Homework 10

$$\mathcal{H} = -J \sum \vec{S}_i \cdot \vec{S}_j$$

For the ground state $\vec{S}_i \parallel \vec{S}_j$ for every i,j. And there are

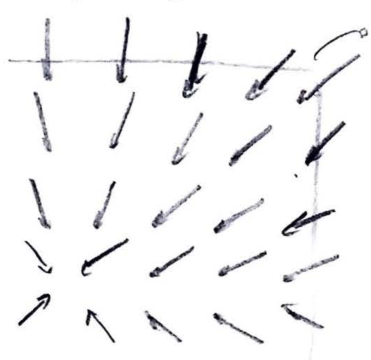
$$\underbrace{\frac{4L^2}{2}}_{\text{4 pairs each spin shared by 2}} - \underbrace{4(L-2)}_{\text{boundary only 3}} - \underbrace{8}_{\text{edges only 2}} = \boxed{2L(L-1)}$$

1 2 3 4 5 6 or L

$L-2$

- To compute vortex energy just 100 for minimization

The energy goes as the logarithm because



at a distance at which the angle between neighboring sites is small $|\varphi_i - \varphi_j| \ll 1$ then energy of the excitation (per site) is

$$\Delta E = -J\{\cos(\varphi_i - \varphi_j) - 1\} = J\frac{\Delta\varphi^2}{2}$$

(ground)

As through the shown path the spin flips all the way

$2\pi \quad \Delta\varphi = 2\pi/L$

$$\Delta E(L) = 4L\frac{\Delta\varphi^2}{2} = \cancel{4}^{2}L\frac{(2\pi)^2}{\cancel{2}L^2} = 8\pi^2\frac{1}{L}$$

Integrating through all shells (of course just from a distance where the approximation is valid)

$$\Delta E = \int_{L \gg \text{core}} dL \, 8\pi^2 \frac{1}{L} + E_{core} \propto \ln L + E_{core}$$

In the program we see indeed for L really close to 0 the logarithmic dependence dissapears.

The energy of the vortex $\Delta E = \pi J_R \log(L) + E_c$ and the entropy (we can put it in L^2 places) is

$$S = K \ln L^2 = 2K \ln L$$

$$\Rightarrow \Delta F = \Delta E - TS = \pi J_R \log(L) + E_c - 2KT \log(L)$$

$$= (\pi J_R - 2KT) \underset{\text{increases with size}}{\underline{\underline{\log(L)}}} + \underset{\text{constant}}{E_c}$$

hence when $T > \dfrac{\pi J_R}{2K}$ vortices form (they lower the free energy $\Rightarrow$ they are stable.

Charge of the vortex antivortex ?