ECE 1505 Convex Optimization Project

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In [440]:
```
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import pandas as pd
import cvxpy as cp
import matplotlib.cm as cm
from datetime import datetime
import scipy.linalg as la
import yfinance as yf
from sklearn.metrics.pairwise import euclidean_distances
import pickle as pkl
sns.set_style('whitegrid')
%matplotlib inline
```

Run the code below to get tickers from wikipedia

```
table=pd.read_html('https://en.wikipedia.org/wiki/List_of_S%26P_500_companies')
df = table[0]
df.to_csv('S&P500-Info.csv')
df.to_csv("S&P500-Symbols.csv", columns=['Symbol'])
```

The next bit of code using the iex api to get historical info for the current S&P 500 stocks. (The same approach adopted in Boyd)

Our period of analysis is will be start January 2012 to end of March 2020

In [441]:
```
sp = pd.read_csv('S&P500-Symbols.csv', index_col=[0])
```

I tried using the iex data provider, however they limit to a 5 year history. Code is below

```
from iexfinance.stocks import Stock
from iexfinance.stocks import get_historical_data

API_TOKEN = '''
def getHistoricalPrices(stock):
    return get_historical_data(stock, start, end,
                               output_format='pandas',
                               token=API_TOKEN)
```

This code extracts the data from yahoo finance

```
data = yf.download(sp.values[:,0].tolist(), start="2012-01-01", end="2020-03-31")
data.to_pickle("stocks.pkl")
```

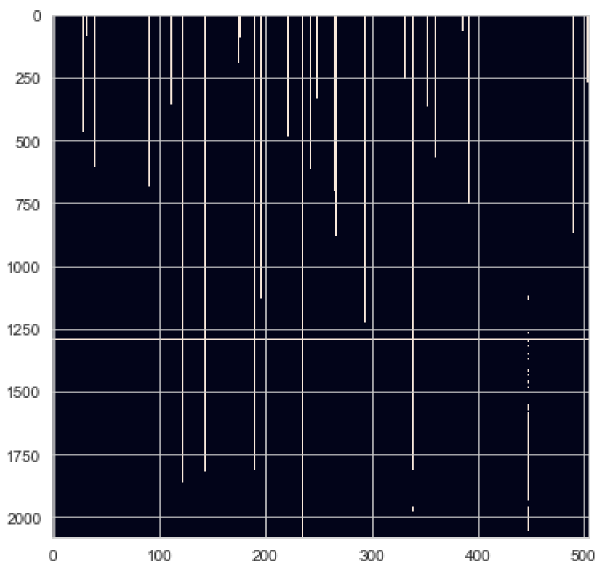
In [442]:
```
#use the adjusted closing prices for the analysis
data = pd.read_pickle("stocks.pkl")
data['Adj Close'].head()
```

Out[442]:

	A	AAL	AAP	AAPL	ABBV	ABC	ABMD	ABT	ACN	ADBE	...	XLNX
Date												
2012-01-03	23.818647	4.826837	67.798805	50.994907	NaN	33.385769	18.250000	21.986044	44.309837	28.570000	...	26.864021
2012-01-04	23.629299	4.741990	69.388268	51.268970	NaN	33.499771	18.110001	21.900763	44.293118	28.280001	...	26.582462
2012-01-05	24.158173	5.156795	69.829826	51.838169	NaN	33.754089	18.270000	21.850374	43.447910	28.480000	...	26.814333
2012-01-06	24.419334	5.279352	70.153603	52.380054	NaN	33.841782	18.139999	21.652693	43.372604	28.719999	...	26.640434
2012-01-09	25.059198	5.392481	70.133995	52.296970	NaN	33.824242	18.209999	21.648811	43.297291	28.530001	...	27.220114

5 rows × 505 columns

In [443]:
```
#### Investigate Missing Values
plt.figure(figsize = (6,6))
plt.imshow(data['Adj Close'].isna().astype(int),aspect='auto');
```



Boyd's Paper drops the series that do not contain continuous observations

paper ctrl-f 'S&P' for details [https://stanford.edu/~boyd/papers/pdf/cvx_portfolio.pdf (https://stanford.edu/~boyd/papers/pdf/cvx_portfolio.pdf)](https://stanford.edu/~boyd/papers/pdf/cvx_portfolio.pdf)

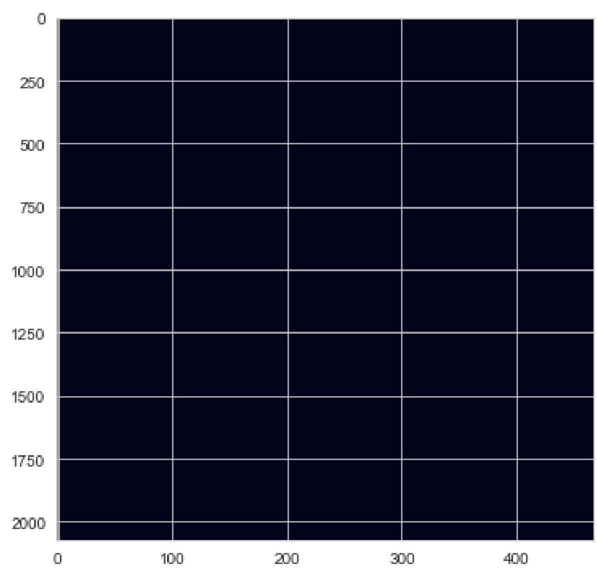
In [444]:
```python
P = data['Adj Close']
P = P[~P.isna().all(axis=1)] #dropping rows with all missing values
P = P.loc[:,~P.isna().any(axis=0)]
P.head()
```

Out[444]:

	A	AAL	AAP	AAPL	ABC	ABMD	ABT	ACN	ADBE	ADI	...	
Date												
2012-01-03	23.818647	4.826837	67.798805	50.994907	33.385769	18.250000	21.986044	44.309837	28.570000	29.164673	...	20.369
2012-01-04	23.629299	4.741990	69.388268	51.268970	33.499771	18.110001	21.900763	44.293118	28.280001	29.124205	...	20.235
2012-01-05	24.158173	5.156795	69.829826	51.838169	33.754089	18.270000	21.850374	43.447910	28.480000	29.253723	...	20.324
2012-01-06	24.419334	5.279352	70.153603	52.380054	33.841782	18.139999	21.652693	43.372604	28.719999	29.059444	...	20.265
2012-01-09	25.059198	5.392481	70.133995	52.296970	33.824242	18.209999	21.648811	43.297291	28.530001	29.626060	...	20.280

5 rows × 468 columns

In [445]:
```python
#### No more missing values
plt.figure(figsize = (6,6))
plt.imshow(P.isna().astype(int),aspect='auto');
```



We will follow the same conventions in the paper 'Detecting change points in VIX and S&P 500: A new approach to dynamic asset allocation' by Nystrup et. al

$r_t = (P_t - P_{t-1})/P_{t-1}$

In [446]:
```python
#returns
R = P.pct_change().dropna(axis=0)
```

Changepoint detection:

Given a series, a window to define the changepoint, use the wasserstein hypothesis estimator to detect the change

If there is a change then we will reoptimize the portfolio, with an ambiguity parameter that is proportional to the size of the estimated change?

Change point detection and hypothesis testing

The paper titled "Robust Sequential Change-Point Detection by Convex Optimization" - Cao et. al frames change point detection as testing ...
to be continued

The above method does not work . . . because you cannot really tell if there is a difference. Gao et. al propose the formulation and solution of the following optimization problem as way to mitigate these issues. This entire cell is taken from Gao et. al . The formulation is outlined below:

Optimal Detectors

$\phi : \Omega \to \mathbb{R}$ associated with a test $T : \Omega \to \{1, 2\}$ (for hypothesis 1 and 2). For $\omega \in \Omega$ (a sample), the worst case risk the max of the type 1 and type 2 errors

$$\inf_{\phi:\Omega\to\mathbb{R}} \max(\sup_{P_1\in\mathcal{P}_1} P_1[\omega : \phi(\omega) < 0], \ \sup_{P_1\in\mathcal{P}_2} P_2[\omega : \phi(\omega) \geq 0]), (\mathrm{P1})$$

Definition 1.

Safe Approximation of problem (3) Risk of a detector for a test $P_1, P_2, l : \mathbb{R} \to \mathbb{R}_+ \cup \infty$

$$\epsilon(\phi|P_1, P_2) = \sup_{P_1\in\mathcal{P}_1, P_2\in\mathcal{P}_2} \mathbb{E}_{P_1}[l \circ (-\phi)(w)] + \mathbb{E}_{P_2}[l \circ (\phi)(w)]$$

$$\Phi(\phi; P_1, P_2) = \mathbb{E}_{P_1}[l \circ (-\phi)(w)] + \mathbb{E}_{P_2}[l \circ (\phi)(w)]$$

The problem below is an upper bound to P1

$$\inf_{\phi:\Omega\to\mathbb{R}} \sup_{P_1\in\mathcal{P}_1, P_2\in\mathcal{P}_2} \Phi(\phi; P_1, P_2), \ (\mathrm{P2})$$

The problem above is near optimality of the nominal problem

Consider $\psi(p) := \min_{t\in\mathbb{R}}[pl(t) + (1-p)l(-t)], \ p \in [0, 1]$

Then whenever there exists a solution to P1 with objective ϵ less than 0.5, then there exists a solution to P2 with objective value less than $\psi(\epsilon)$

In P2; we can exchange the sup and inf operators

$$\inf_{\phi:\Omega\to\mathbb{R}} \sup_{P_1\in\mathcal{P}_1, P_2\in\mathcal{P}_2} \Phi(\phi; P_1, P_2) = \sup_{P_1\in\mathcal{P}_1, P_2\in\mathcal{P}_2} \inf_{\phi:\Omega\to\mathbb{R}} \Phi(\phi; P_1, P_2)$$

Best detector for a simple test can be found first and the least favorable distribution can be found after. This is tricky because there is no sion minmax theorem that would help (see the paper for more details)

Theorem 2: Optimal detector

For any P_1 and P_2, let $\frac{dP_k}{d(P_1+P_2)}$ be the radon nikodym derivative of P_k w.r.t $P_1 + P_2$ then

$$\inf_{\phi:\Omega\to\mathbb{R}} \Phi(\phi; P_1, P_2) = \int_\Omega \psi(\frac{dP_k}{d(P_1 + P_2)})d(P_1 + P_2)$$

Let $\Omega_0(P_1, P_2) := \{\omega \in \Omega | 0 < \frac{dP_k}{d(P_1+P_2)}(\omega) < 1, k = 1, 2\}$ and suppose

$$t^*(\omega) = \mathrm{argmin}_{t\in\mathbb{R}}\big[\frac{dP_2}{d(P_1 + P_2)}l(t) + \frac{dP_1}{d(P_1 + P_2)}l(-t)]]$$

then $\phi(\omega) := -t^*(\omega)$

Remarks on Theorem 2

- $\int_\Omega \psi(\frac{dP_k}{d(P_1+P_2)})d(P_1 + P_2)$ is a measure of closeness (- divergence measure . . .) meaning that the worst case probability distribution in terms of optimal detection is one where the distribution is close. A major problem is now, how can one actually solve these problems given samples of data.

Tractable Convex Formulation

Given two empirical distributions $Q_k = \frac{1}{n_k}\sum_{i=1}^{n_k} \delta_{\bar{\omega}}, \ k = 1, 2$ for $l = 1, \ldots, n_1 + n_2$

$$\omega^l = \begin{cases} \bar{\omega}^l, & 1 \leq l \leq n_1 \\ \bar{\omega}^{l-n_1}, & n_1 + 1 \leq l \leq n_1 + n_2 \end{cases}$$

Set $\bar{\Omega} := \{\omega^l : l = 1, \ldots, n_1 + n_2\}$

Theorem 3

$$\sup_{P_1 \in \mathcal{P}_1, P_2 \in \mathcal{P}_2} \int_\Omega \psi\left(\frac{dP_k}{d(P_1+P_2)}\right) d(P_1+P_2)$$

where $\mathcal{P}_k$ are wasserstein ambiguity sets, can be equivalently reformulated as a finite dimensional convex program

$$\begin{aligned}
\max_{\substack{p_1, p_2 \in \mathbb{R}_+^{n_1+n_2} \\ \gamma_1, \gamma_2 \in \mathbb{R}_+^{n_1+n_2} \times \mathbb{R}_+^{n_1+n_2}}} \quad & \sum_{l=1}^{n_1+n_2} (p_1^l + p_2^l)\psi\left(\frac{p_1^l}{p_1^l + p_2^l}\right) \\
\text{s.t.} \quad & \sum_{l=1}^{n_1+n_2} \sum_{m=1}^{n_1+n_2} \gamma_k^{lm} ||\omega^l - \omega^m|| \leq \theta_k, \quad k = 1, 2 \\
& \sum_{m=1}^{n_1+n_2} \gamma_1^{lm} = \frac{1}{n_1},\ 1 \leq l \leq n_1, \quad \sum_{m=1}^{n_1+n_2} \gamma_1^{lm} = 0,\ n_1 + 1 \leq l \leq n_1 + n_2 \\
& \sum_{m=1}^{n_1+n_2} \gamma_2^{lm} = 0,\ 1 \leq l \leq n_1, \quad \sum_{m=1}^{n_1+n_2} \gamma_2^{lm} = \frac{1}{n_2},\ n_1 + 1 \leq l \leq n_1 + n_2 \\
& \sum_{l=1}^{n_1+n_2} \gamma_k^{lm} = p_k^m,\ 1 \leq m \leq n_1 + n_2,\ k = 1, 2
\end{aligned}$$

The table below provides the main ingredients to construct and solve the robust hypothesis testing problem. (See Table 1 in Gao et. al)

alt text

I emailed the authors to figure out how they actually implemented the test and they used n_1 and n_2 = 10

"we use n1 = n2 = 10 as the size of empirical samples. Fig 3(a) shows the results for a subsequence with 100 samples, and at each time t, we use n1 samples before time t and n2 samples after time t as the empirical samples for k=1,2, respectively. Then we apply the obtained optimal detector for the sample at time t; this serves as our detection statistic. The intuition is that when activity changes at time t, then we expect a significant difference between the obtained p1*(x_t) and p2*(x_t); thus we can detect the change."

In [8]:

```python
t = len(R)-80
n = 30
theta_val = 0.1

def robust_hypothesis(t, n, theta_val, R, rand = False):

    Q_1  = R.values[t-n:t,:]
    Q_2 = R.values[t+1:t+n+1,:]
    n_1 = len(Q_1)
    n_2 = len(Q_2)

    if rand:
        Q_1  = np.random.choice(R.values, size=n_1, replace=True, p=None)
        Q_2 = np.random.choice(R.values, size=n_2, replace=True, p=None)
    #print(len(Q_1))
    #print(len(Q_2))
    #have to modify this to make it for multi dimensional inputs
    Q = np.concatenate((Q_1, Q_2))

    A = euclidean_distances(Q, Q)
    b_1 = np.concatenate((1/n_1*np.ones(n_1), np.zeros(n_2)))
    b_2 = np.concatenate((np.zeros(n_2), 1/n_2*np.ones(n_2)))

    ## Construct the problem.
    theta = cp.Parameter()
    p1 = cp.Variable(n_1+n_2)
    p2 = cp.Variable(n_1+n_2)

    gamma_1 = cp.Variable((n_1+n_2,n_1+n_2))
    gamma_2 = cp.Variable((n_1+n_2,n_1+n_2))

    theta.value = theta_val

    constraints = [cp.sum(cp.multiply(gamma_1,A)) <= theta,
                   cp.sum(cp.multiply(gamma_2,A)) <= theta,
                   cp.sum(gamma_1, axis = 1) ==  b_1,
                   cp.sum(gamma_2, axis = 1) ==  b_2,
                   cp.sum(gamma_1, axis = 0) ==  p1,
                   cp.sum(gamma_2, axis = 0) ==  p2,
                   p1 >= 0,
                   p2 >= 0,
                   gamma_1 >= 0,
                   gamma_2 >= 0]

    #objective = cp.Minimize((p1+p2)*2*cp.minimum(p1/(p1+p2),p2/(p1+p2) ))
    objective = cp.Minimize(-2*cp.sum(cp.minimum(p1,p2)))

    prob = cp.Problem(objective,
                      constraints)
    try:
        prob.solve()
    except Exception:
        try:
            prob.solve(verbose = False, solver=cp.SCS, max_iters=20000);
        except Exception:
            pass
    t = cp.sum(cp.multiply(gamma_1,A))
    #print(t.value)
    return [p1, p2, gamma_1, gamma_2, prob, Q]

[p1,p2, gamma_1, gamma_2, prob, Q] = robust_hypothesis(t, n, theta_val, R)
```

Detection Algorithm as per the paper titled 'Robust Sequential Change-Point Detection by Convex Optimization' by Cao et. al

$T_1 = \inf\{t > 0 | \max_{1 \le k \le t} \sum_{i=k}^{t} (-\phi(\omega^i)) \ge b\}$ this is known as the CUSUM detection method. T is the first time

From the hypothesis testing paper it also follows by the properties of the generating functions l that the type one error is bounded by the expected loss function evaluated under P_1. That is:

$\mathbb{P}_1(\phi(\omega) < 0) \le \mathbb{E}^{\mathbb{P}_1}[l(-\phi(\omega))]$

The formula above is only for one observation... therefore to evaluate/get a bound for the type one error for the CUSUM type detector one can follow the logic from appendix a of Cao et. al. The proof is as follows:

First note: $T = \inf\{t > 0 | \sum_{i=1}^{t} -\phi(\omega^i) > b\}$ is the same procedure as T_1 ... therefore the arguments about T hold for T_1

$$\begin{aligned}\mathbb{P}_1(T \le m) &\le \mathbb{P}_1\Big(\bigcup_{k=1}^{m}\Big\{\sum_{i=1}^{k} -\phi(\omega^i) > b\Big\}\Big)\\ &\le \sum_{k=1}^{m} \mathbb{P}_1\Big(\sum_{i=1}^{k} -\phi(\omega^i) > b\Big)\\ &= \sum_{k=1}^{m} \mathbb{P}_1\Big(\sum_{i=1}^{k} (-\phi(\omega^i) - \frac{b}{k}) > 0\Big)\end{aligned}$$

Letting $\phi_k^* = \phi + \frac{b}{k}$, implies:

$$\mathbb{P}_1(-\phi_k^*(\omega) > 0) \le \mathbb{E}^{\mathbb{P}_1}[l(-\phi_k^*(\omega))]$$

Therefore in the case of k repeated observations:

$$\mathbb{P}_1\Big(\sum_{i=1}^{k} (-\phi(\omega^i) - \frac{b}{k}) > 0\Big) \le \mathbb{E}^{\mathbb{P}_1}[l(-\phi_k^*(\omega))]^k$$

Then it follows that

$$\mathbb{P}_1(T \le m) \le \sum_{k=1}^{m} \Big(\mathbb{E}^{\mathbb{P}_1}[l(-\phi_k^*(\omega))]\Big)^k$$

Note that expressions depend on the form of the generating function. For example Goldenshluger et al use $\phi = \log(p_1/p_2)$ as a detector with $l(.) = \exp(.)$ and find that $\mathbb{E}^{\mathbb{P}_1}[\exp(-\phi^*(\omega))] \le \exp(\frac{-b}{k})\epsilon*$ which allows Cao et. al to derive that $\mathbb{P}_1(T < \infty) = \exp(-b) * \frac{\epsilon^*}{(1-\epsilon^*)}$ and since $\mathbb{E}^{\mathbb{P}_1}[T] \ge \frac{1}{\mathbb{P}_1(T<\infty)}$ the result follows

As discussed by Gao et. al: the detector $\phi(\omega) = \mathrm{sign}\{p_1(\omega) - p_2(\omega)\}$ has the smallest optimality gap. Therefore, we would like to use that detector.

Evaluate $\mathbb{E}^{\mathbb{P}_1}[l(-\phi_k^*(\omega))]$ for $k = 1 \ldots m$ in the case of hinge loss

$$\begin{aligned}\mathbb{E}^{\mathbb{P}_1}[l(-\phi_k^*(\omega))] &= \mathbb{E}^{\mathbb{P}_1}[(1 - \mathrm{sign}(p_1(\omega) - p_2(\omega)) - \frac{b}{k})^+]\\ &= \sum_{i=1}^{n_1+n_2} p_1^i(1 - \mathrm{sign}(p_1^i - p_2^i) - \frac{b}{k})^+ \quad \text{which implies} \sum_{k=1}^{m} \mathbb{P}_1\Big(\mathbb{E}^{\mathbb{P}_1}[l(-\phi_k^*(\omega))]^k\Big) \quad \text{is finite because}\\ &\sum_{i=1}^{n_1+n_2} p_1^i(1 - \mathrm{sign}(p_1^i - p_2^i) - \frac{b}{k})^+ \le \sum_{i=1}^{n_1+n_2} p_1^i(1 - \mathrm{sign}(p_1^i - p_2^i))^+ \le \epsilon^*\end{aligned}$$

Although, a closed form expression for the sum $\sum_{i=1}^{n_1+n_2} p_1^i(1 - \mathrm{sign}(p_1^i - p_2^i) - \frac{b}{k})^+$(and hence the ARL) is not clear, the sum can be numerically approximated with ease. Note that it is tempting to evaluate $\mu = \sum_{i=1}^{n_1+n_2} p_1^i(1 - \mathrm{sign}(p_1^i - p_2^i))^+$ and determine $\lim_{m\to\infty} \mathbb{P}_1[T < m]$ to be less than $\mu/(1-\mu)$ however, this bound is too loose to determine any average run length. If we want a fixed number of computations then we can define $M = \inf\{k | b/k < \text{tolerance}\}$ and evaluate the partial sum up to M and then use the geometric approximation to provide an upper bound for the remainder.

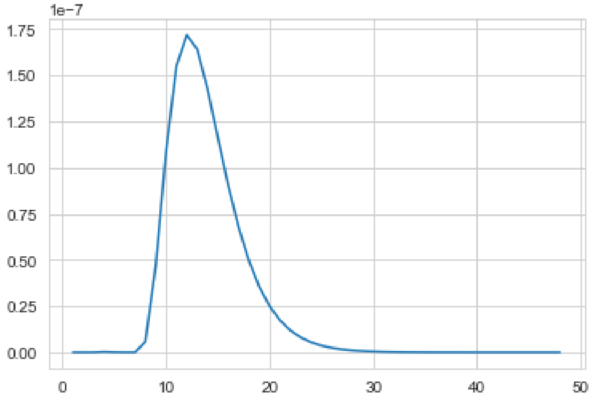
i.e:

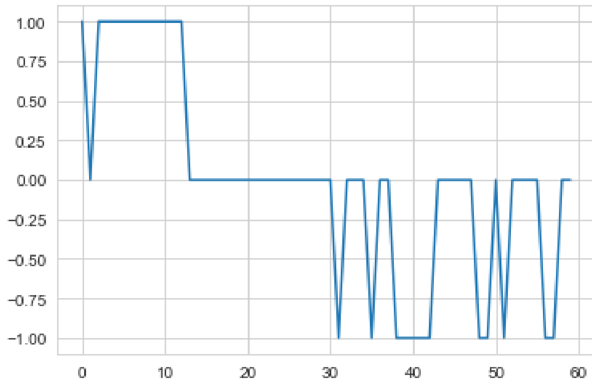
$$\sum_{k=1}^{\infty} \Big(\mathbb{E}^{\mathbb{P}_1}[(1 - \mathrm{sign}(p_1(\omega) - p_2(\omega)) - \frac{b}{k})^+]\Big)^k \le \sum_{k=1}^{M} \Big(\mathbb{E}^{\mathbb{P}_1}[(1 - \mathrm{sign}(p_1(\omega) - p_2(\omega)) - \frac{b}{k})^+]\Big)^k + \sum_{k=M+1}^{\infty} \Big(\mathbb{E}^{\mathbb{P}_1}[(1 - \mathrm{sign}(p_1(\omega) -$$

The sum is evaluated below $\sum_{i=1}^{n_1+n_2} p_1^i\,(1 - \mathrm{sign}(p_1^i - p_2^i) - \frac{b}{k})^+$

In [9]:
```
b = 7
signal = p1.value - p2.value
signal[np.abs(signal) < 10**(-3)*np.max(signal)] = 0
phi = np.sign(signal)

def detect_change(phi, b):
    S = np.zeros_like(phi)
    for t in range(len(phi)):
        max_sum = 0
        for k in range(t):
            if np.sum(-1*phi[k:t]) > max_sum:
                max_sum = np.sum(-1*phi[k:t])
        S[t] = max_sum
        if max_sum >= b:
            break
    return t , S
    #evaluate the max over k of the partial sum
T, S = detect_change(phi, b)

#The approach is to break the sum up into two parts
# The first part is to up k where b/k is actually large
# Second part takes care of the remaining infinity number
#of terms using the geometric approximation
def calculate_ARL(p1, p2, phi, b, plots=False ):
    l = []
    sum_ = 0
    k = 1
    err = 10**(-9)

    mu = np.dot(p1.value, (np.maximum(1 - np.sign(p1.value - p2.value), 0)))
    #print(mu)
    if mu >= 1:
        M = 100000
        ARL = 0
        return ARL
    M = np.round(np.log(err*(1-mu))/np.log(mu))
    #print(M)
    while (k <= M):
        term = np.dot(p1.value, np.maximum(1 - phi - b/k, 0))
        l.append(term**k)
        sum_ = sum_ +term**k
        k = k+1
    if plots:
        plt.plot(range(1, k), l)
    if err/sum_ > 0.01:
        print("Adjusted due to relative error")
        sum_ = sum_ + (mu**M)/(1-mu)
    ARL = 1/sum_
    return ARL

print(calculate_ARL(p1, p2, phi, b, plots = True))
plt.figure()
plt.plot(phi)
print(T)
```

```
811518.4396710892
43
```

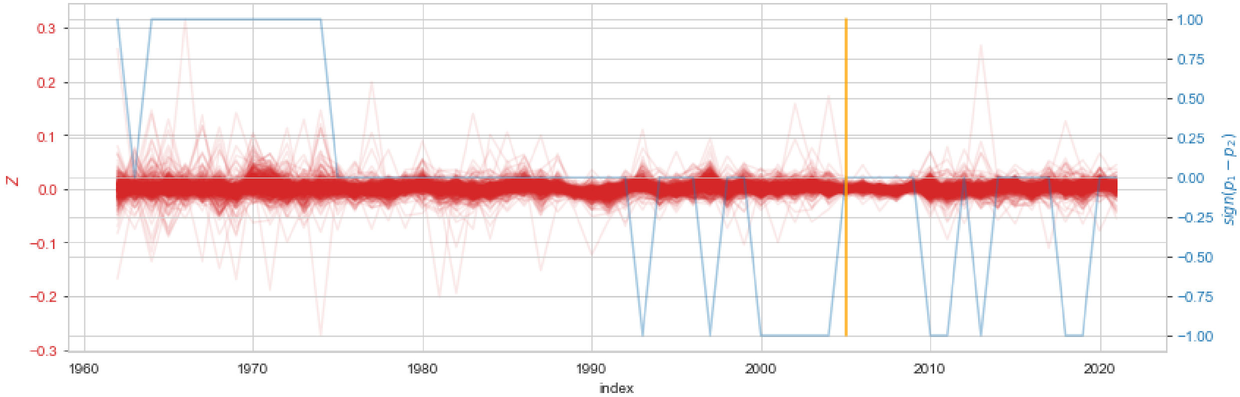




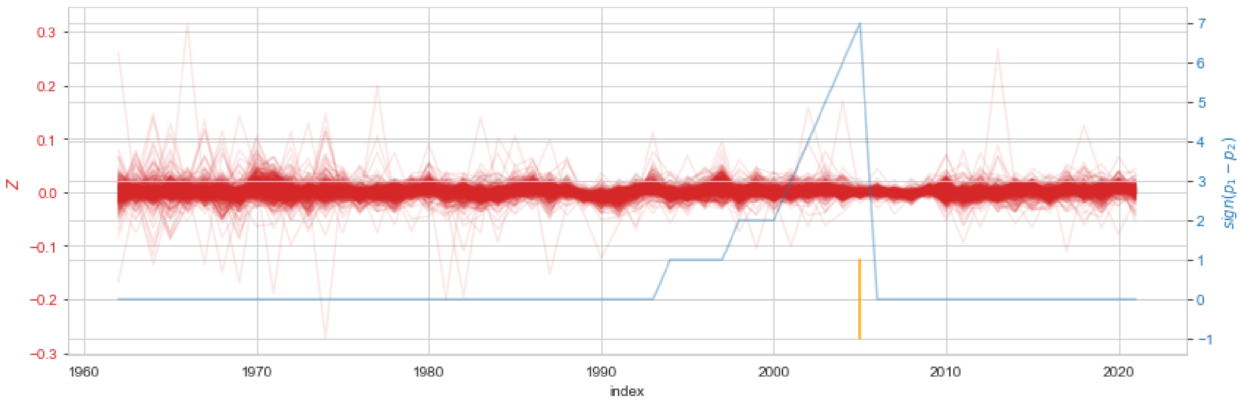
In [12]:

```python
#A function to make the plots
def overlap_plt(data1, data2, label1, label2, x_index, T = -1):
    fig, ax1 = plt.subplots(figsize = (12,4))

    color = 'tab:red'
    ax1.set_xlabel('index')
    ax1.set_ylabel(label1, color=color)
    ax1.plot(x_index, data1, color=color, alpha = 0.1)
    ax1.tick_params(axis='y', labelcolor=color)

    ax2 = ax1.twinx()  # instantiate a second axes that shares the same x-axis

    color = 'tab:blue'
    ax2.set_ylabel(label2, color=color)  # we already handled the x-label with ax1
    ax2.plot(x_index,data2, color=color, alpha = 0.4)
    ax2.tick_params(axis='y', labelcolor=color)
    ax2.plot([x_index[T],x_index[T]], [-1,1], 'orange')
    fig.tight_layout()  # otherwise the right y-label is slightly clipped
    plt.show()
```

```
In [13]: x_index = range(t-n,t+n)
         overlap_plt(Q, phi, '$Z$', '$sign(p_1-p_2)$', x_index, T)
```




```
In [14]: overlap_plt(Q, S, '$Z$', '$sign(p_1-p_2)$', x_index, T)
```



Graphic of Changepoint Detection Scheme

alt text

alt text

Changepoint Detection Algorithm Below

In [483]:
```python
### from the series extract a list of changepoints
times = {}
ns = [60, 90]
theta_vals = [0.05, 0.075, 0.1, 0.2, 0.5, 1]
ave_run_length = [1000, 10000]
limit = int(len(R)/30)
def get_list_of_trade_times(theta_val, ARL_target, n, limit=2*int(len(R)/n)):

    t_mid = n+1
    t_mids = []
    Ts = []
    ARLs = []
    balance = []
    (n1, n2) = (n,n)
    k = 0
    while t_mid <= len(R)- n2 - 1 and k < limit: #k < 2*int(len(R)/n):
        t_mids.append(t_mid)
        t_now = t_mid+n2
        #single step change point algorithm
        [p1,p2, gamma_1, gamma_2, prob, Q] = robust_hypothesis(t_mid, n, theta_val, R)
        signal = p1.value - p2.value
        signal[np.abs(signal) < 10**(-3)*np.max(signal)] = 0
        phi = np.sign(signal)

        for b in range(1,n):
            arl = calculate_ARL(p1, p2, phi, b, plots = False)
            if arl > ARL_target:
                break

        T, S = detect_change(phi, b)
        t_mid = t_now + T - n2

        if T <= n1+n2: #if there was a change then store it
            balance.append(t_now)
            Ts.append(T)
            ARLs.append(arl)

        k = k + 1
        if k == limit:
            print("Too many rebalancing points")

    return {"times":balance, "mid_points": t_mids, "Ts": Ts, "ARL":ARLs}
```

```python
import itertools
for (theta, arl_t, n) in itertools.product(theta_vals, ave_run_length, ns):
    res = get_list_of_trade_times(theta, arl_t, n, limit);
    with open(str(theta)+"_"+str(arl_t)+"_"+str(n)+".pkl", 'wb') as f:
        pkl.dump(res, f)
```

The two code snippets below generate the set of rebalancing times

```python
theta = 0.5
arl_t = 1000
n = 60
res = get_list_of_trade_times(theta, arl_t, n);
with open(str(theta)+"_"+str(arl_t)+"_"+str(n)+".pkl", 'wb') as f:
    pkl.dump(res, f)
```

```
import itertools
for (theta, arl_t, n) in itertools.product(theta_vals, ave_run_length, ns):
    res = get_list_of_trade_times(theta, arl_t, n);
    with open(str(theta)+"_"+str(arl_t)+"_"+str(n)+".pkl", 'wb') as f:
        pkl.dump(res, f)
```

The code below opens one of the results and get's the rebalancing times and a-posteriori changpoints

It also includes the capability of including a required estimation window. For example, lets say we detect at time 70 that a change occured at time 60 but we want at least 30 observations to estimate the mean and covariance ... then we would wait until time 60+30 to rebalance. If there was a detected change before time 90 then we missed the rebalancing opportunity and will try to wait again.

post_detection_times_updated is an array of times whereby the distribution is estimated to have changed

res_times_updated is an array of times that form the corresponding rebalancing times.

In [574]:

```
def retrieve_rebalancing_info(filename, min_estimation=0):
    '''This function takes the results and extracts
       the hypothesis model parameters and outputs
       The function takes in a minimum estimation window
       and applies some logic to output a set of updated
       rebalancing times that ensures the estimated quantities
       are estimated over a window in which there are no
       detected changes'''
    with open(filename, 'rb') as f:
        res = pkl.load(f)

    [delta, arl_t, n] = [float(x) for x in filename[filename.find("/")+1:filename.find(".pkl")].split(
"_")]
    n = int(n)
    res_times = np.array(res['times'])
    res_times = res_times[np.array(res['ARL']) >arl_t]

    post_detections =  np.array(res['Ts'])
    post_detections = post_detections[np.array(res['ARL']) >arl_t]

    post_detection_times = res_times - 2*n + post_detections - 1
    if len(res['mid_points']) > 2*int(len(R)/n):
        print("Do not use this file: the hypothesis testing algorithm did not terminate")

    res_times_updated = np.minimum(np.maximum(res_times - post_detection_times, min_estimation) + post
_detection_times, len(R)-1);
    post_detection_times_updated = post_detection_times
    i = 0
    k = 1
    leng = len(res_times)

    while i < leng:
        deleted = False
        for j in range(k,len(res_times)):
            if res_times_updated[i] > res_times[j]:
                res_times_updated = np.delete(res_times_updated, i)
                post_detection_times_updated = np.delete(post_detection_times_updated, i)
                deleted = True
                break
        if not deleted:
            i = i + 1
        k = k + 1
        leng = len(res_times_updated)
    return res, res_times, post_detection_times, post_detections, delta, arl_t, n, post_detection_time
s_updated, res_times_updated

def plot_times(R,post_detection_times, res_times):
    '''makes a plot of all the stock returns along
    with the rebalancing times and estimation/posterior changepoint times'''
    R.plot(legend =False, alpha = 0.02, color = 'red', figsize = (12,4), ylim = (-0.5, 0.5))

    for i in post_detection_times:
        plt.axvline(x=R.index[i], color = 'orange', linewidth = 2)

    for i in res_times:
        plt.axvline(x=R.index[i], linewidth =2)
```

Follow through each of these cells to see how the estimation window update works

Rebalancing Rules to test:

Results/0.075_10000_60.pkl: 24 rebalancing points --> rebalance every 86 days

Results/0.075_1000_60.pkl: 26 rebalancing points --> rebalance every 80 days

Results/0.075_1000_90.pkl: 12 rebalancing points --> rebalance every 173 days

Results/Results/1_10000_60.pkl: 7 rebalancing points --> rebalance every 296 days

In [662]:
```
filename = 'Results/1_10000_60.pkl'
estimation_window = 100;
res, res_times, post_detection_times, post_detections, delta, arl_t, n, post_detection_times_updated,
res_times_updated = retrieve_rebalancing_info(filename, estimation_window)
```

In [663]:
```
len(res_times)
```

Out[663]:
```
7
```

In [664]:
```
len(R)/len(res_times)
```

Out[664]:
```
296.0
```

In [665]:
```
res_times_updated
```

Out[665]:
```
array([ 219,  338,  576, 1409, 1528, 1647, 1885])
```

In [666]:
```
post_detections
```

Out[666]:
```
array([119, 119, 119, 119, 119, 119, 119])
```

In [667]:
```
res_times - post_detection_times
```

Out[667]:
```
array([2, 2, 2, 2, 2, 2, 2])
```

In [668]:
```
np.maximum(res_times - post_detection_times, 65)
```

Out[668]:
```
array([65, 65, 65, 65, 65, 65, 65])
```

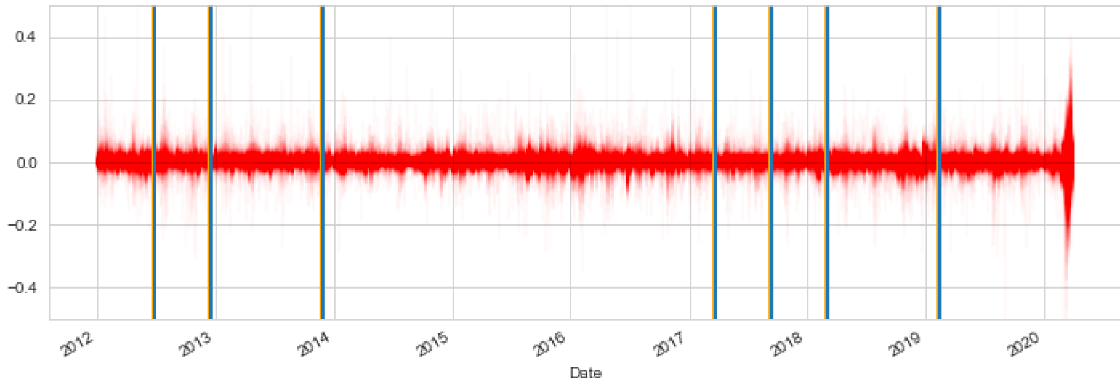
In [669]:
```
np.maximum(res_times - post_detection_times, 65) + post_detection_times
```

Out[669]:
```
array([ 184,  303,  541, 1374, 1493, 1612, 1850])
```

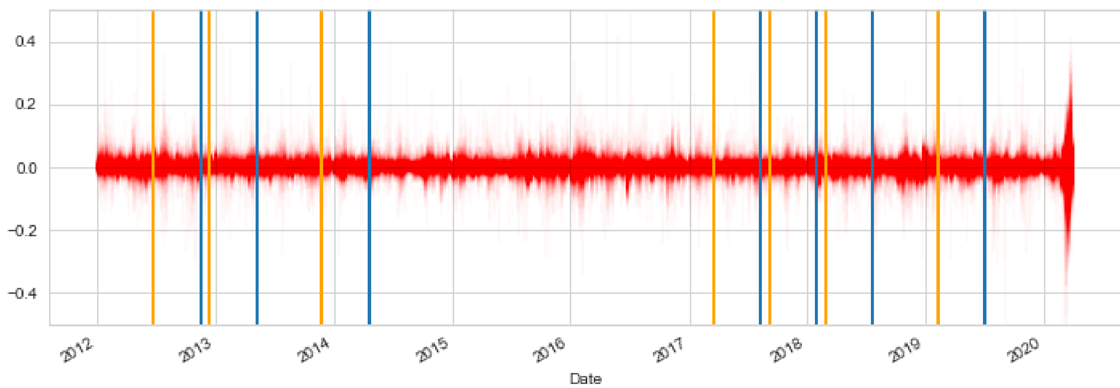
In [670]:
```
res_times_updated
```

Out[670]:
```
array([ 219,  338,  576, 1409, 1528, 1647, 1885])
```

In [671]:
```
#orange is detection time and blue is rebalance time
plot_times(R,post_detection_times, res_times)
```



In [672]:
```
#orange is detectino time and blue is reblance time shifted to allow for an estimation interval.
plot_times(R,post_detection_times_updated, res_times_updated)
```



Robust mean variance implementation

In [673]:
```
def blanchet_mvo(R, delta_val, alpha_val):
    cov_ = R.cov()
    mean_ = np.transpose(R.mean().values)
    cov_half = la.sqrtm(cov_)
    (T, m) = R.shape
    x = cp.Variable(m)
    delta = cp.Parameter(nonneg=True)
    alpha = cp.Parameter(nonneg=True)

    delta.value = delta_val
    alpha.value = alpha_val

    constraints = [x >= 0,
                   cp.sum(x) == 1,
                   cp.sum(cp.multiply(x,mean_)) >= alpha + delta*cp.norm(x,2)]

    #objective = cp.Minimize((p1+p2)*2*cp.minimum(p1/(p1+p2),p2/(p1+p2) ))
    objective = cp.Minimize(cp.square(cp.norm(cp.matmul(cov_half, x),2) + delta*cp.norm(x,2)))

    prob = cp.Problem(objective,
                      constraints)
    return [x, prob]
#test
delta_val, alpha_val = (0.05, 0.05)
x, prob = blanchet_mvo(R, delta_val, alpha_val)
```

In [ ]:
```

```