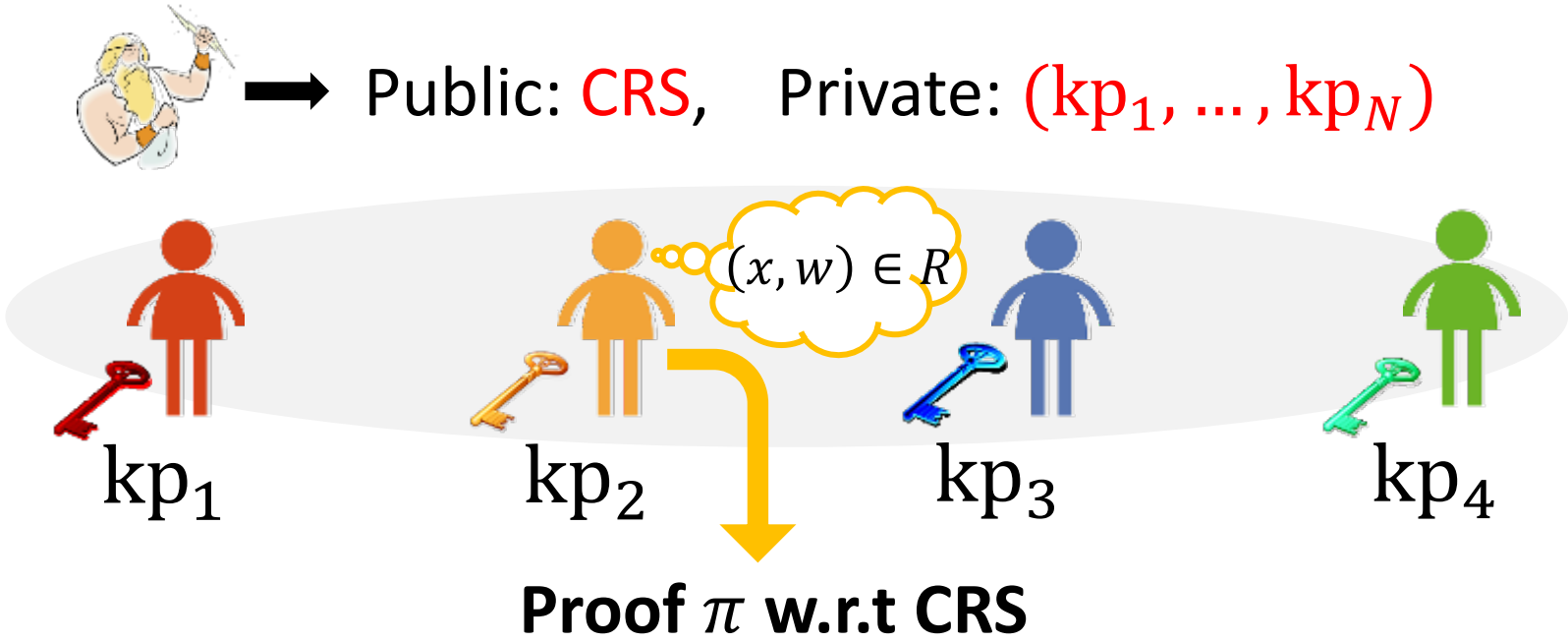


- ✓ **Zero-Knowledge**: π leaks no information even with corruption.
- ✓ **Soundness**: $x \notin L$ cannot be proven even with corruption.
- ✓ **Anonymity**: Information of who generated π is not leaked even with corruption.

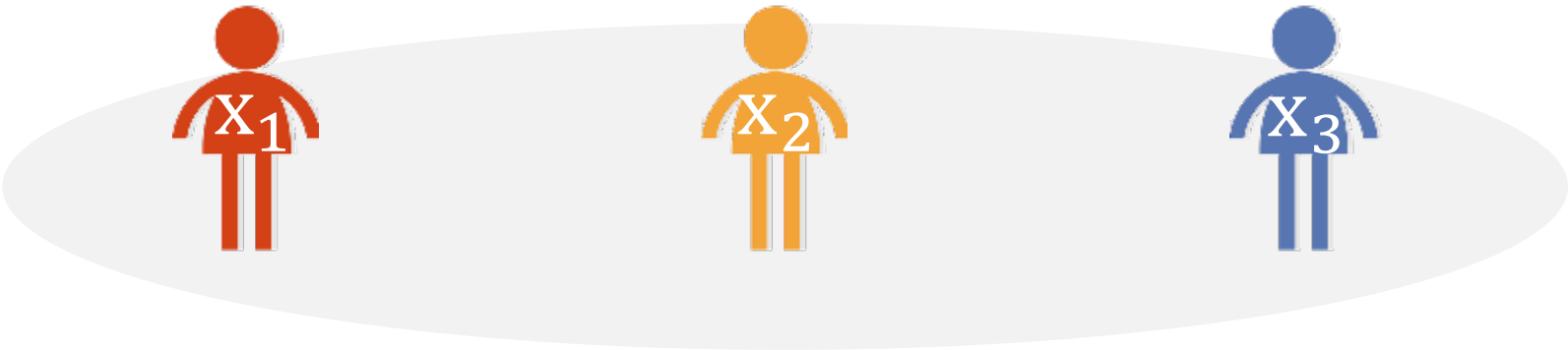
How to Construct MU-DP-NIZK??

The Plan

1. Review Attribute-based Signatures (ABS).
2. Compile ABS into MU-DP-NIZK using the technique developed in [KW18].

Attribute-based Signatures (ABS)

$$(\mathrm{mpk}, \mathrm{msk}) \leftarrow \mathrm{ABS.Setup}(1^k)$$



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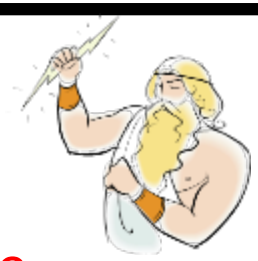
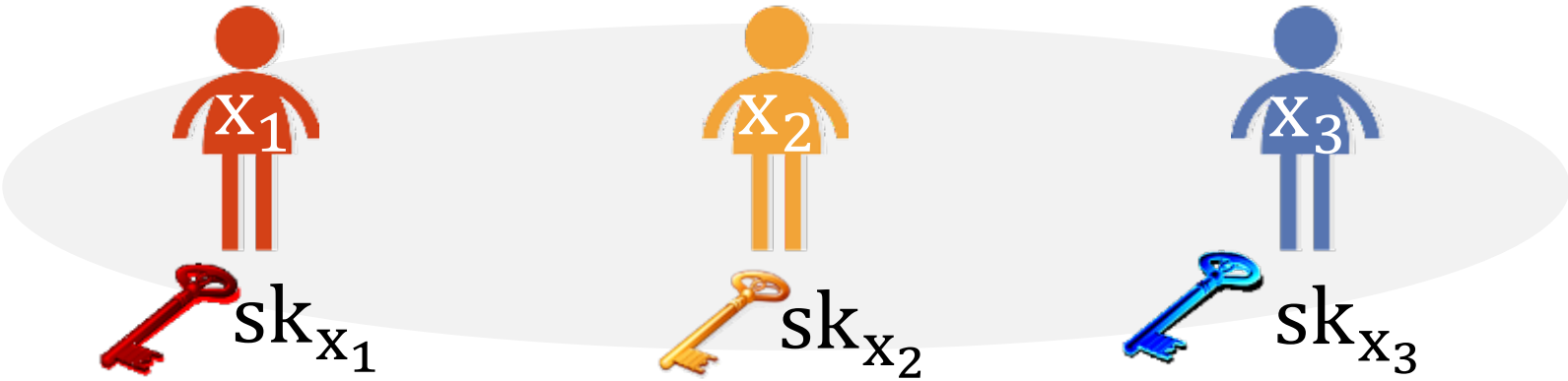


*attribute



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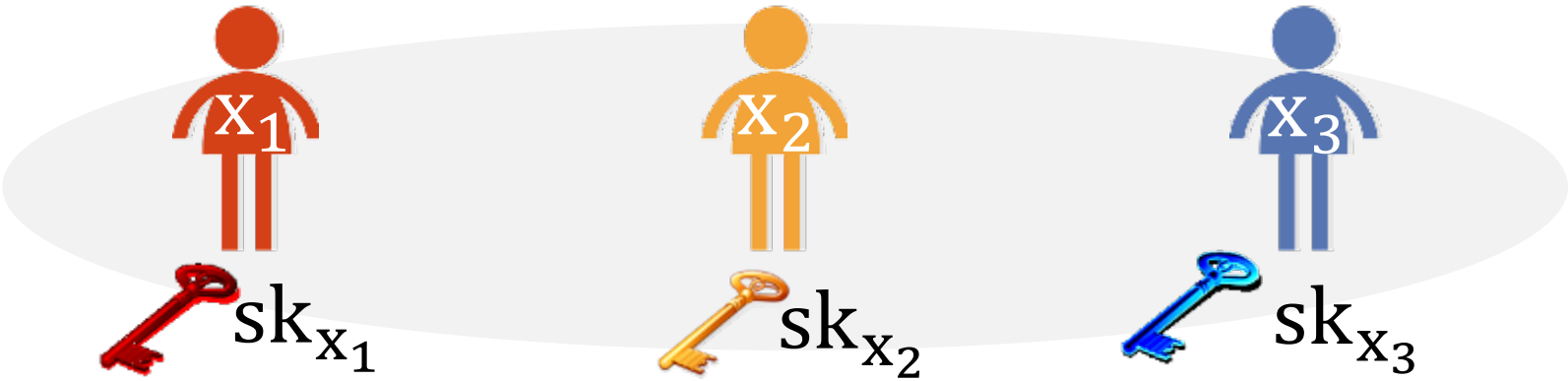

$$\sigma \leftarrow \mathrm{ABS.Sign}(\mathrm{mpk}, \mathrm{sk}_{\mathrm{x}}, \underline{\mathrm{C}}, \mathrm{M})$$

*policy

Can sign on a policy C iff $\mathrm{C}(\mathrm{x}) = 1$.

*attribute satisfies policy.

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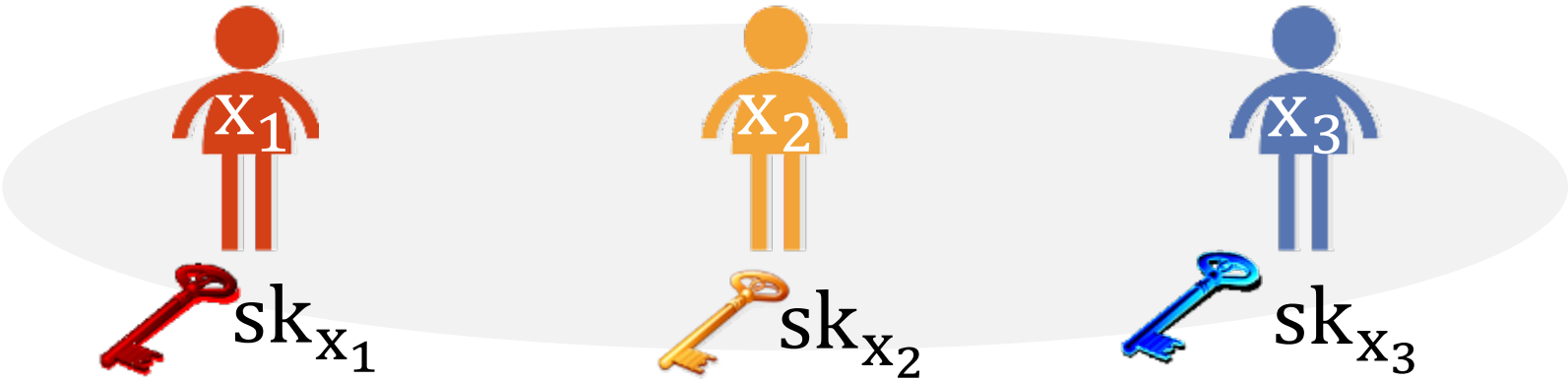

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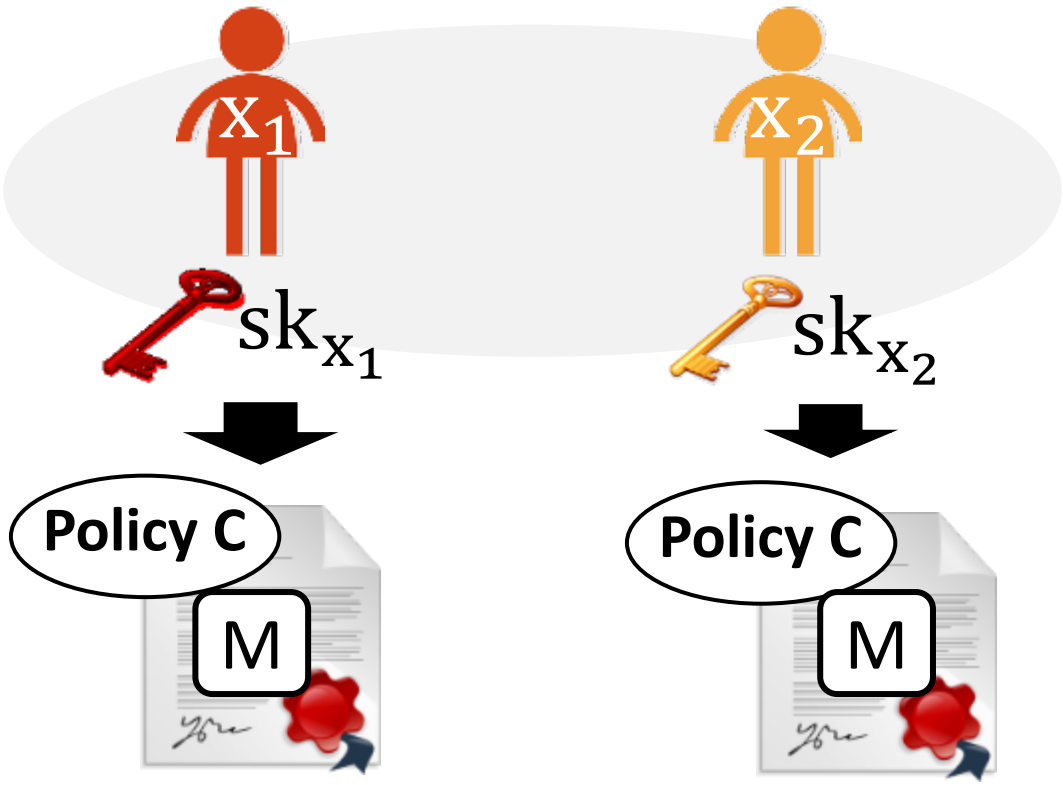
$$\top/\bot \leftarrow \mathrm{ABS.Verify}(\mathrm{mpk}, \mathrm{C}, \sigma, \mathrm{M})$$

Security of ABS: Anonymity



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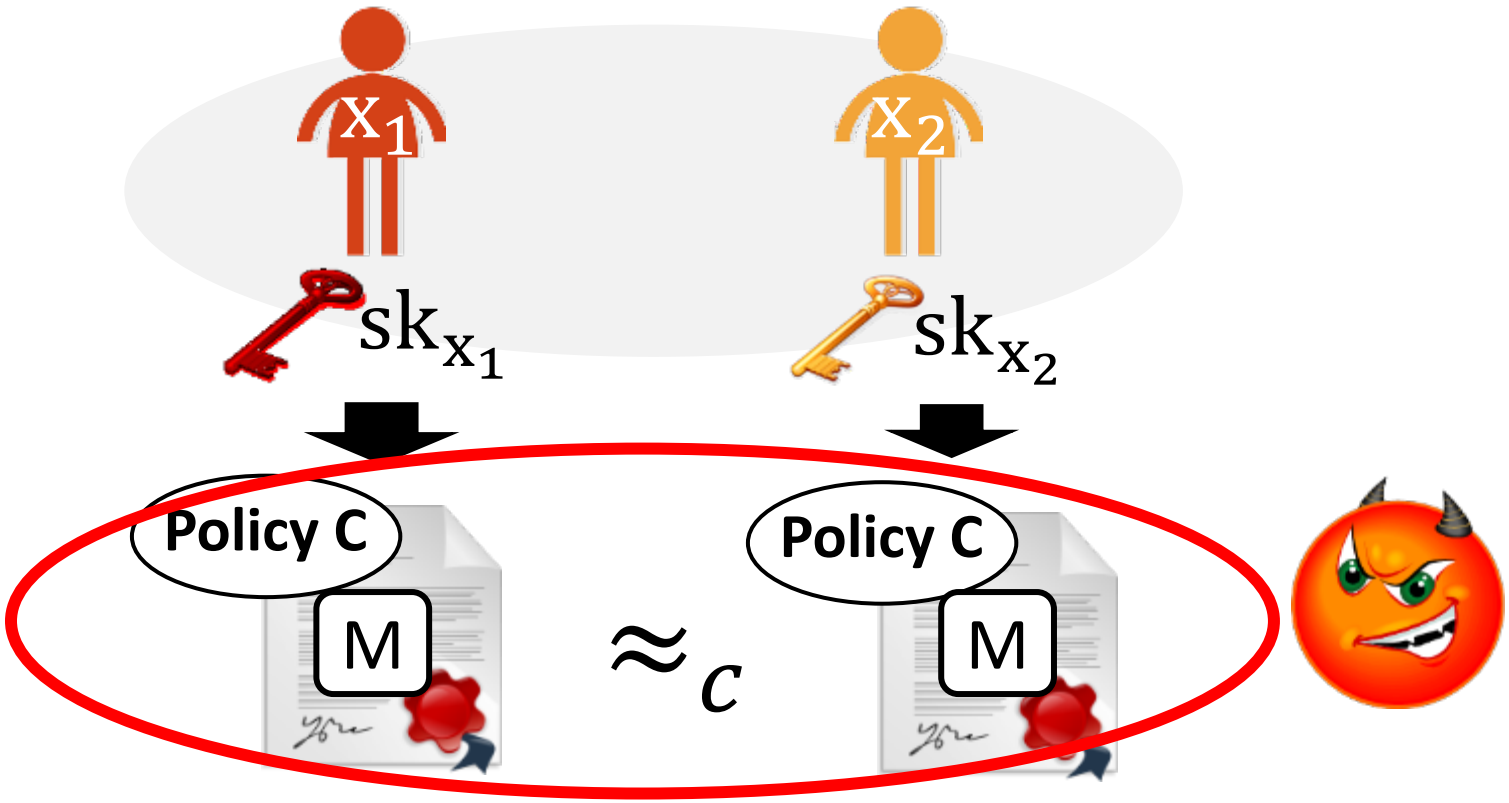

If attribute x_1 and x_2 satisfy $C(x_1) = C(x_2) = 1$, then the signatures are indistinguishable.

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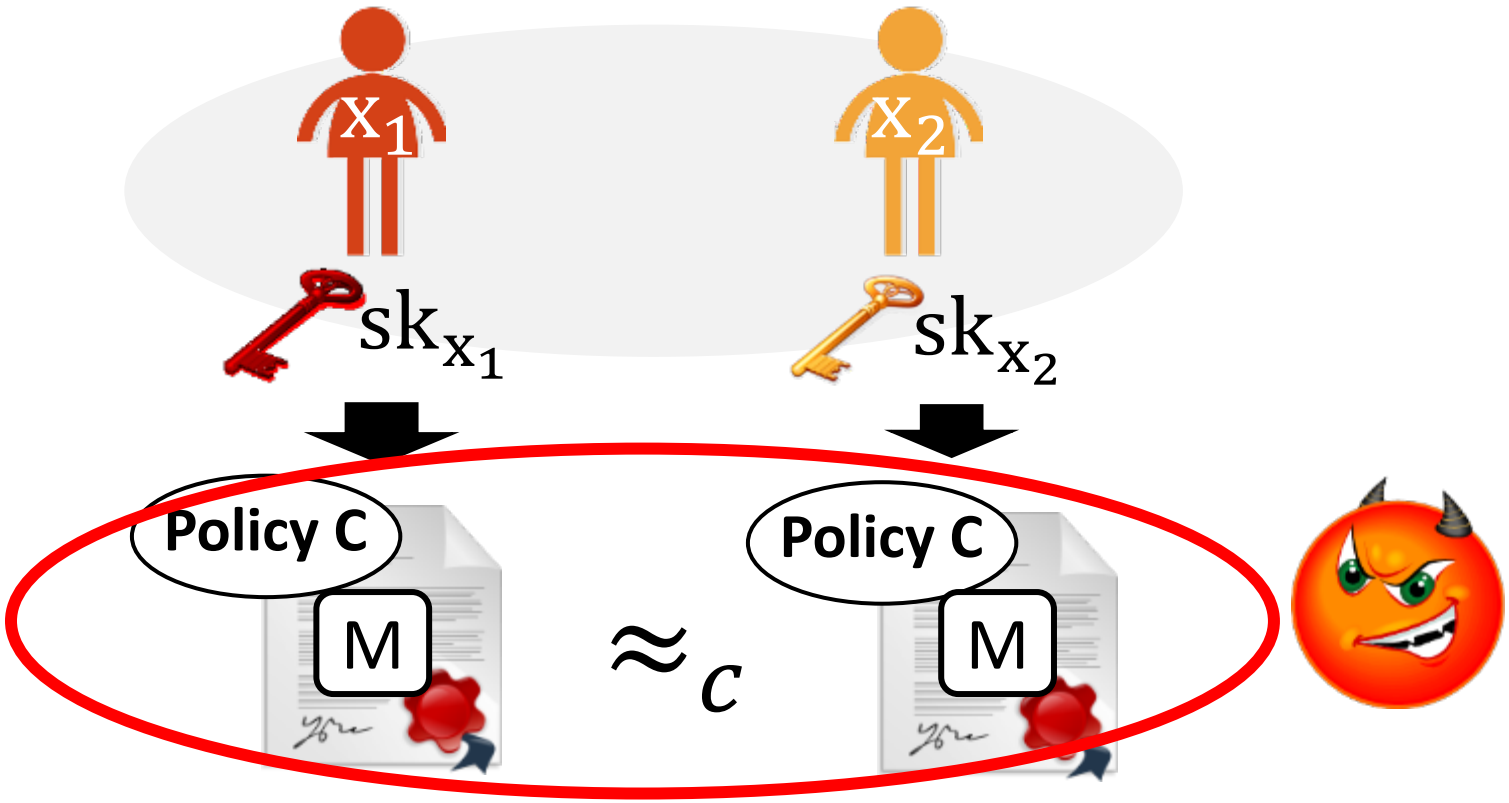
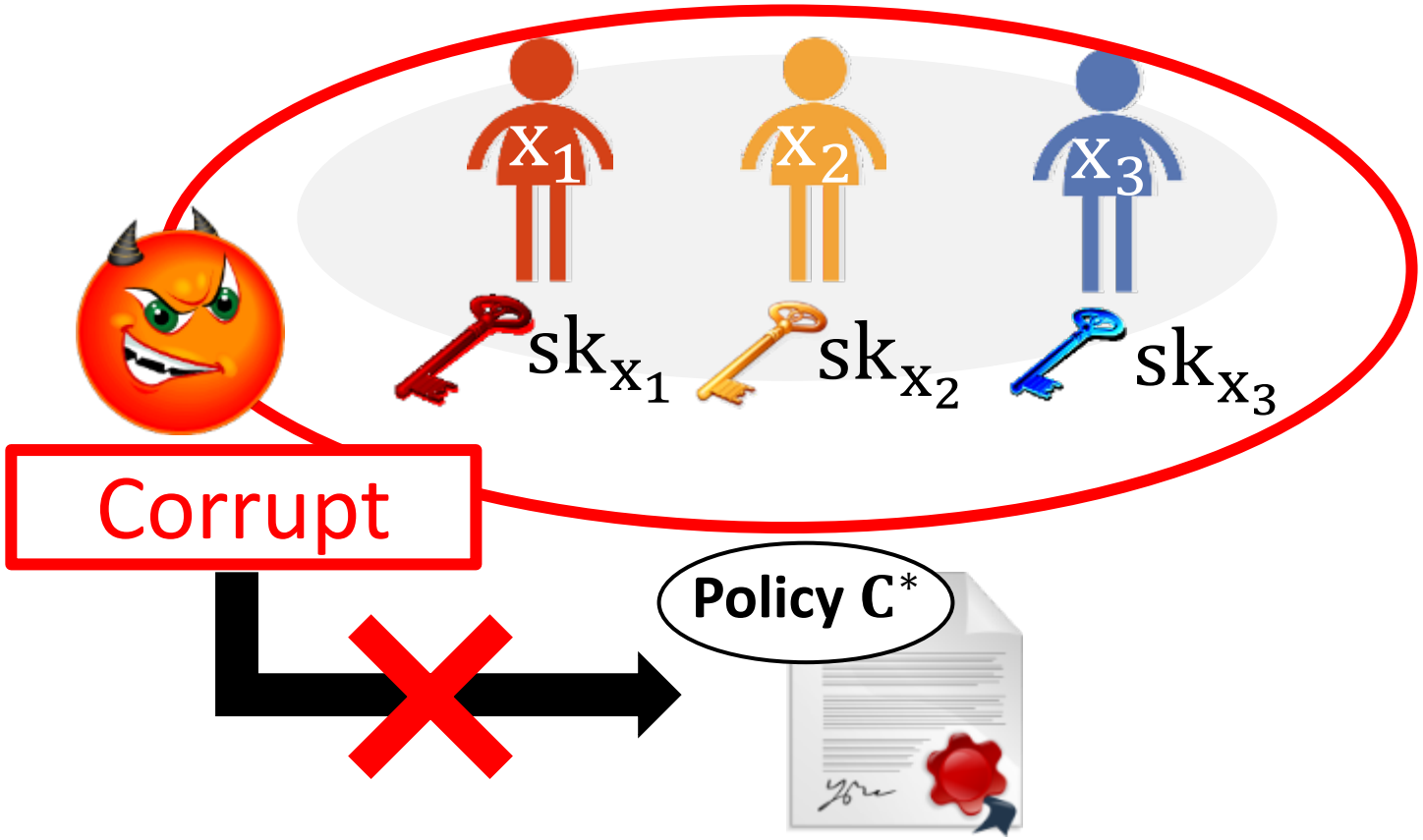


If attribute x_1 and x_2 satisfy $C(x_1) = C(x_2) = 1$, then the signatures are indistinguishable.

*Signature only leaks that the signer had a satisfying attribute x for policy C.

Security of ABS: Unforgeability

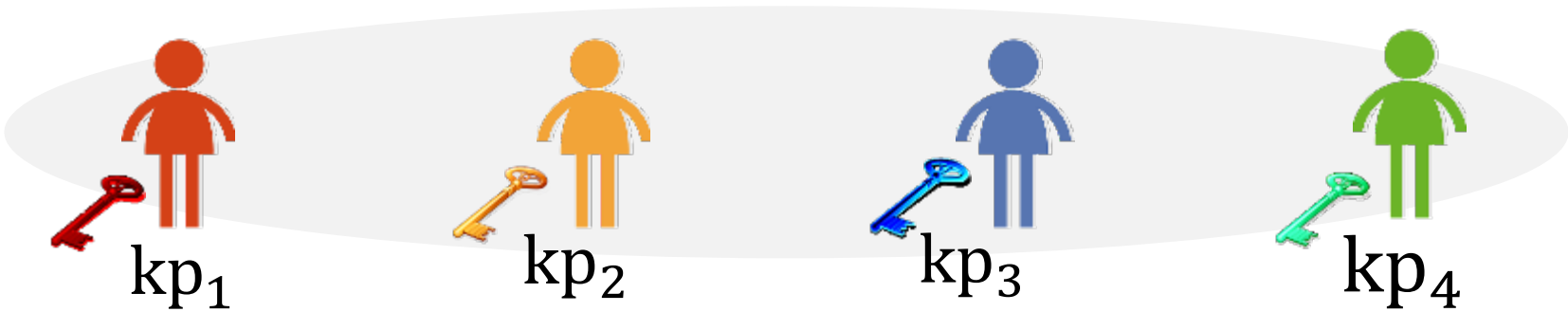


Hard to forge a signature on C^* even if given signing keys $\{sk_x\}$ that are not allowed to sign on C^* (i.e., $C^*(x) = 0$)

*Secret key sk_x can only be used with respect to C such that $C(x) = 1$.

MU-DP-NIZK from ABS (+ SKE)

Apply the idea of [KW18] to ABS instead of FHS.



$= (\mathrm{K}_1, \mathrm{sk}^{\mathrm{ABS}}_{\mathrm{K}_1})$ ..... $= (\mathrm{K}_4, \mathrm{sk}^{\mathrm{ABS}}_{\mathrm{K}_4})$

SKE secret key

ABS signing key viewing K_1 as attribute

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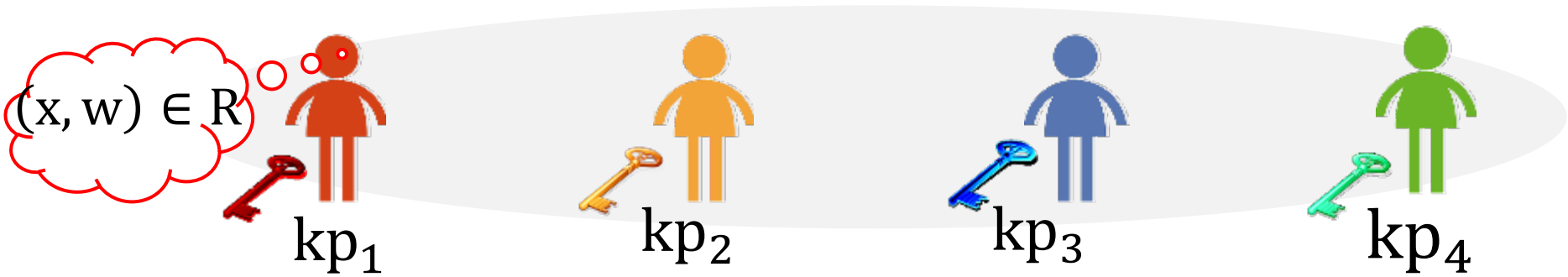

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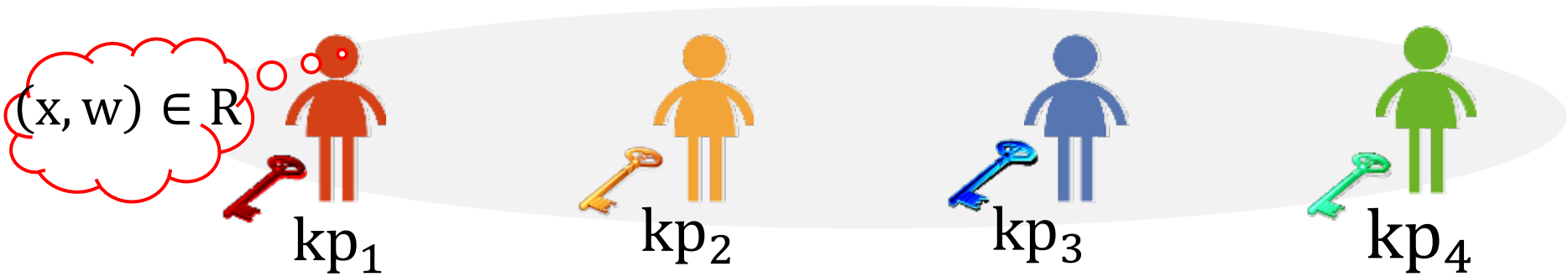

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Public Function

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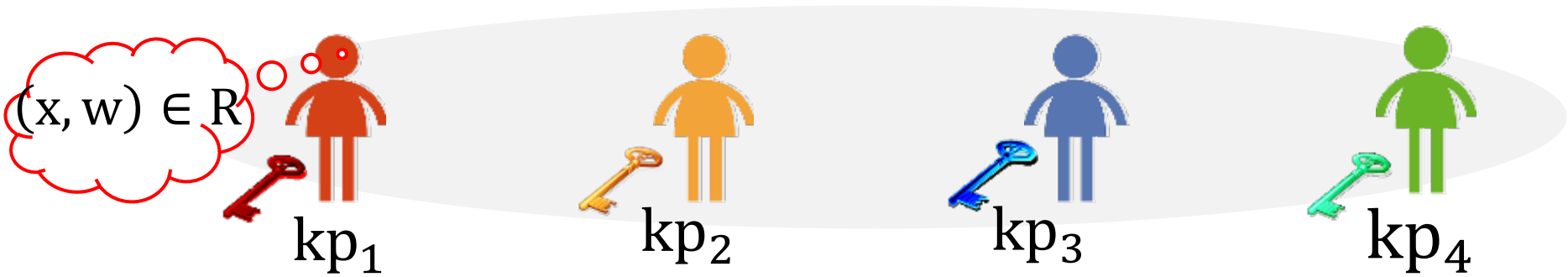

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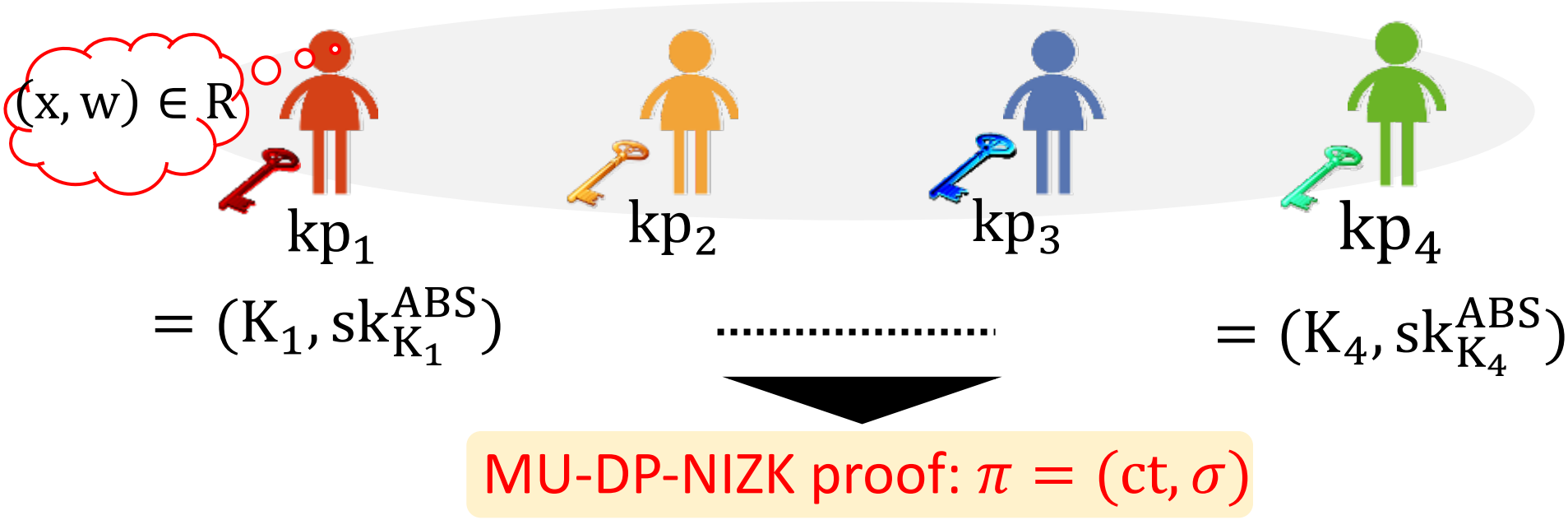
3. $\pi := (ct, \sigma)$

Public Function

Security: Soundness



Soundness

If x is not in the language, then for all ct and K,

$$C_{x,ct}(K) := R\big(x, SKE.Dec(K, ct)\big) = 0.$$

Hence, unforgeability of ABS implies soundness ☺

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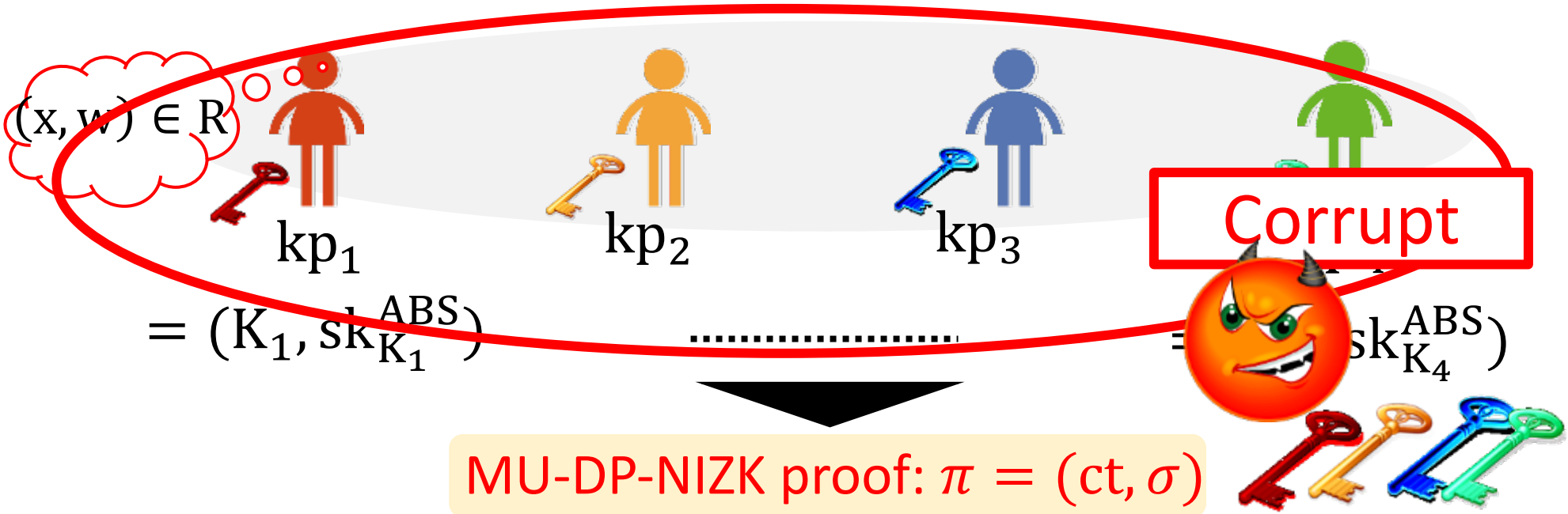

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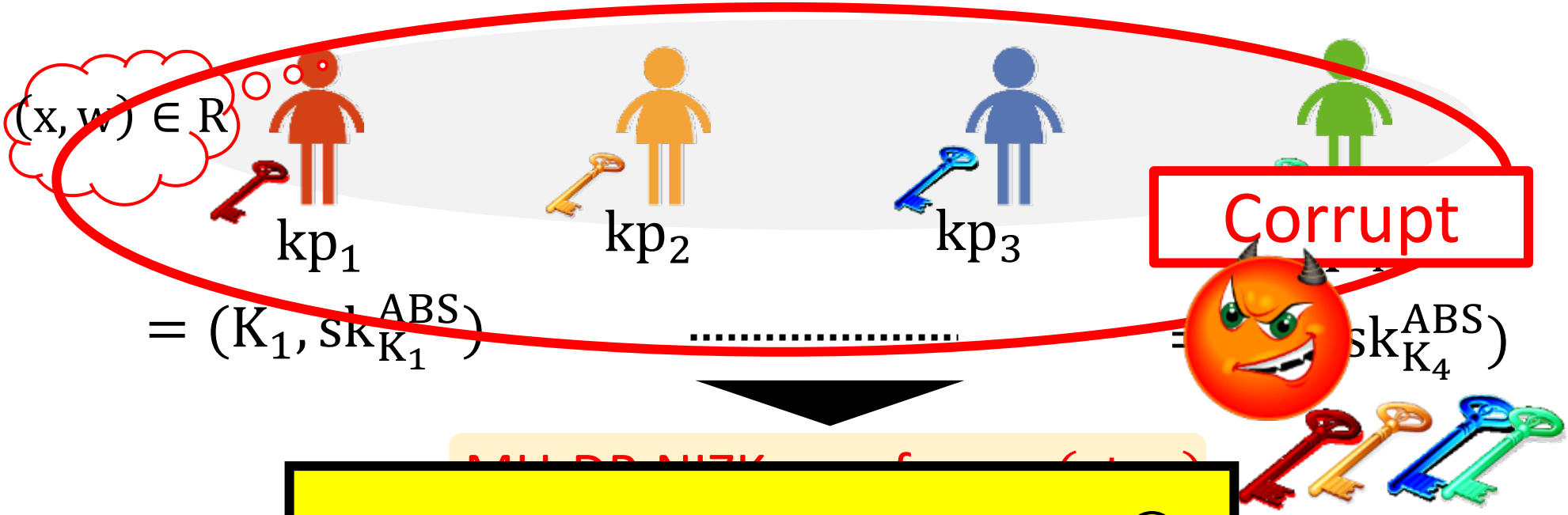

Soundne

Sound even with corruption ☺
(∵ ABS is unforgeable even with corruption)

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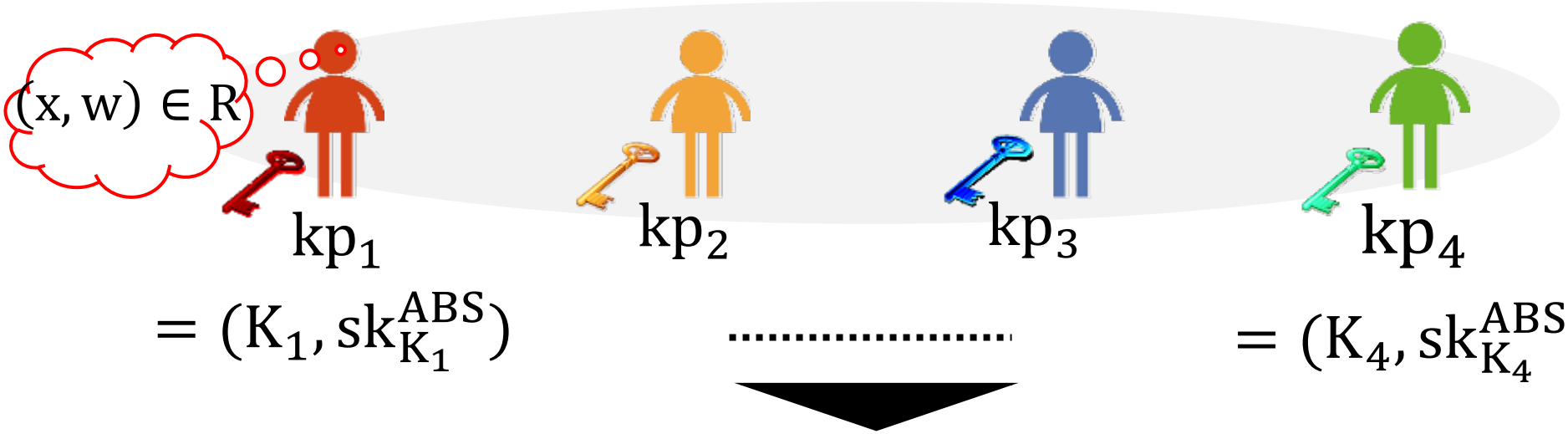
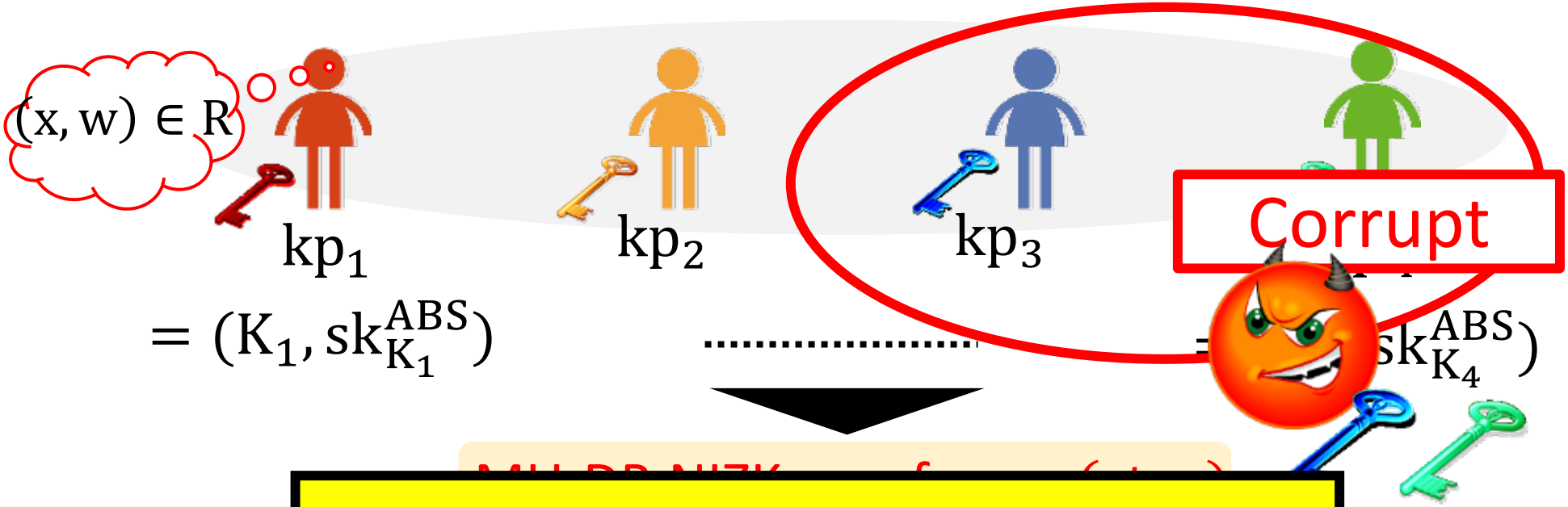


MU-DP-NIZK proof: $\pi = (ct, \sigma)$

ZK/Anonymity

- ct$= \mathrm{SKE.Enc}(K, w)$: Does not leak w due to security of SKE.
- σ is a ABS signature using sk_K^{ABS}: Does not leak (attribute) K due to anonymity of ABS.

Security: Zero-Knowledge/Anonymity



ZK/Anon

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ZK and Anonymous even with corruption ☺
(∵SKE Keys are independent and ABS is anonymous)

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Piecing Everything Together

Plug in MU-DP-NIZK in place of CRS-NIZK in the "Sign-then-Enc-and-Prove" paradigm [BMW03, CG04].

$$\text{SKE} + \text{Signature} + \underbrace{\text{Multi-User DP-NIZK}}_{\text{SKE} + \text{ABS} \Longrightarrow} \Longrightarrow \text{GS}$$

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$$\text{SKE} + \text{ABS} \Longrightarrow$$

Straightforward to instantiate SKE and Signature using existing constructions (SIS, LWE, LPN,…).

How about Attribute-based Signature??

Instantiating ABS

Instantiation 1:

[Tsabary17] gives ABS, but needs complexity leveraging for our purpose $\Rightarrow$ Need (subexp) SIS (due to mismatch of security notions).

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[Tsabary17] gives ABS, but needs complexity leveraging for our purpose ⇒ Need (subexp) SIS (due to mismatch of security notions).

Instantiation 2:

Weaken the security requirements for ABS by the following observations

- Bounded key queries is sufficient
- Attributes for signing key can be determined before the setup of system (They are SKE keys)

⇒Directly construction from (poly) SIS.

Conclusion of [KY19]

① Construct the **first group signatures from lattices in the standard model**.

② Consider a new type of Multi-User DP-NIZK and construct it from ABS.

③ Constructions from various assumptions.

- ✓ SIS w/ subexp-modulus.
- ✓ LWE w/ poly-modulus.
- ✓ SIS w/ poly-modulus + LPN w/ const. noise rate

Some Open Questions

① Construct **group signatures** based on **poly SIS**.

*Ours require sub-exp SIS.

② Construct **ring signatures** w/o setup with **logarithmic signature size from lattices**.

*Linear size known from SIS.

③ Any other interesting notions for NIZKs??
(e.g., malicious DV-NIZK [QRW19])

④ Explore other links between various types of NIZK and signatures.