Homework 8

1. The energy in some site (denote by the subindex 0) is

$$E|_0 = -\sigma_0 \left(J \sum_i \sigma_j + h \right) = -\sigma_0 h_{weiss}$$

if we compute the expectation value of σ_0

$$\langle \sigma_0 \rangle = \frac{e^{\beta h_{weiss}} - e^{-\beta h_{weiss}}}{e^{\beta h_{weiss}} + e^{-\beta h_{weiss}}} = \tanh(\beta h_{weiss})$$

- $$\boxed{\langle \sigma_0 \rangle = \tanh(\beta h_{weiss})}$$

In the mean field aproximation we replace h_{weiss} by $\langle h_{weiss} \rangle$ and require the consistency condition $m = \langle \sigma_0 \rangle = \langle \sigma_j \rangle \ \forall_j$. then the above equation becomes

$$m = \tanh(m \beta q J + \beta h)$$

- $$\boxed{m = \tanh(\beta (Jqm + h))}$$

A possible generalization is by abandoning the consistency condition and let $m_j = \langle \sigma_j \rangle$. This can be usefull for instance to study long range correlation. the equation then becomes

$$m_k = \tanh\left(\beta \left(J \sum_{\substack{j \\ \text{neighbors} \\ \text{of } \sigma_k}} m_j + h\right)\right)$$

For instance in 1D it becomes $\boxed{m_k = \tanh(\beta(J(m_{k+1} + m_{k-1}) + h))}$

For $m \ll 1$ we can approximate the hyperbolic tangent by a power series. (we take $h=0$)

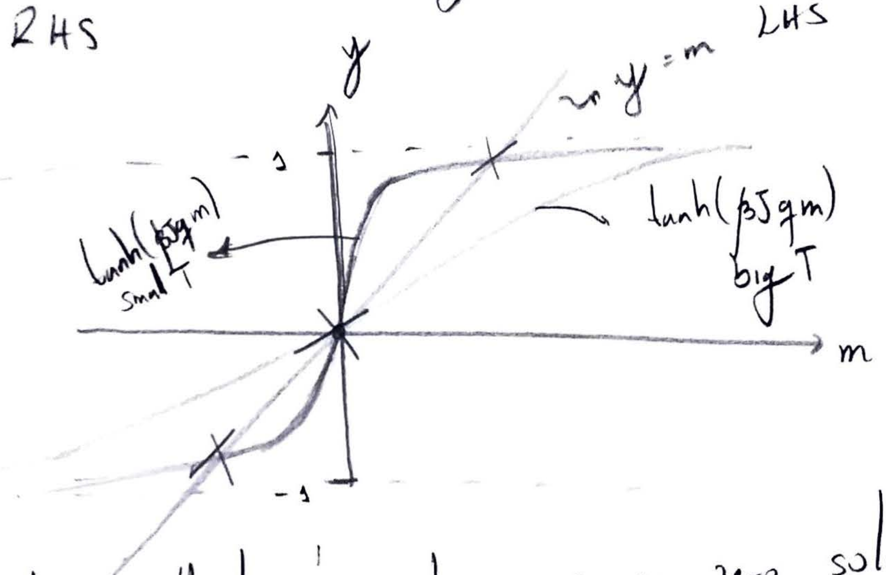
$$m = \tanh(\beta J q m) \quad \dagger$$

$$m = \beta J q m - \frac{(\beta J q m)^3}{3} \quad \ddagger$$

as the second term in the RHS has the oposite sign of m, then equation has a solution with $m \neq 0$ iff

$$\beta J q > 1.$$

This can also been seen by plotting the LHS and the RHS



And noting that we have a non-zero solution iff the slope of $\tanh(\beta J q m)$ at 0 is bigger than 1.

We thus define a critical temperature $(k=1)$

$$\beta_c J q = 1 \Rightarrow T_c = \frac{1}{\beta_c} = Jq$$

Then equations $\dagger$, $\ddagger$ become

$$m = \tanh\left(\frac{\beta}{\beta_c} m\right), \quad m = \frac{\beta}{\beta_c} m - \left(\frac{\beta}{\beta_c}\right)^3 \frac{1}{3} m^3$$

d we know $m=0$ for $\beta<\beta_c$ $(T>T_c)$

Then let $T<T_c$

$$\cancel{m} = \frac{T_c}{T}\cancel{m} - \left(\frac{T_c}{T}\right)^3 \frac{1}{3} m^{\cancel{3}\,2}$$

$$\Rightarrow m^2 = 3\left(\frac{T}{T_c}\right)^3\left(\frac{T_c}{T}-1\right)$$

$$\boxed{m = \pm\sqrt{3}\left(\frac{T}{T_c}\right)^{3/2}\left(\frac{T_c}{T}-1\right)^{1/2}}$$

- Near T_c $(T-T_c \ll T)$ $\left(\frac{T}{T_c}\right)^{3/2}\sim 1$

and Then $m \sim \pm \text{constant}\left(\frac{T_c}{T}-1\right)^{1/2}$

which is equation (2) with $\beta = 1/2$

3 The given program solves the problem by founding a fix point mathematica file does it faster.

- Lattice Self consistency: Ascence of fluctuations.

1. We just have to pick

$$q_i = \frac{\sum_{\langle mn\rangle} m_j}{m_i} \quad \text{then eq (3) becomes}$$

$$m_i = \tanh(\beta J q_i m_i + h)$$

2 Second program terribly innefficient.

For $T>T_c$ shows $m\sim 0$ and in particular gets really small at the middle.

For $T<T_c$ breaks

..le just set random from 1 to avoid
..anging m(0).

I will make this properly

(DONE)