Writing Portfolio I

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Proof Count: 18

Theorem 1.1. *If x is an even integer, then x^2 is even.*

Proof. Let x is even thus $x = 2k$ such that $k \in \mathbb{Z}$ then it follows that $x^2 = (2k)^2$ then $x^2 = 4k^2$ then $x^2 = 2(2k^2$ and $2k^2 \in \mathbb{Z}$ so $2(2k^2) = 2(a)$ where $a \in \mathbb{Z}$ so if follows that $x^2 = 2(i)$ so x^2 is even. $\square$

Theorem 1.2. *If x is an odd integer, then x^3 is odd.*

Proof. Let if x is odd then $x = 2k + 1$ such that $k \in \mathbb{Z}$ then it follows that $x^3 = (2k + 1)^3$ then $x^3 = 8k^3 + 12k^2 + 6k + 1$, $2(4k^3 + 6k^2 + 3k) + 1$ where $4k^3 + 6k^2 + 3k \in \mathbb{Z}$ so $2(4k^3 + 6k^2 + 3k) + 1 = 2(a) + 1$ where $a \in \mathbb{Z}$ so if follows that $x^3 = 2(a) + 1)$ so x^3 is odd. $\square$

Theorem 1.4. *Suppose $x, y \in \mathbb{Z}$. If x and y are odd, then xy is odd.*

Proof. Let $x = 2k + 1$ if x is odd, same for y without loss of generality so it follows that $xy = (2k + 1)(2l + 1)$ then $xy = 4kl + 2k + 2l + 1$ so $xy = 2(2kl2 + k + l) + 1$ and $2kl + k + l \in \mathbb{Z}$ so $2kl + k + l = a$ such that $a \in \mathbb{Z}$ so it follows that $xy = 2(a) + 1$ so xy is odd. $\square$

Theorem 1.5. *Suppose $x, y \in \mathbb{Z}$. If x is even, then xy is even.*

Proof. Let $x = 2k$ if x is even, same for y without loss of generality so it follows that $xy = (2k)(2l)$ then $xy = 4kl$ so $xy = 2(2kl)$ and $2kl \in \mathbb{Z}$ so $2kl = a$ such that $a \in \mathbb{Z}$ so it follows that $xy = 2(a)$ so xy is even. $\square$

Theorem 1.7. *Suppose $a, b \in \mathbb{Z}$. If $a \mid b$, then $a^2 \mid b^2$.*

Proof. Let if $a, b \in \mathbb{Z}$ then $a^2, b^2 \in \mathbb{Z}$ Let $b = a * k$ such that $k \in \mathbb{Z}$ then is follows that $b^2 = a^2 * k^2$ since $b^2/a^2 = k^2$ and $k^2 \in \mathbb{Z}$ than a^2 must divide b^2 therefor if $a \mid b$ then $a^2 \mid b^2$. $\square$

Theorem 1.16. *If two integers have the same parity, then their sum is even.*

Proof. Let if integers a and b have the same parity, both are even or odd. Let $a = 2k$ or $a = 2l + 1$ same for b without loss of generality. Then it follows that $a + b = 4kl$ or $a + b = 2k + 2l + 2$ such that $k, l \in \mathbb{Z}$ then $a + b = 2(2kl)$ and $a + b = 2(k + l + 1)$ and $(2kl), (k + l + 1) \in \mathbb{Z}$ then $2kl = p$ and $k + l + 1 = q$ such that $p, q \in \mathbb{Z}$ it follows that $a + b = 2(p)$ and $a + b = 2(q)$ and 2 times and integer is even, it is shown that $a + b$ is even when a and b, have the same parity. □

Theorem 1.17. *If two integers have opposite parity, then their product is even.*

Proof. Let if two integers, a and b, have opposite parity, one is even and one is odd. Let $a = 2k$ and $b = 2l + 1$ same for b without loss of generality. Then it follows that and $ab = 4kl + 2k$ such that $k, l \in \mathbb{Z}$ then $ab = 2(2kl + k)$ where $(2kl + k) \in \mathbb{Z}$ then $2kl + k = p$ and such that $p \in \mathbb{Z}$ it follows that $ab = 2(p)$ and 2 times and integer is even, so it is shown that ab is even when a and b, have the opposite parity. □

Theorem 1.20. *If a is an integer and $a^2 \mid a$, then $a \in \{-1, 0, 1\}$.*

Proof. Let if $a^2 \mid a$ then $a = a^2k$ so $a/a^2 = k$ if $a = 0$ then $0 = 0^2k$, $0 = 0$. if $a \neq 0$ then $1 = ak$ so $1 = ak$, than only $a = 1, -1$ and $k = 1, -1$ are solutions so the only solutions are $\{-1, 0, 1\}$. It is shown that if a is an integer and a^2/a, then $a \in \{-1, 0, 1\}$. Counterexample if a is an integer and $a = 2$ then $a^2 = 4$ and $4/2 = 2$ and $2 \notin \{-1, 0, 1\}$ so it is not true that if a is an integer and a^2/a, then $a \in \{-1, 0, 1\}$. □

Theorem 1.22. *Every odd integer is a difference of two squares.*

Proof. Let an odd integer a is odd if $a = 2k + 1$ such that $k \in \mathbb{Z}$ and the difference of squares is $(x - y)(x + y)$ so if it is true that for every odd integer is a difference of two squares then let $a = (x - y)(x + y)$ and let $x = k + 1$ and $y = k$ then $a = (k + 1 - k)(k + 1 + k)$, $a = (1)(2k + 1)$ so $a = 2k + 1$ then it is shown that every odd integer is a difference of two squares. □

2. 2

Theorem 2.7. *The number $\sqrt{5}$ is irrational, that is, $\sqrt{5} \notin Q$*

Proof. Let $\sqrt{5}$ be rational then $5 = p/q$ such that p, q are co-prime numbers or have no common factors other than 1. Then $\sqrt{5} = p/q, 5 = p^2/q^2$, $5q^2 = p^2$ then it follows that p^2 is divisible by 5 and p is divisible by 5 as well under Euclid's lemma, that is $p*p$ is divisible by 5 then p or p must be divisible by 5 or in other words p is divisible by 5. This is because if integers a, or b are divisible by p and p is prime then a/p or b/p then ab/p. So $p = 5k$ then $p^2 = 25k^2$ substitute that in gives $5q^2 = 25k^2$, $q^2 = 5k^2$ this implies that q is divisible by 5 by Euclid's lemma. Hence, it means that p and q have a common factor of 5 contradicts p, q are co-prime numbers or have no common factors other than 1 so the $\sqrt{5}$ must be irrational. $\square$

Theorem 3.12. *If $a, b, c, d \in \mathbb{R}$ such that $c, d \in [a, b], then (c, d) \subseteq [a, b]$.*

Proof. If a and b are a closed interval and c and d are an open interval that is an element of the closed interval a and b then by the nested interval theorem $c, d \subseteq [a, b]$ $\square$

Theorem 4.3. *If $m, n \in \mathbb{Z}$, then $\{k \in \mathbb{Z} : mn \mid k\} \subseteq \{k \in Z : m \mid k\} \cap \{k \in \mathbb{Z} : n \mid k\}$.*

Proof. Let $a \in \{k \in \mathbb{Z} : mn \mid k\}$ such that $a \in \mathbb{Q}$ then $a = k/mn$, $anm = k$ than $(an)m = k$, $an = k/m$ then $an \in \{k \in Z : m \mid k\}$ and $(am)n = k$, $am = k/n$ then $am \in \{k \in \mathbb{Z} : n \mid k\}$ since $mn \mid k$ implies $n \mid k$ and $m \mid k$ thus $m, n \in \mathbb{Z}$, then $\{k \in \mathbb{Z} : mn \mid k\} \subseteq \{k \in Z : m \mid k\} \cap \{k \in \mathbb{Z} : n \mid k\}$ $\square$

Theorem 5.12. $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.

Proof. Let $x \in A$ then $x \notin B \; or \; C$ this implies that $x \in A \setminus B$ and $x \in A \setminus C$ thus $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$. $\square$

Theorem 6.1. $(A \cup B) \times C = (A \times C) \cup (B \times C)$.

Proof. Let the ordered pair$(x, y) \in (A \cup B) \times C$ then $x \in (A \cup B)$ and $y \in C$ since $x \in (A \cup B)$ it must be that $x \in A$ or $x \in b$ or both. this implies that $x \in (A \times C)$ or $x \in (B \times C)$ thus $(A \cup B) \times C = (A \times C) \cup (B \times C)$. Also the Cartesian product is distributive so this is true by definition. □

Theorem 7.4. *Let X be a set. If $A, B \in P(X)$, then $A \setminus B \in P(X)$.*

Proof. if X is a set then $P(X)$ is the power set of X if A, B are elements of the power set than $A \subseteq X$ and $B \subseteq X$. Let $n \in A$ then $n \notin B$ but if $n \in A$ then $n \in X$ and $n \in P(X)$ so $n \in A \setminus B$ and $A \setminus B \in P(X)$ thus $A, B \in P(X)$, then $A \setminus B \in P(X)$. □

Theorem 9.6. $p \vee (q \vee r) = (p \vee q) \vee r$ *and* $p \wedge (q \wedge r) = (p \wedge q) \wedge r$.

Proof. The table below shows that $p \vee (q \vee r) = (p \vee q) \vee r$.

p	q	r	$(q \vee r)$	$(p \vee q)$	$p \vee (q \vee r)$	$(p \vee q) \vee r$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T
F	F	F	F	F	F	F

As we can see from the truth table $p \vee (q \vee r) = (p \vee q) \vee r$ □

Theorem 9.6. $p \vee (q \vee r) = (p \vee q) \vee r$ *and* $p \wedge (q \wedge r) = (p \wedge q) \wedge r$.

Proof. The table below shows that $p \wedge (q \wedge r) = (p \wedge q) \wedge r$.

p	q	r	$(q \wedge r)$	$(p \wedge q)$	$p \wedge (q \wedge r)$	$(p \wedge q) \wedge r$
T	T	T	T	T	T	T
T	T	F	F	T	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	T	F	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

As we can see from the truth table $p \wedge (q \wedge r) = (p \wedge q) \wedge r$. □

Theorem 5.6. *$A \cup (B \cup C) = (A \cup B) \cup C$ and $A \cap (B \cap C) = (A \cap B) \cap C$.*

Proof. Let $x \in B$ then $x \in (B \cup C)$ then $x \in A \cup (B \cup C)$ and if $x \in B$ then $x \in (A \cup B)$ then $x \in (A \cup B) \cup C$ thus $A \cup (B \cup C) = (A \cup B) \cup C$

Let $y \in A, B, C$ then $y \in (B \cap C)$ and $y \in A \cap (B \cap C)$ and if $y \in A, B, C$ then $y \in (A \cap B)$ and $y \in (A \cap B) \cap C$ thus $A \cap (B \cap C) = (A \cap B) \cap C$.

□

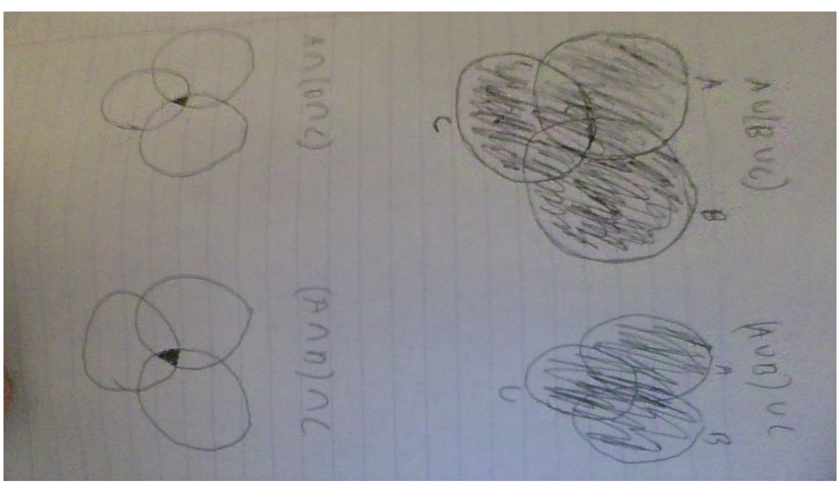

FIGURE 5.6.1. Venn Diagram