Simulation and Validation of Models for Interest Rate Risk

Johan Hagenbjörk, Jörgen Blomvall

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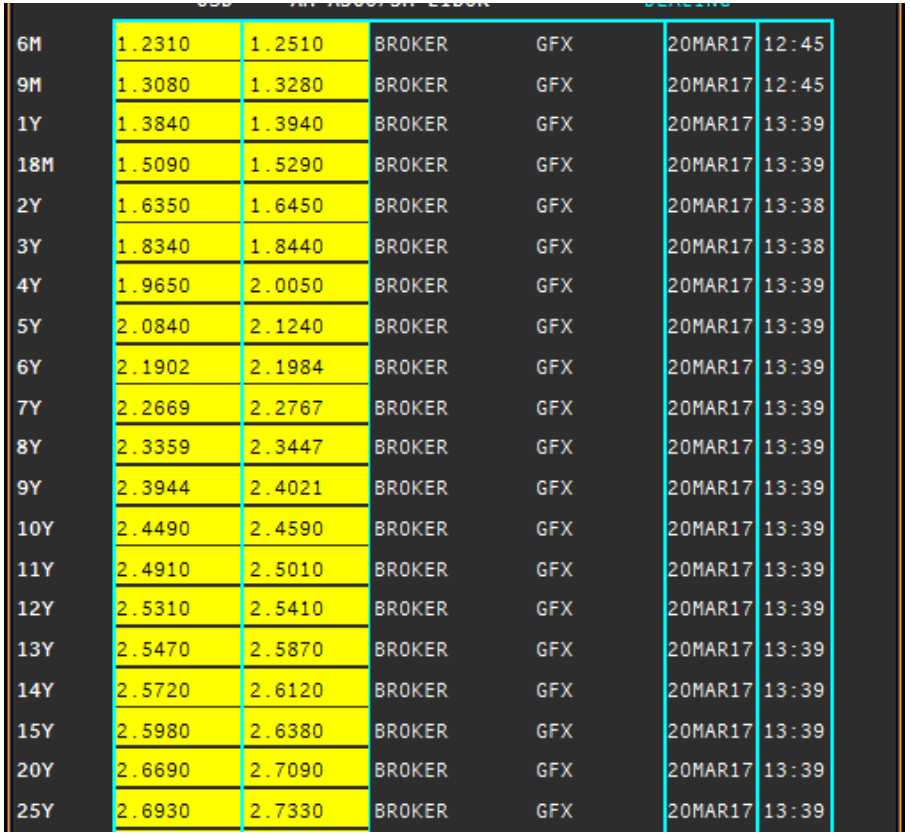
The Interest Rate Market

- Equity Market: 57,000 BUSD.
- Bond Market: 49,000 BUSD.
- Interest Rate Swap Market: 514,000 BUSD (notional).



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9M	1.3080	1.3280	BROKER	GFX	20MAR17	12:45
1Y	1.3840	1.3940	BROKER	GFX	20MAR17	13:39
18M	1.5090	1.5290	BROKER	GFX	20MAR17	13:39
2Y	1.6350	1.6450	BROKER	GFX	20MAR17	13:38
3Y	1.8340	1.8440	BROKER	GFX	20MAR17	13:38
4Y	1.9650	2.0050	BROKER	GFX	20MAR17	13:39
5Y	2.0840	2.1240	BROKER	GFX	20MAR17	13:39
6Y	2.1902	2.1984	BROKER	GFX	20MAR17	13:39
7Y	2.2669	2.2767	BROKER	GFX	20MAR17	13:39
8Y	2.3359	2.3447	BROKER	GFX	20MAR17	13:39
9Y	2.3944	2.4021	BROKER	GFX	20MAR17	13:39
10Y	2.4490	2.4590	BROKER	GFX	20MAR17	13:39
11Y	2.4910	2.5010	BROKER	GFX	20MAR17	13:39
12Y	2.5310	2.5410	BROKER	GFX	20MAR17	13:39
13Y	2.5470	2.5870	BROKER	GFX	20MAR17	13:39
14Y	2.5720	2.6120	BROKER	GFX	20MAR17	13:39
15Y	2.5980	2.6380	BROKER	GFX	20MAR17	13:39
20Y	2.6690	2.7090	BROKER	GFX	20MAR17	13:39
25Y	2.6930	2.7330	BROKER	GFX	20MAR17	13:39

Contents

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- *Modelling*
- *Evaluation*
- *Simulation*
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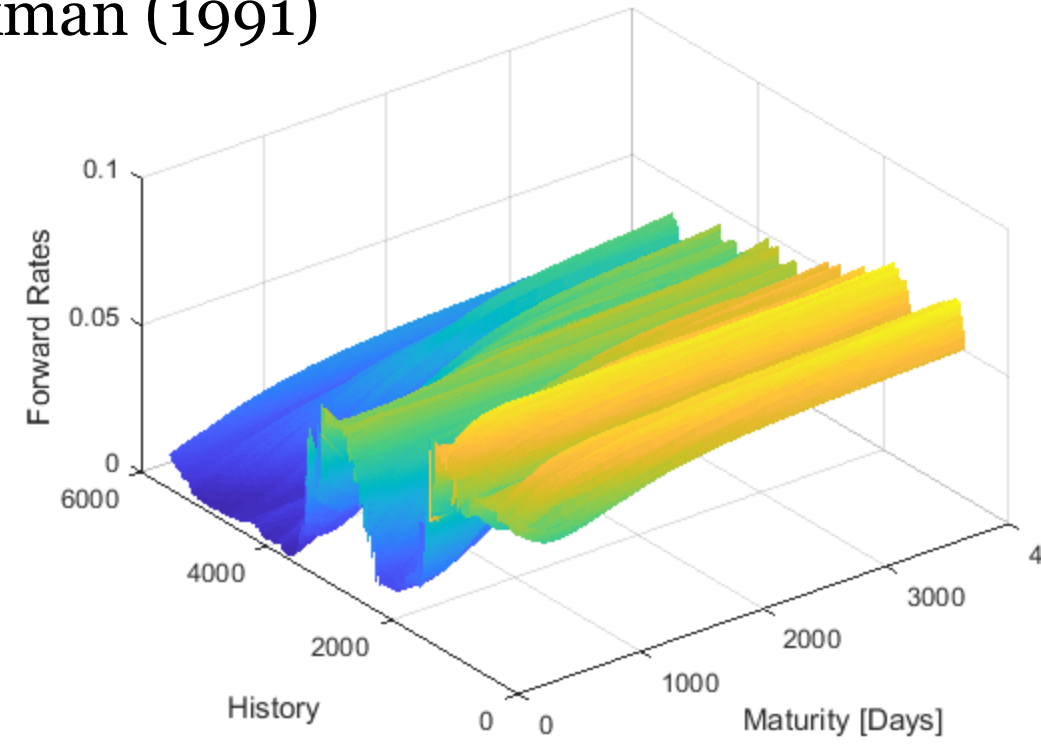
- *Conclusions*

Background

- Our hedging study using a rather primitive interest rate risk model proved successful.
  - Can we improve this further?
- Important to be able to simulate risk factors in finance (CVA, VaR).
- Improve our theoretical understanding of interest rate risk.

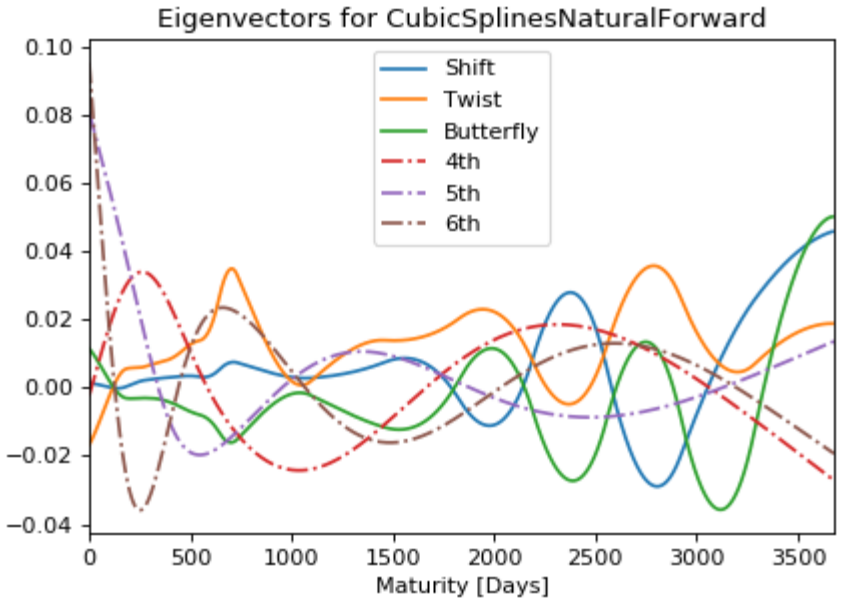
Interest Rate Risk

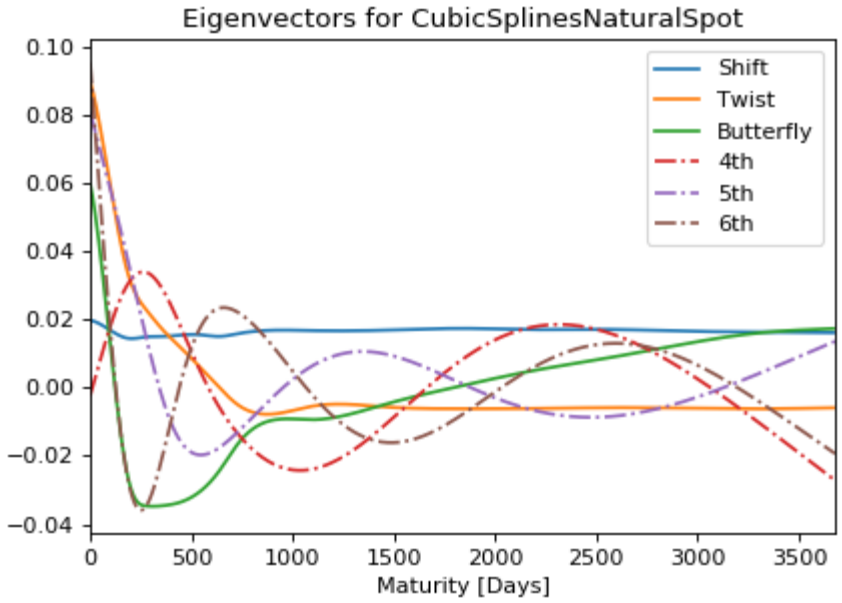
- Principal Component Analysis (PCA) Litterman and Scheinkman (1991)
  - Day to day changes.
- Eigenvectors become our risk factors.
- Principal component time series describe daily shocks to these risk factors.



Risk Factors for Cubic Splines

- Cubic splines are standard methodology in practical finance.
- Spot rates are often used for PCA.





Yield Curve Estimation using Blomvall (2017)

Regularization: roughness in the estimated curves

Price errors for included instruments

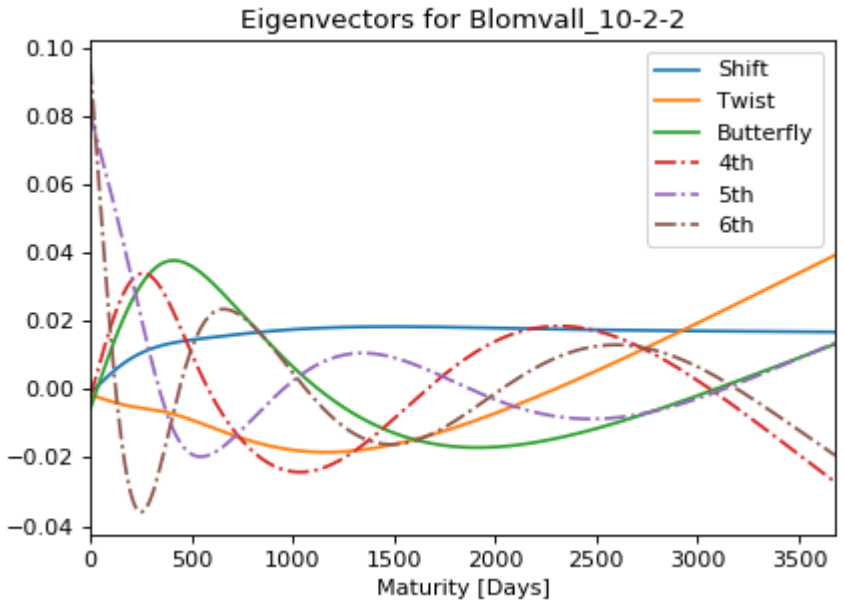
$$\min_f \; h(f) + \frac{1}{2} z_e^T E z_e$$

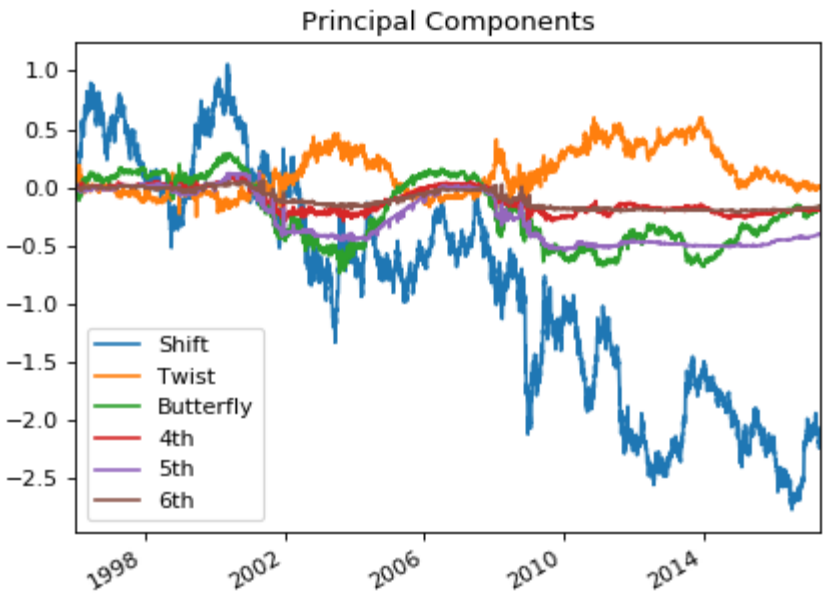
$$\text{s.t.} \quad g_e(f) + F_e z_e = p_{market}$$

Pricing function
Yield curve → price for different instruments

$$h(f) = \int_{T_0}^{T_n} a_0(t) \left(f(t) - \bar{f}(t) \right)^2 dt + \sum_{k=1}^{K} \int_{T_0}^{T_n} a_k(t) \left(f^{(k)}(t) - \bar{f}^k(t) \right)^2 dt$$

Risk Factors for Blomvall Forward Curves





Modelling

- *How to simulate the distribution of our risk factors?*

Monte Carlo Simulation

- Using a model for the volatility together with our risk factors we can simulate scenarios.
- Since our factors are approximately independent we use univariate models for each factor.
  - Constant Volatility:

$$\sigma_t = c$$

  - GARCH

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{o} \gamma_i \epsilon_{t-j}^2 I_{\epsilon_{t-j}<0} + \sum_{k=1}^{q} \beta_k \sigma_{t-k}^2$$

Monte Carlo Simulation

- We can also use different models for the mean while simulating principal component values y_t:
  - Zero

$$y_t = \epsilon_t$$

  - Constant

$$y_t = \mu + \epsilon_t$$

  - Autoregressive

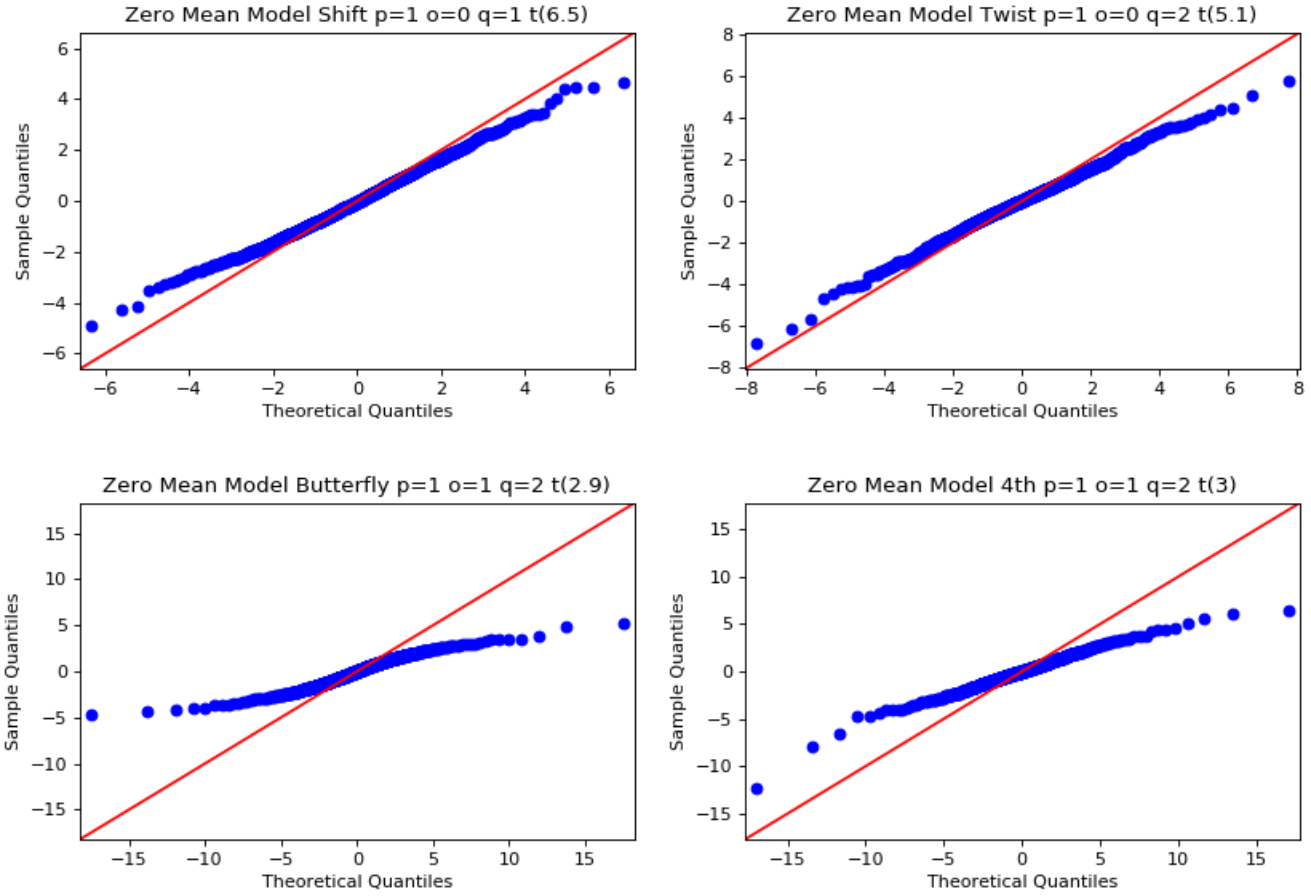
$$y_t = \mu + \sum_{i=1}^{p} \Phi_{L_i} y_{t-L_i} + \epsilon_t$$

Monte Carlo Simulation

- The residuals are drawn from either $N(0, \sigma)$ or Student-t$(0, \sigma, \nu)$ using our volatility models.
- The Akaike information criterion (AIC) is used to select the appropriate distribution and order of the GARCH-models.
- Using the inversion principle, uniformly distributed random variables are used to generate scenarios

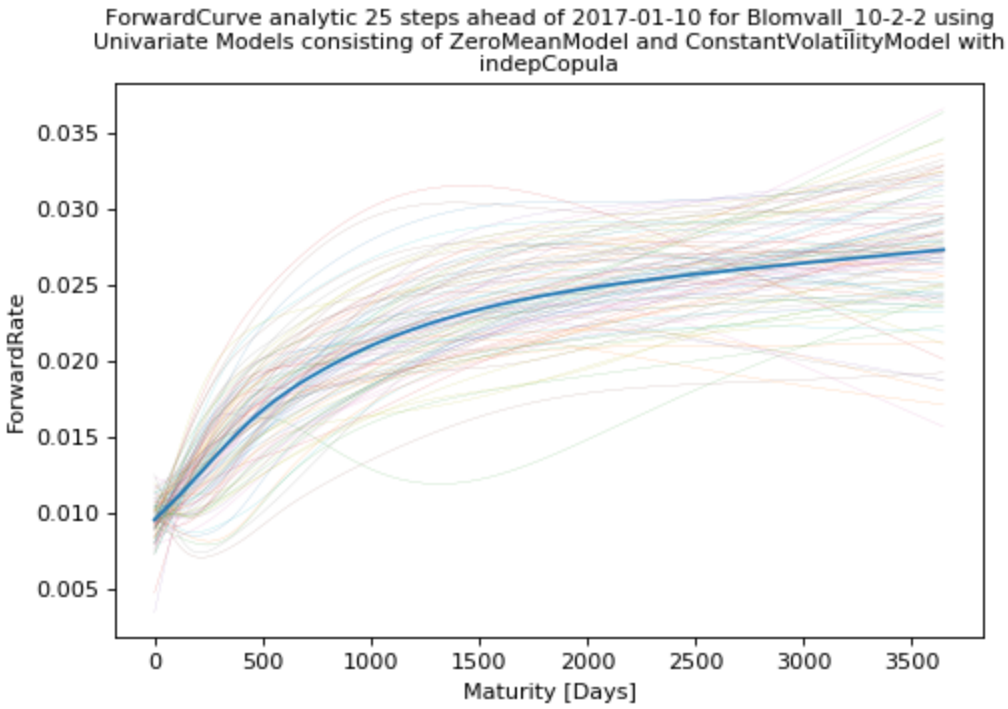
$$X \sim U(0,1)$$
$$\epsilon = F^{-1}(X)$$

Out-of-Sample QQ-plots for Blomvall_10-2-2 using analytic Univariate Models consisting of ZeroMeanModel and GARCHModel with indepCopula



Term Structure Scenarios

- Using risk factors together with random variables from our models we can generate term structure scenarios.



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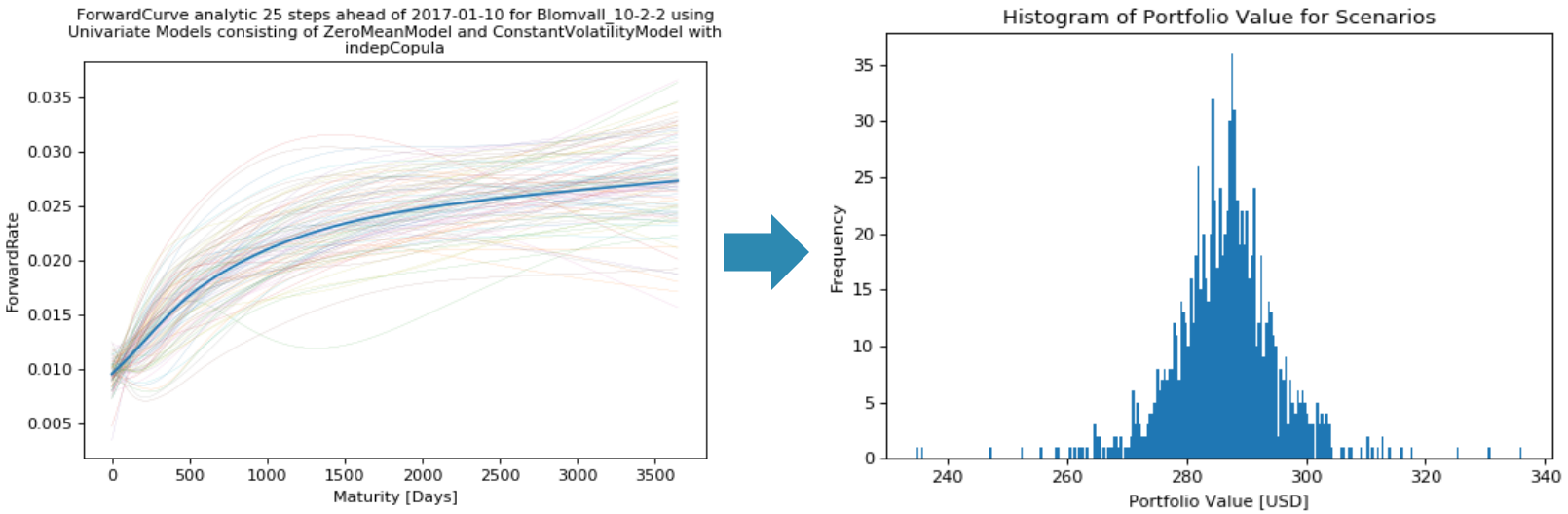
Evaluation

- *How do we evaluate our simulation methods?*

Evaluation

- The realized term structure could be compared to the scenarios
  - Very high dimension!
- In practice we are interested in using the generated scenarios for pricing.
  - We construct a portfolio to valuate using all scenarios and the realized term structure.
  - Dimension is now one!

Illustration of Scenario Generation



- One problem remains: not a continuous distribution!

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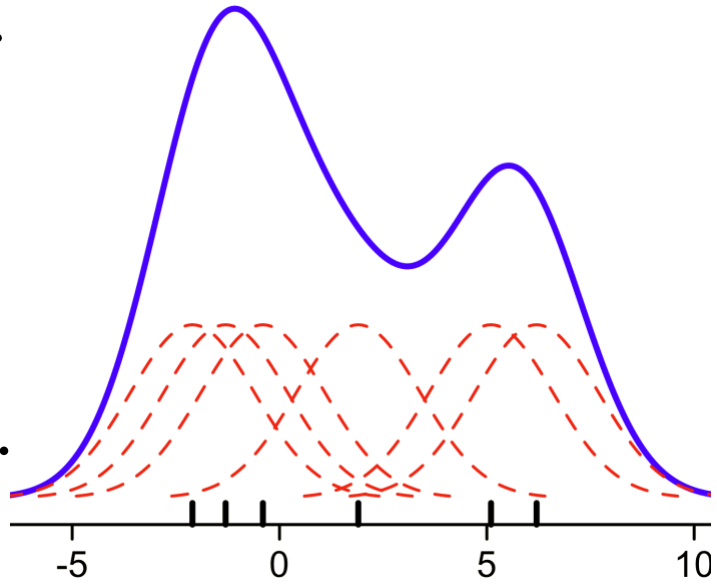
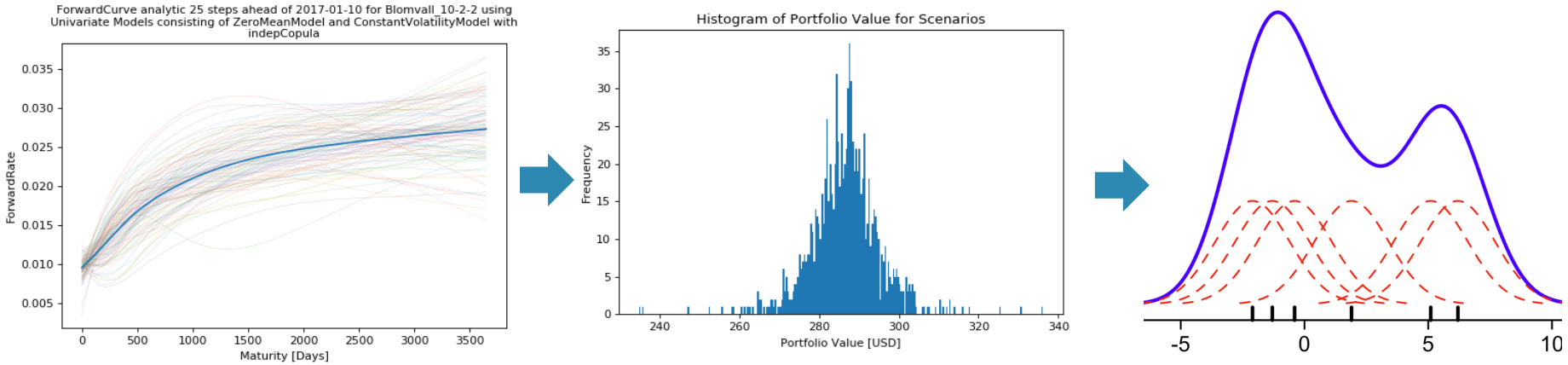
Estimation of a Continuous Distribution

- Use a kernel estimator as described in Kristensen, Shin (2012)
  - Place a Gaussian distribution centered in each observation and normalize.
  - Use $\left(\frac{4s^5}{3n}\right)^{\frac{1}{5}}$ as variance, where s is the sample standard deviation and n is the number of samples.



A Statistical Test



- Diebold, Gunter and Tay (1998)
  - If realized outcomes transformed by the cdf are uniformly distributed, then the cdf will be preferred independently of the loss function.

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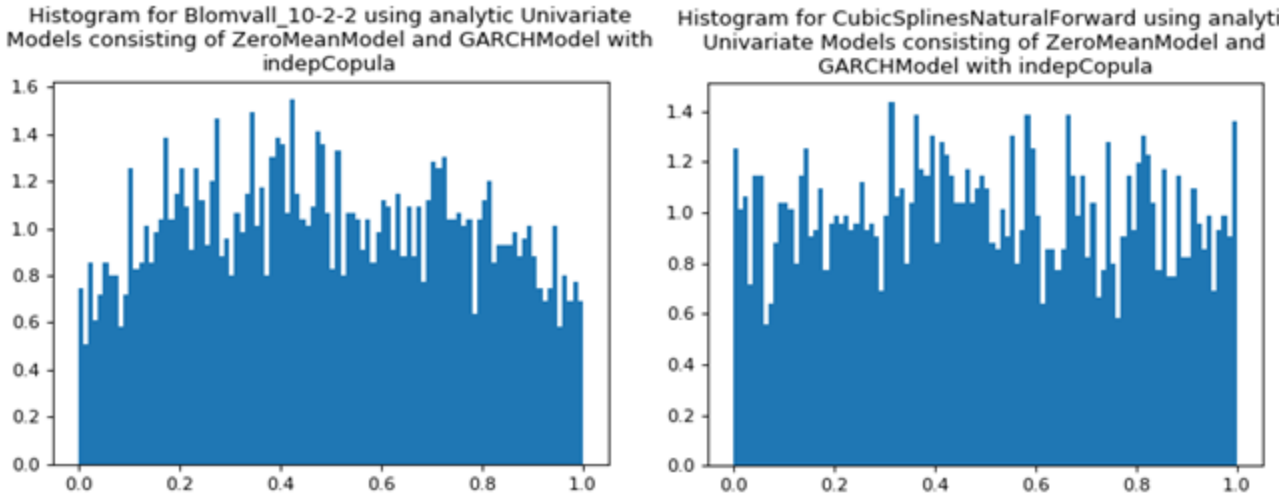
A Statistical Test



- Giacomini and White (2006)
  - Wald type test statistic to compare the log likelihood ratio between two models using $\chi^2(1)$.

[redacted]

Simulation

- *How is the simulation carried out?*

SWAP Portfolio Generation

- Randomize the Notional amount USD $100x$, $x \sim N(0,1)$ (paying/receiving).
- 2001-2002
  - Buy 1-10Y each day
- 2002-
  - Buy 10Y each day.
- This results in approximately one cash flow each future day with around 2500 active instruments.

SWAP simulation

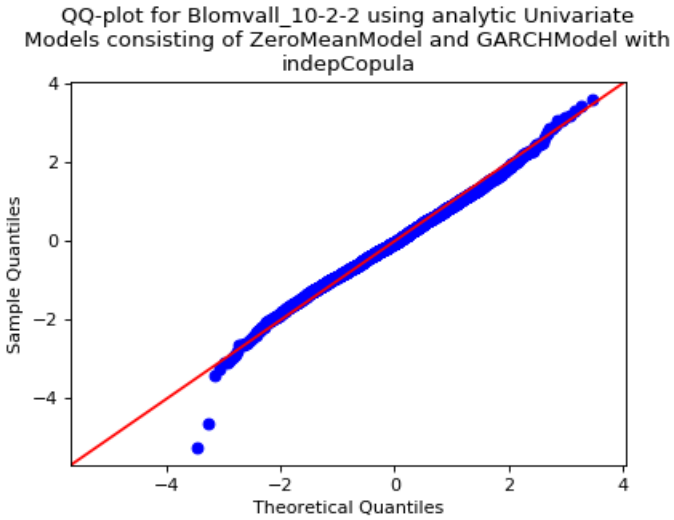
- In sample estimation 1996-2002
- Out-of-sample backtest 2002-2017 (3755 days)
- Simulate one day ahead.
- 1000 scenarios each day.
- Recalibration of models to newest data every year.

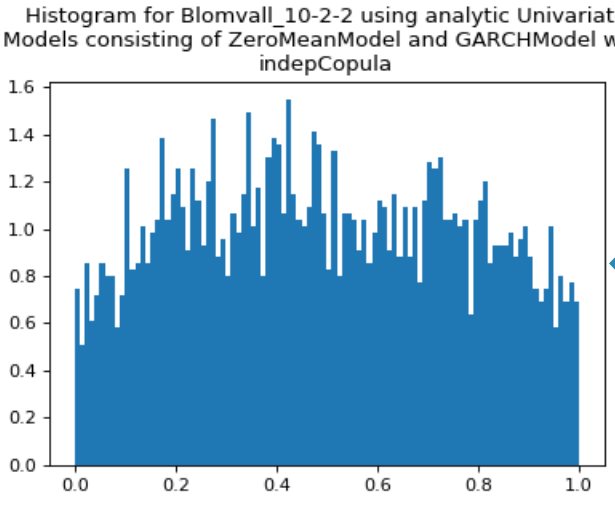
- Using a customized version of QuantLib (C++) we can simulate one day in 1,5 seconds per model.

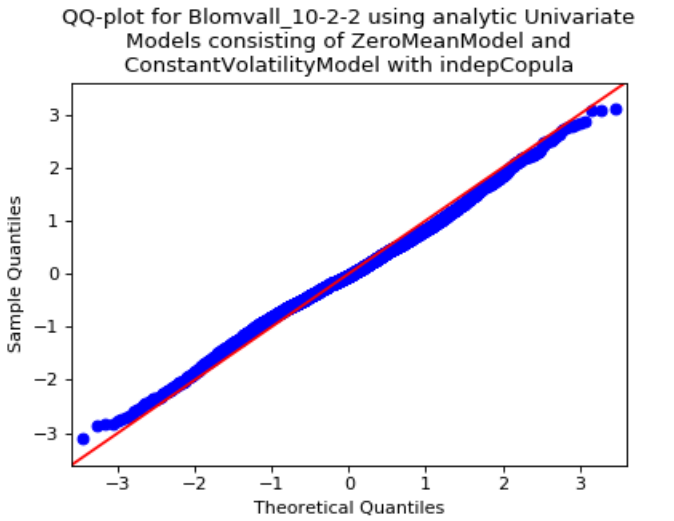
Results

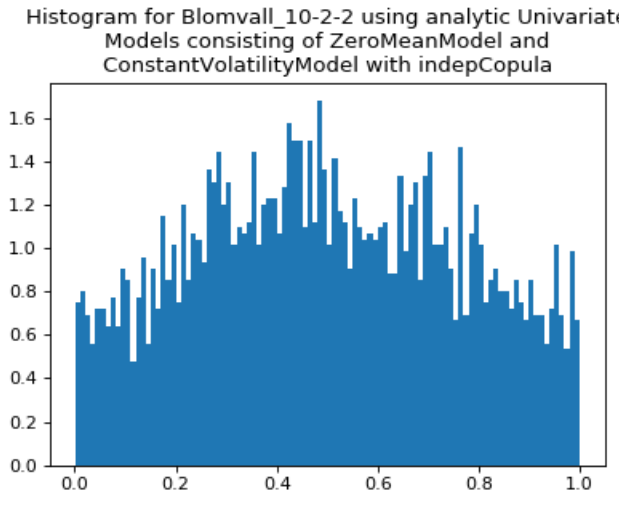
- *How well did our models capture the interest rate risk?*

Blomvall – GARCH vs Constant Vol



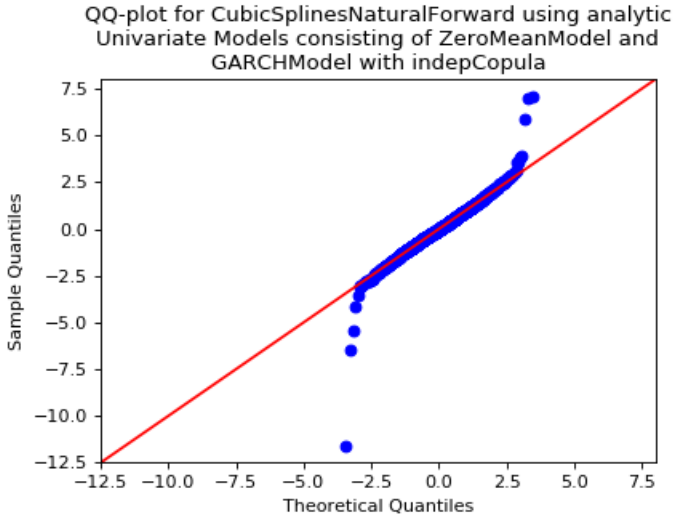


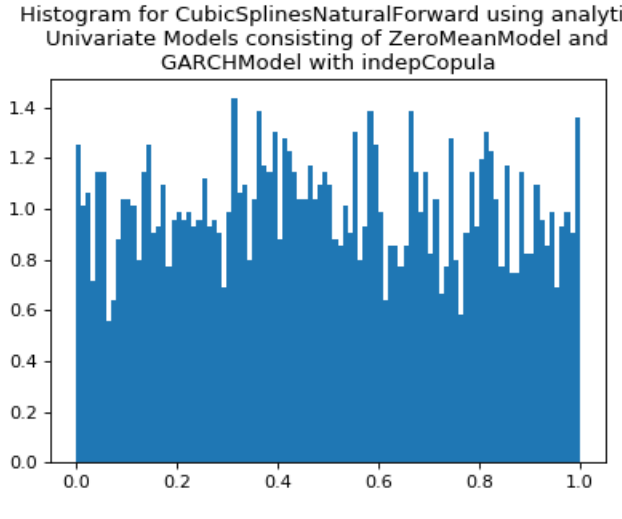


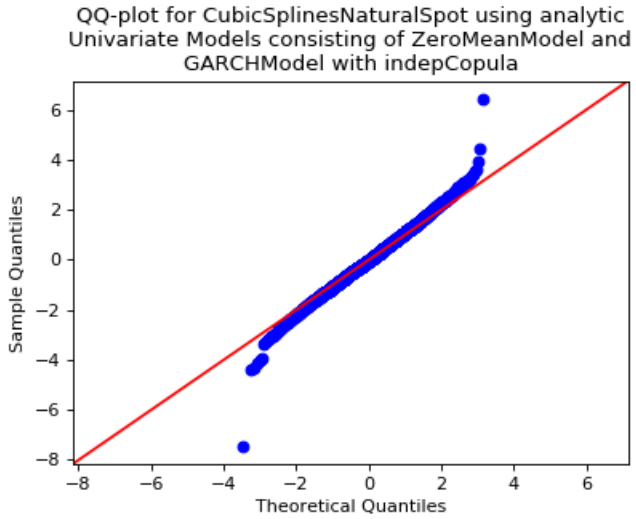


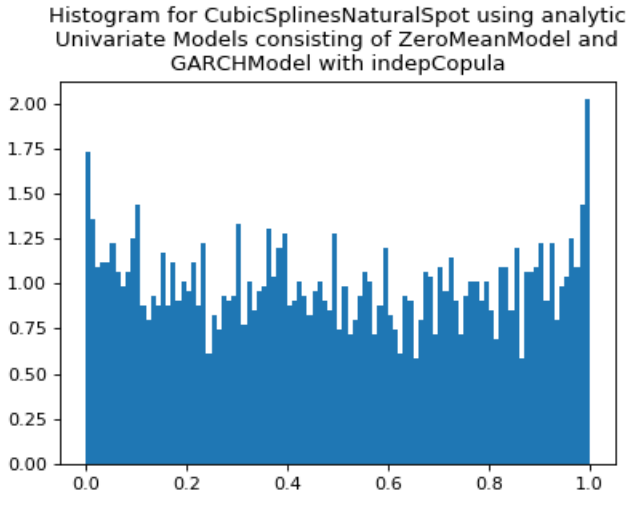
T-stat	102,285
P-value	4.80e-24

Cubic Splines – Forward vs Spot Rates



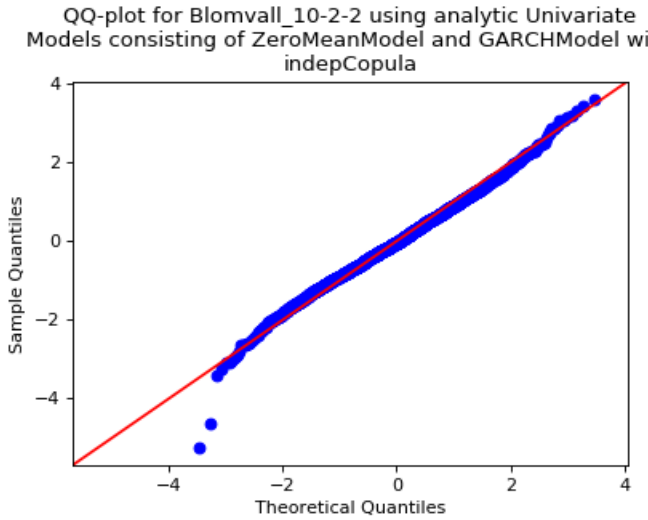


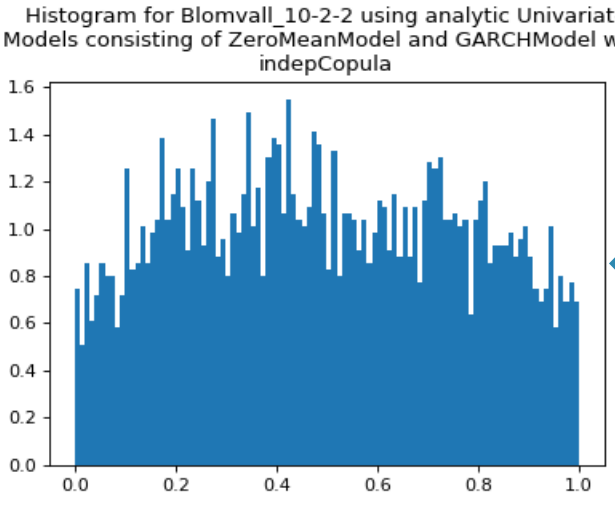


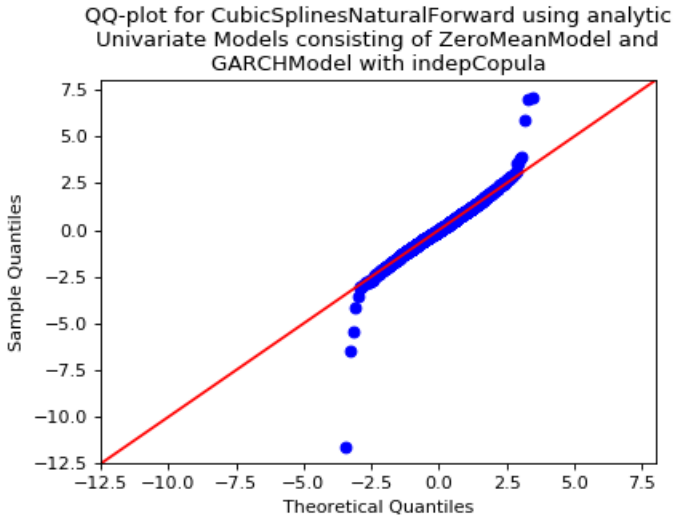


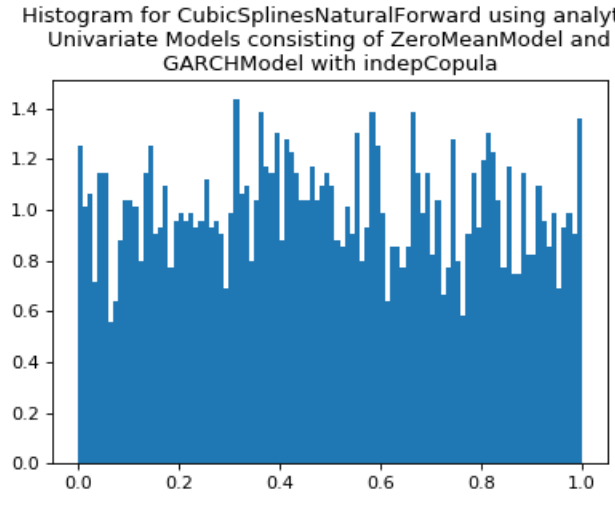
T-stat	0.04067
P-value	0.84

Forward Rates – Blomvall vs Cubic Splines







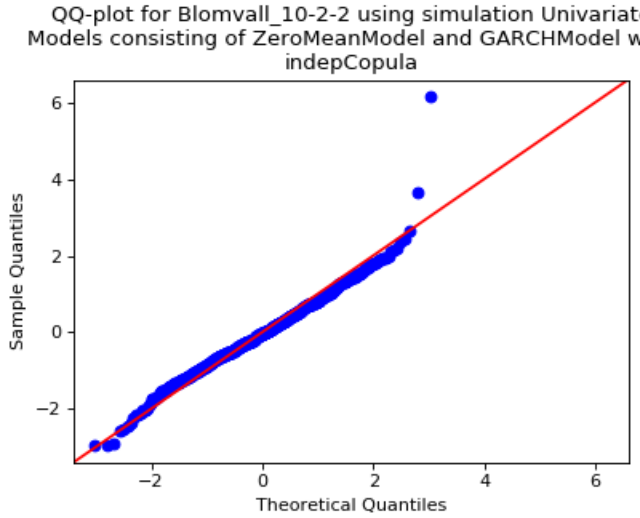


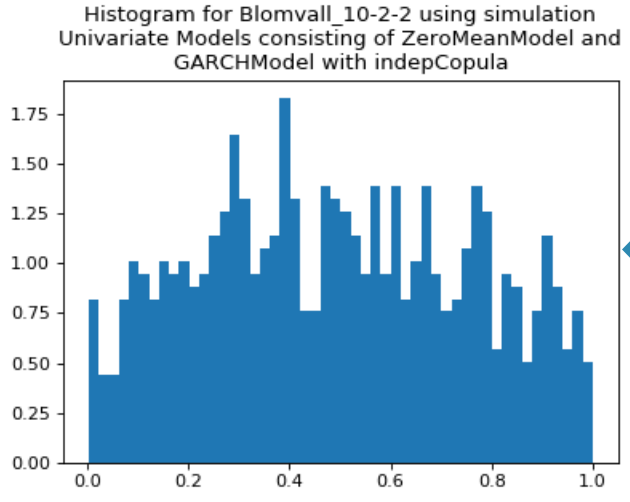
T-stat	57,57
P-value	3.257e-14

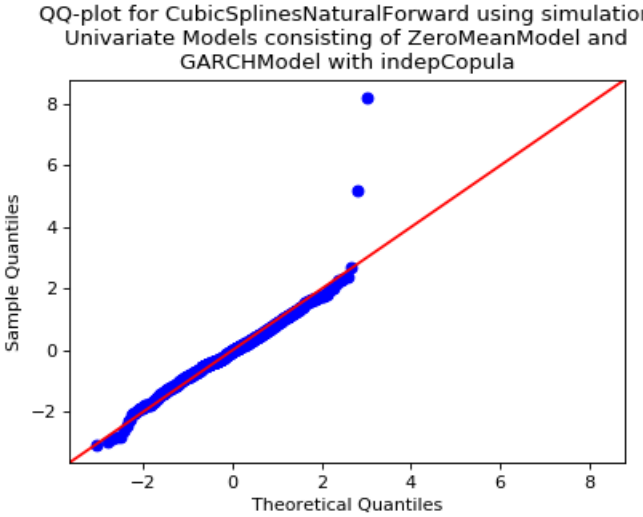
Simulations for longer horizons

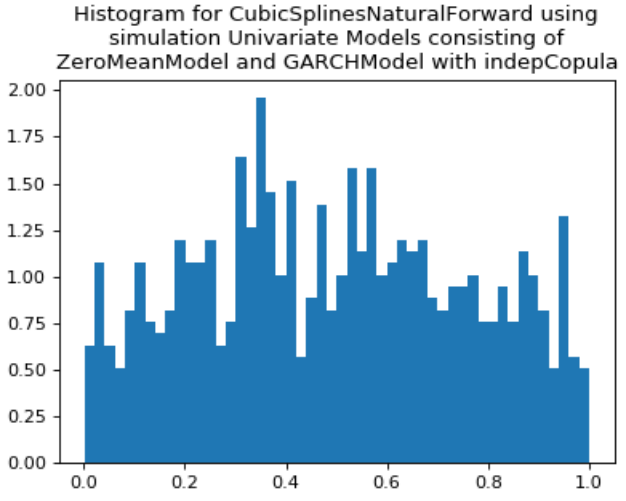
- Two possibilities to simulate our models
  - Analytical forecast: set shocks to 0
    - Mean reverting.
  - Simulation forecast: simulate shocks
    - Path dependent trajectories.
    - Computationally and memory intensive!
- Term structure paths must be simulated in order to set appropriate fixing rates for all scenarios.
- Statistical test requires independent observations!

Weekly – Blomvall vs Cubic Splines







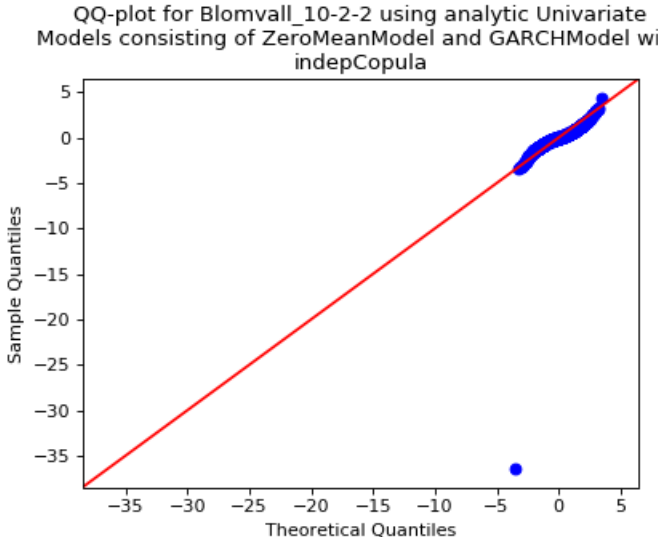


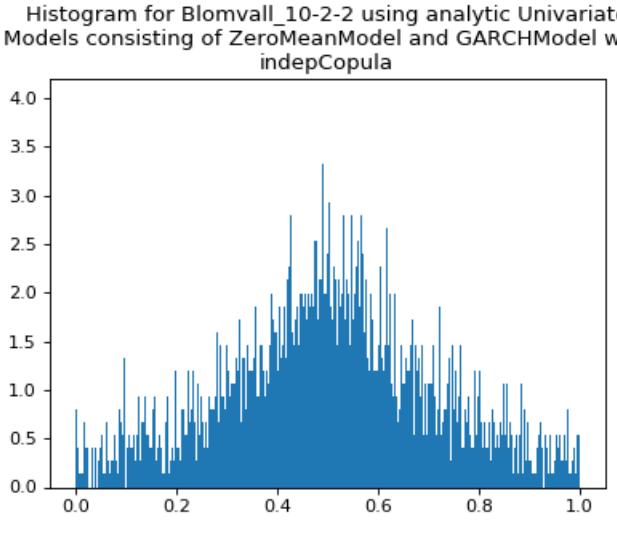
T-stat	12.256
P-value	0.0805207

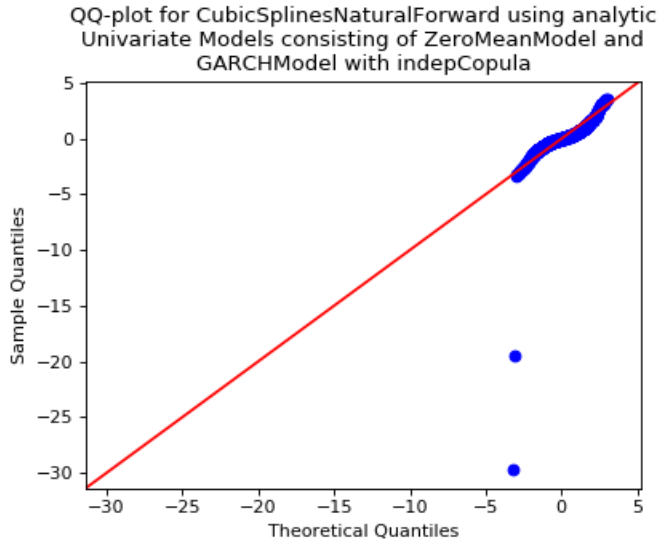
Forward Rate Agreements (FRA)

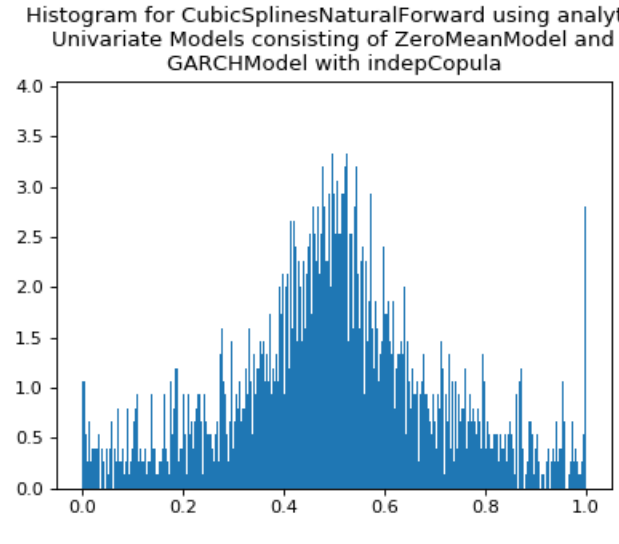
- Forward Rate Agreements are more sensitive to the forward rate than SWAPs.
- Use the same technique but build portfolio using FRA.
  - Faster simulation due to shorter maturity.

FRA – Blomvall vs Cubic Spline









T-stat	2067
P-value	0

Conclusions

- *What have we learnt so far?*

Conclusions

- Huge market where risks needs to be both measured and handled!
- Yield curve estimation is important!
- With high statistical significance, we have proved that cubic splines give an inappropriate distribution for interest rate risk.
  - PCA on spot rate changes does not help.

References

- Blomvall, Jörgen. "Measurement of interest rates using a convex optimization model." *European Journal of Operational Research* 256.1 (2017): 308-316.
- Diebold, F. X., Gunther, T. A. and Tay, A. S. (1998) *International Economic Review*, 39, 863-883.
- Giacomini, Raffaella, and Halbert White. "Tests of conditional predictive ability." *Econometrica* 74.6 (2006): 1545-1578.
- Kristensen, D. and Shin, Y., Estimation of dynamic models with nonparametric simulated maximum likelihood. *J. Econom.*, 2012, 167, 76–94.
- Litterman, R., Scheinkman, J., 1991. Common factors affecting bond returns. *Journal of Fixed Income* 1, 54–61.

Johan Hagenbjörk

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