Testing pmt calibration

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Abstract

Toy MC test of pmt calibration method.

1 Methodology

The ability to fit data with MC distributions of NPE (number of photoelectron) is examined.

The MC NPE distribution is assumed to poisson with mean μ_M. The data NPE distributions are assumed to poisson with mean μ_d with a tail fraction (tailF) of events draw from a poisson distribution with a larger mean μ_t. The single photoelectron resolution is taken to be 0.5 PE. The parameters and results of each configuration are shown in Table 3 (Table numbering is screwed up by latex, I don't know why). The best fit result is determined by a simple iterative scan of the χ^2. The scanning is not optimized. The χ^2 defined in Section 2.

2 χ^2 construction

Consider the response of a single PMT in the one-ton detector filled with water.

- Let d_j = the sum over data events with the measured number of photoelectrons between e_j and e_{j+1} (in other words, the content of the j^{th} bin),
- Let m_j = the sum over MC events with the simulated number of observed photoelectrons between e_j and e_{j+1} which is equal to the number of generated photons times the probability a that a generated photon creates a photoelectron in the PMT.
- $m_j = \sum_k^j c \times a \times g_k$ where k = event number, g_k = number of generated photons in the k^{th} event and $\sum_k^j$ means the sum over all entries with $f \times a \times g_k$ in the j^{th} bin. f = is the *calibration factor* defined such that the overall probability that a generated photon creates a photoelectron is the same as the data.
- Let $M \equiv \sum_j m_j$ = the total number of MC events
- and $D \equiv \sum_j d_j$ = the total number of data events for a single PMT.

For a single PMT, determine the *calibration factor* f by defining the $\chi^2(f)$ as

$$\chi^2(f) \equiv \sum_j (\frac{d_j - m_j \frac{D}{M}}{\sigma_j})^2 \tag{1}$$

σ_j can be calculated as follows. Let $y_j = d_j - m_j \frac{D}{M}$, then

$$\begin{aligned} {\sigma_j}^2 \equiv \delta y_j^2 &= (\tfrac{\partial y_j}{\partial d_j}\delta d_j)^2 + (\tfrac{\partial y_j}{\partial m_j}\delta m_j)^2 && (2)\\ &= (\delta d_j)^2 + (\tfrac{D}{M}\delta m_j)^2 && (3)\\ &= (\sqrt{d_j})^2 + (\tfrac{D}{M}\sqrt{m_j})^2 && (4)\\ &= d_j + (\tfrac{D}{M})^2 m_j && (5) \end{aligned}$$

Note that σ_j depends on f, the calibration factor.

Note that a sum must be taken over the number of photoelectrons in the MC events $m_j = \sum_k^j f \times h_k$ to evaluate $\chi^2(f)$, where $h_k \equiv a \times g_k$ and $\sum_k^j$ was defined above.

3 Results

Figures are provided showing the data and MC NPE distributions, best fit results, random fit results and the $\chi^2(f)$ where f the calibration factor. In general the fitted calibration factor is an unbiased estimator of the expected calibration factor for tail fractions up to 5%. Performance may differ if the NPE distributions are poisson or combinations of poisson distributions as approximated in this study.

config	nData	nMC	μ_d	μ_M	μ_t	tailF	f_{exp}	f_{best}	χ^2_{min}	nBin
0	10000	100000	8.30	8.30	40.00	0.00	1.00	1.00	17.70	19
1	10000	100000	8.30	6.00	40.00	0.00	1.38	1.35	669.50	19
2	10000	100000	8.30	7.00	40.00	0.00	1.19	1.16	172.63	20
3	10000	100000	8.30	8.00	40.00	0.00	1.04	1.04	24.21	19
4	10000	100000	8.30	9.00	40.00	0.00	0.92	0.93	38.30	20
5	10000	100000	8.30	10.00	40.00	0.00	0.83	0.84	140.57	19
6	10000	100000	8.30	6.00	40.00	0.01	1.38	1.37	693.45	20
7	10000	100000	8.30	7.00	40.00	0.01	1.19	1.18	253.10	20
8	10000	100000	8.30	8.00	40.00	0.01	1.04	1.04	97.81	20
9	10000	100000	8.30	9.00	40.00	0.01	0.92	0.93	136.91	19
10	10000	100000	8.30	10.00	40.00	0.01	0.83	0.84	255.79	19
11	10000	100000	8.30	6.00	40.00	0.05	1.38	1.37	1030.07	19
12	10000	100000	8.30	7.00	40.00	0.05	1.19	1.18	658.83	19
13	10000	100000	8.30	8.00	40.00	0.05	1.04	1.04	508.18	19
14	10000	100000	8.30	9.00	40.00	0.05	0.92	0.93	588.49	20
15	10000	100000	8.30	10.00	40.00	0.05	0.83	0.84	651.68	19
16	10000	100000	16.60	14.00	40.00	0.00	1.19	1.18	229.40	31
17	10000	100000	16.60	15.00	40.00	0.00	1.11	1.10	85.20	31
18	10000	100000	16.60	16.00	40.00	0.00	1.04	1.03	49.33	31
19	10000	100000	16.60	17.00	40.00	0.00	0.98	0.98	26.70	32
20	10000	100000	16.60	18.00	40.00	0.00	0.92	0.92	59.92	31
21	10000	100000	16.60	14.00	40.00	0.01	1.19	1.19	235.18	32
22	10000	100000	16.60	15.00	40.00	0.01	1.11	1.11	172.72	32
23	10000	100000	16.60	16.00	40.00	0.01	1.04	1.04	107.73	31
24	10000	100000	16.60	17.00	40.00	0.01	0.98	0.98	112.93	33
25	10000	100000	16.60	18.00	40.00	0.01	0.92	0.92	130.53	31
26	10000	100000	16.60	14.00	40.00	0.05	1.19	1.18	635.45	50
27	10000	100000	16.60	15.00	40.00	0.05	1.11	1.11	544.71	49
28	10000	100000	16.60	16.00	40.00	0.05	1.04	1.04	522.63	50
29	10000	100000	16.60	17.00	40.00	0.05	0.98	0.98	499.06	51
30	10000	100000	16.60	18.00	40.00	0.05	0.92	0.93	537.80	51

Table 1: Different configurations and results. μ_d = mean PE in data, μ_M = mean PE in MC, μ_t = mean PE in the tail, tailF = tail fraction, f_{exp} = expected calibration factor, f_{best} = best fit calibration factor, χ^2_{min} = value of χ^2 at minimum and nBin = number of bins in histogram.

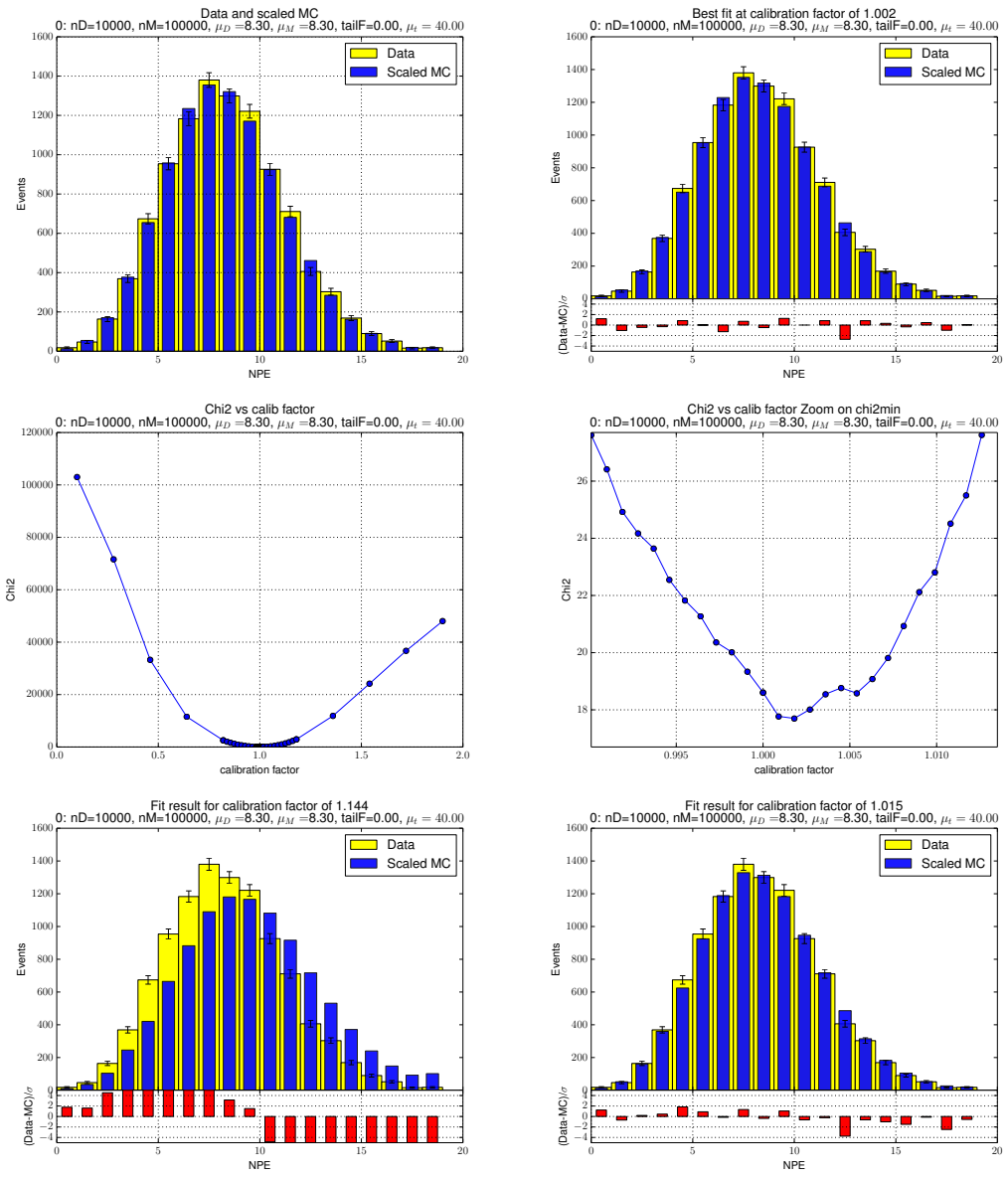

Figure 1: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 00. Data compared to MC scaled by two randomly chosen calibration factors.

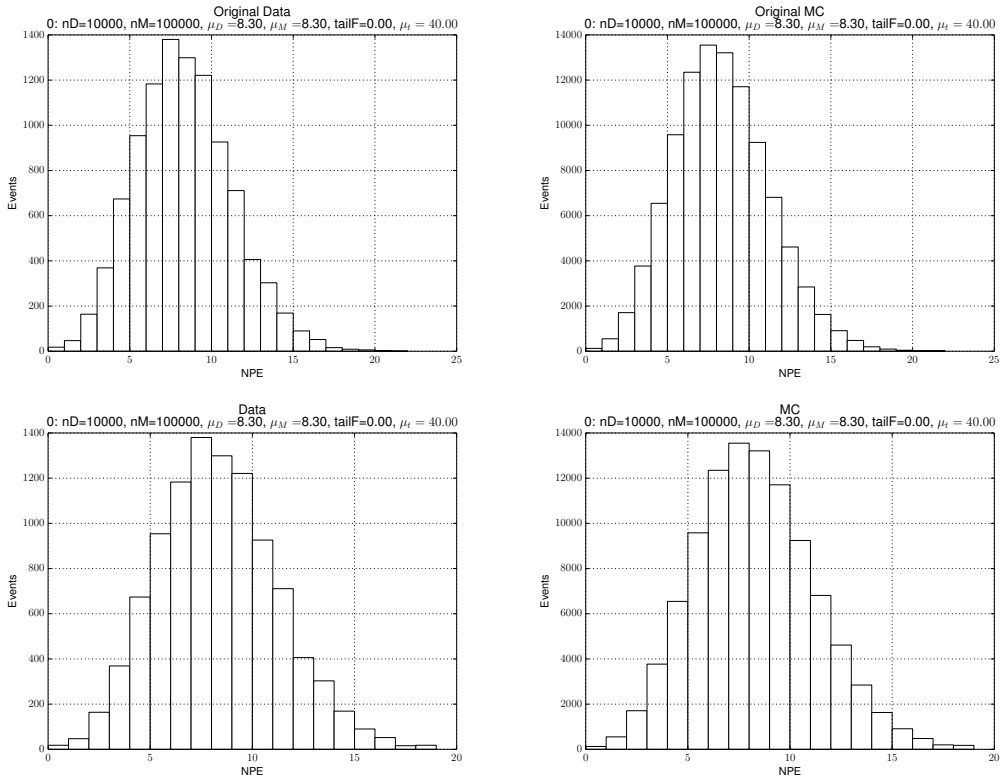

Figure 2: NPE histograms for data and MC for configuration 00. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

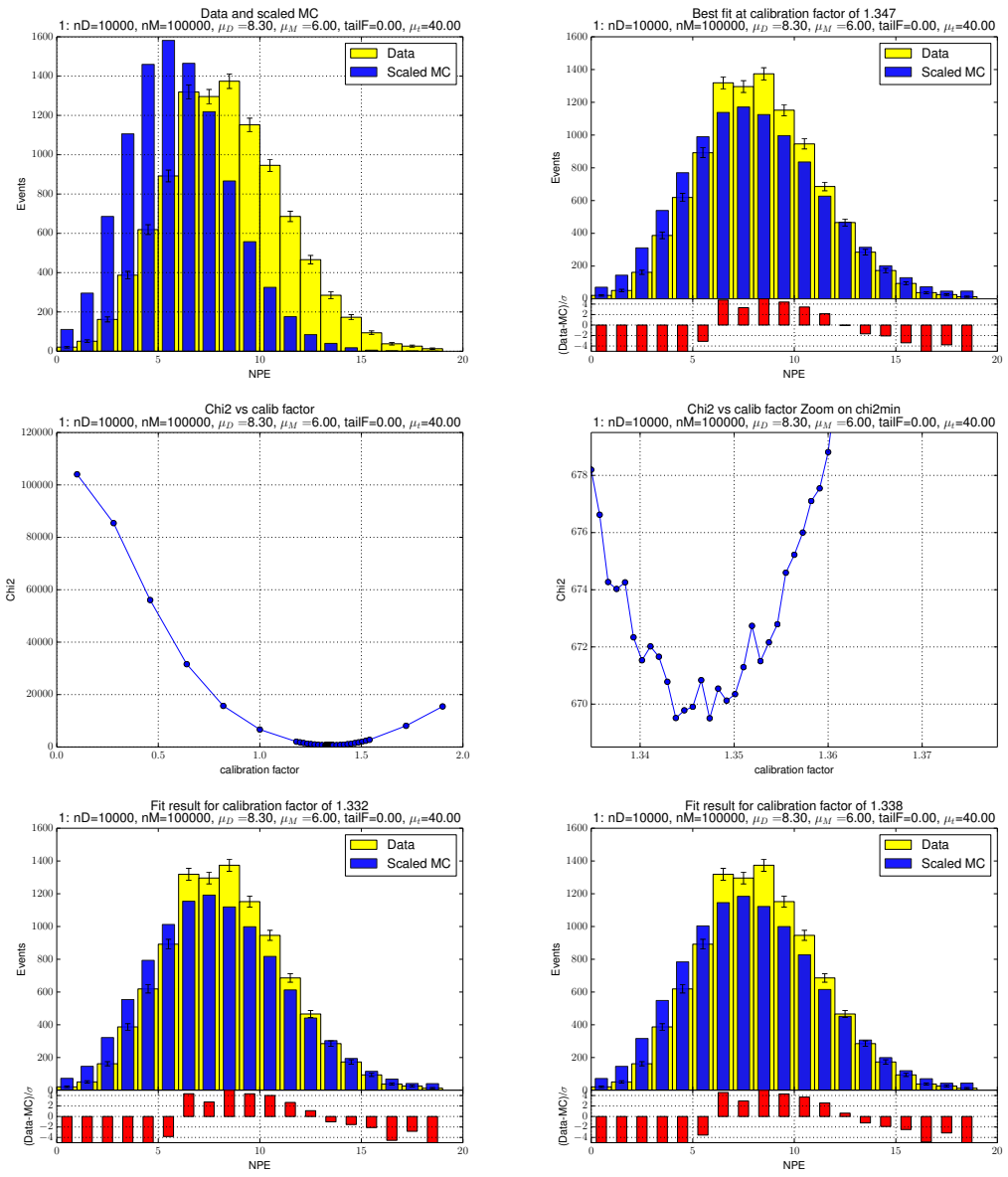

Figure 3: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 01. Data compared to MC scaled by two randomly chosen calibration factors.

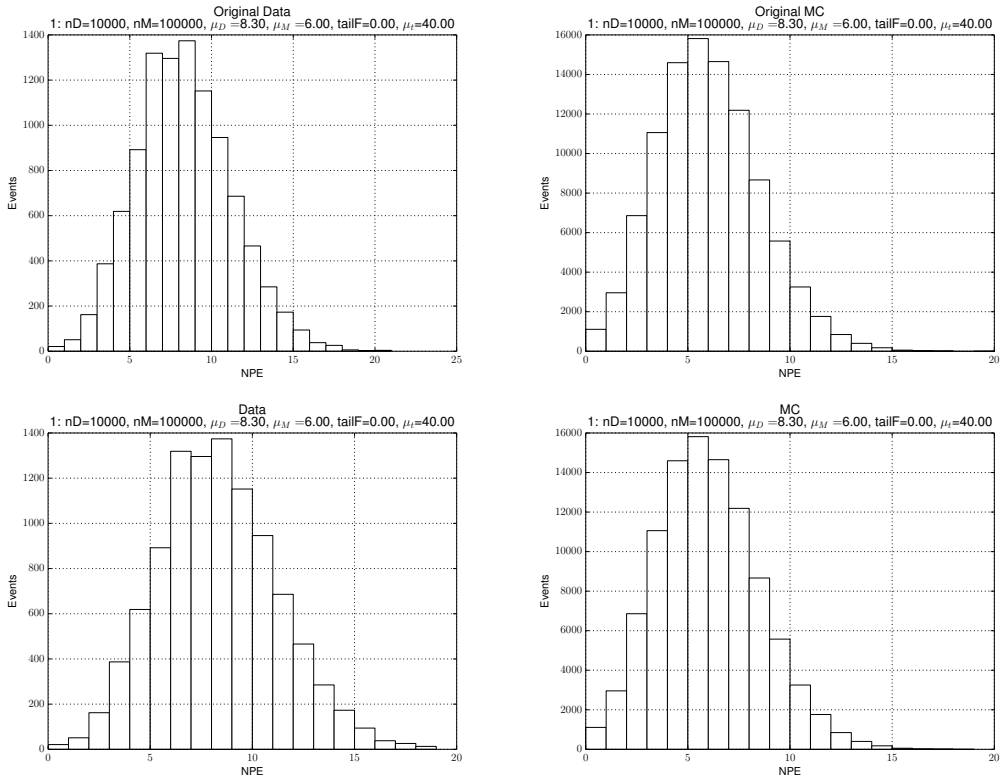

Figure 4: NPE histograms for data and MC for configuration 01. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

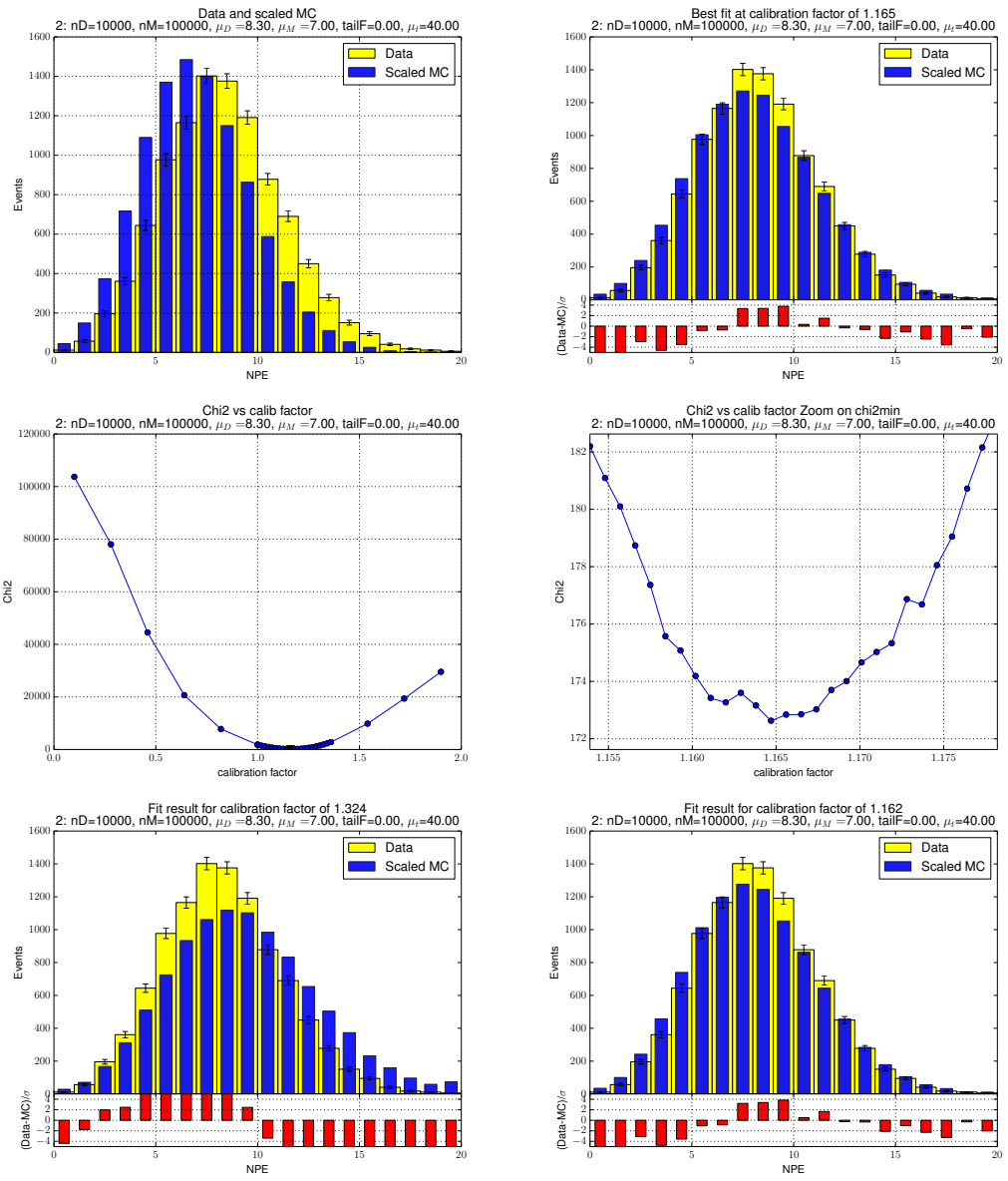

Figure 5: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 02. Data compared to MC scaled by two randomly chosen calibration factors.

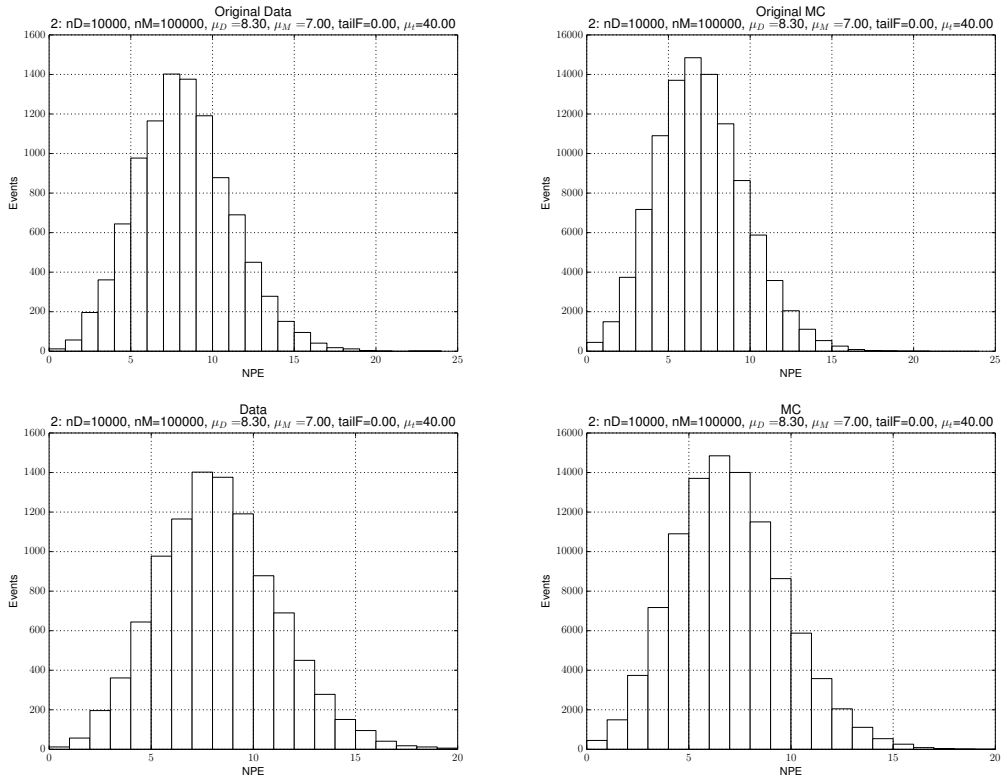

Figure 6: NPE histograms for data and MC for configuration 02. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

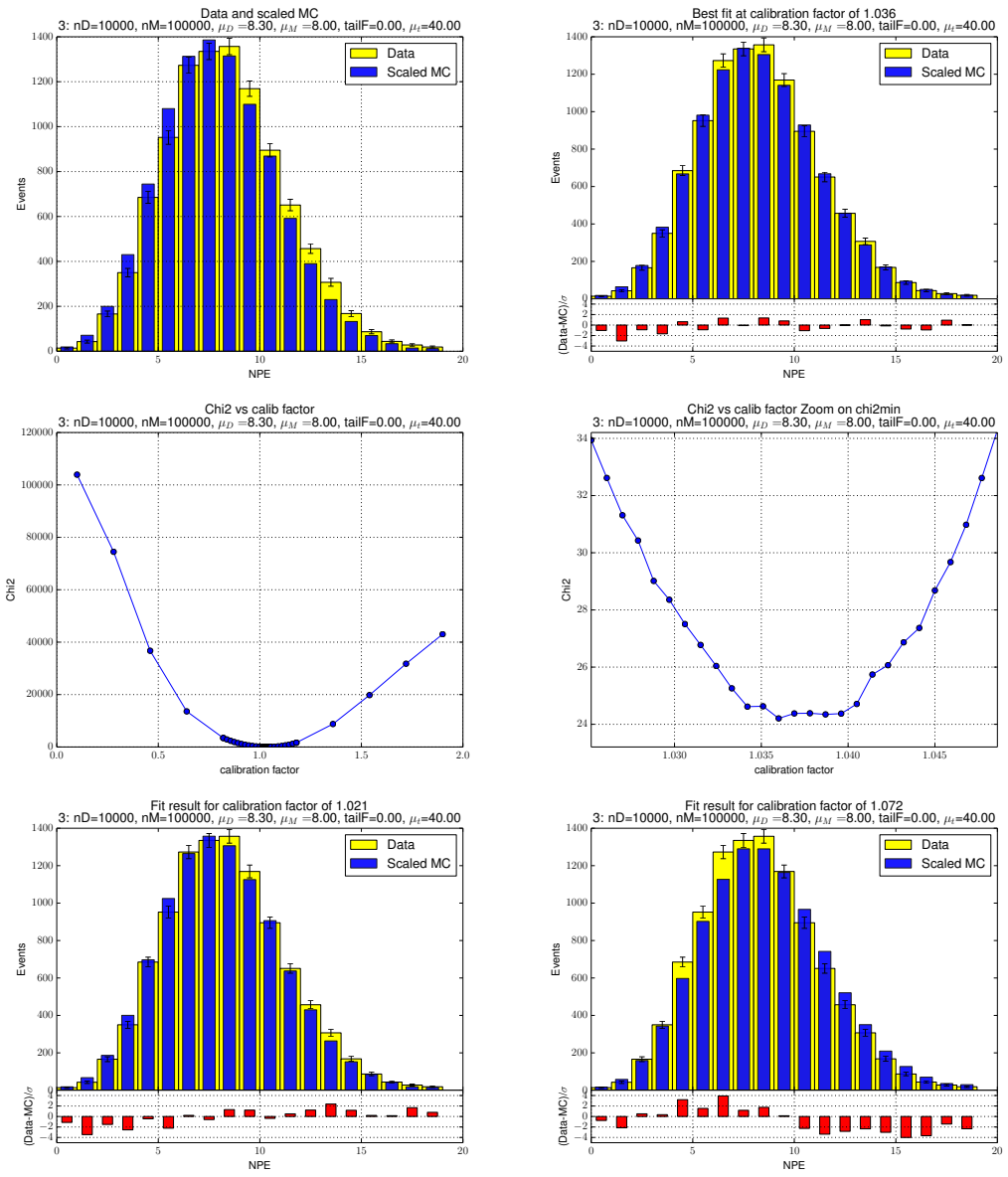

Figure 7: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 03. Data compared to MC scaled by two randomly chosen calibration factors.

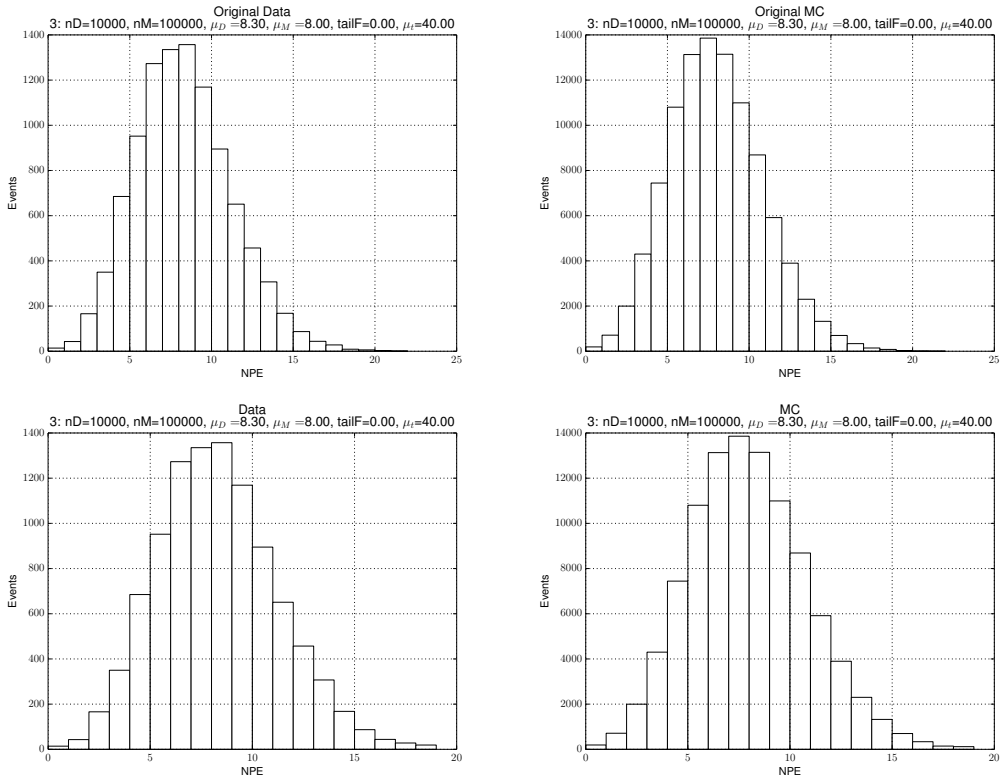

Figure 8: NPE histograms for data and MC for configuration 03. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

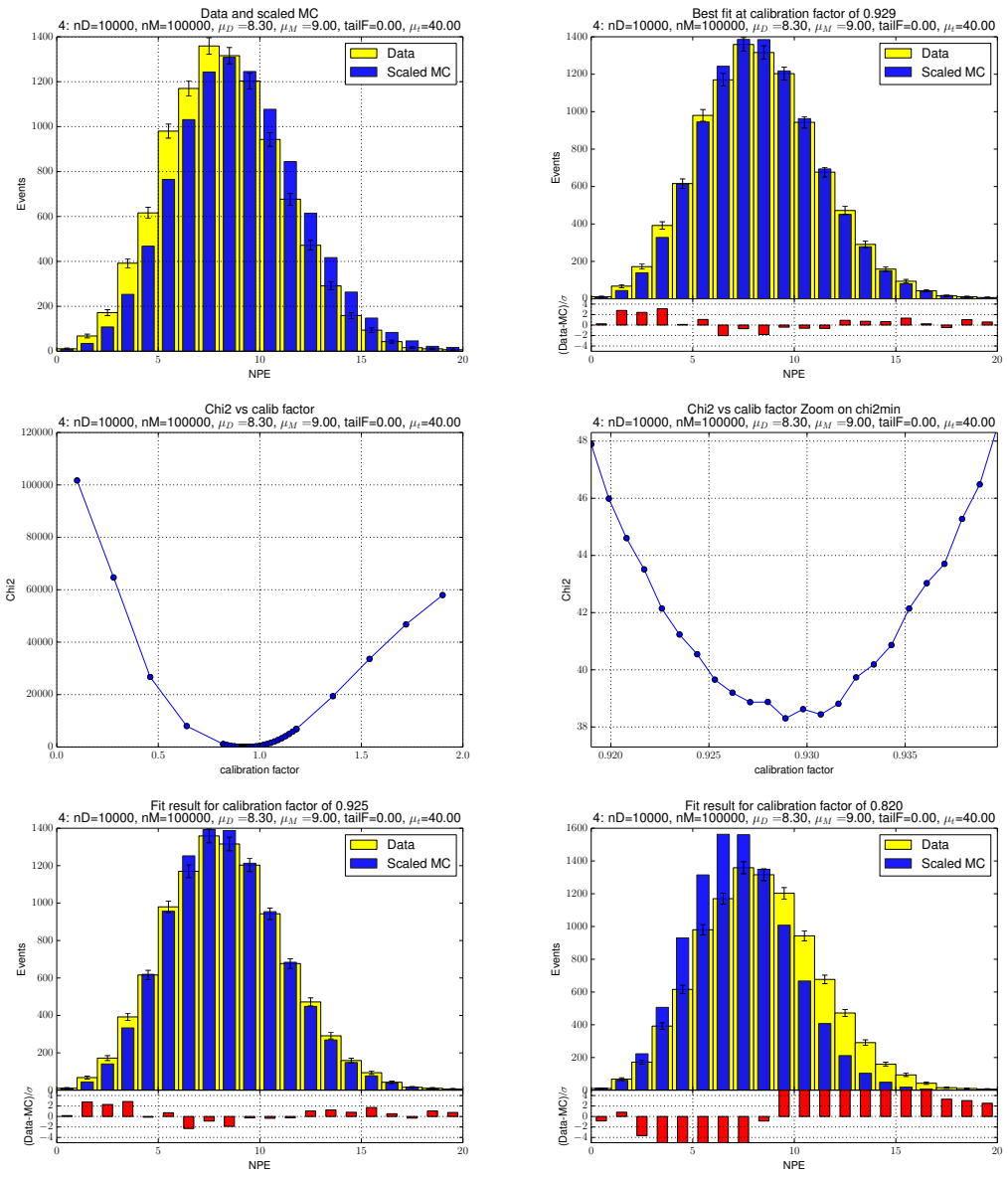

Figure 9: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 04. Data compared to MC scaled by two randomly chosen calibration factors.

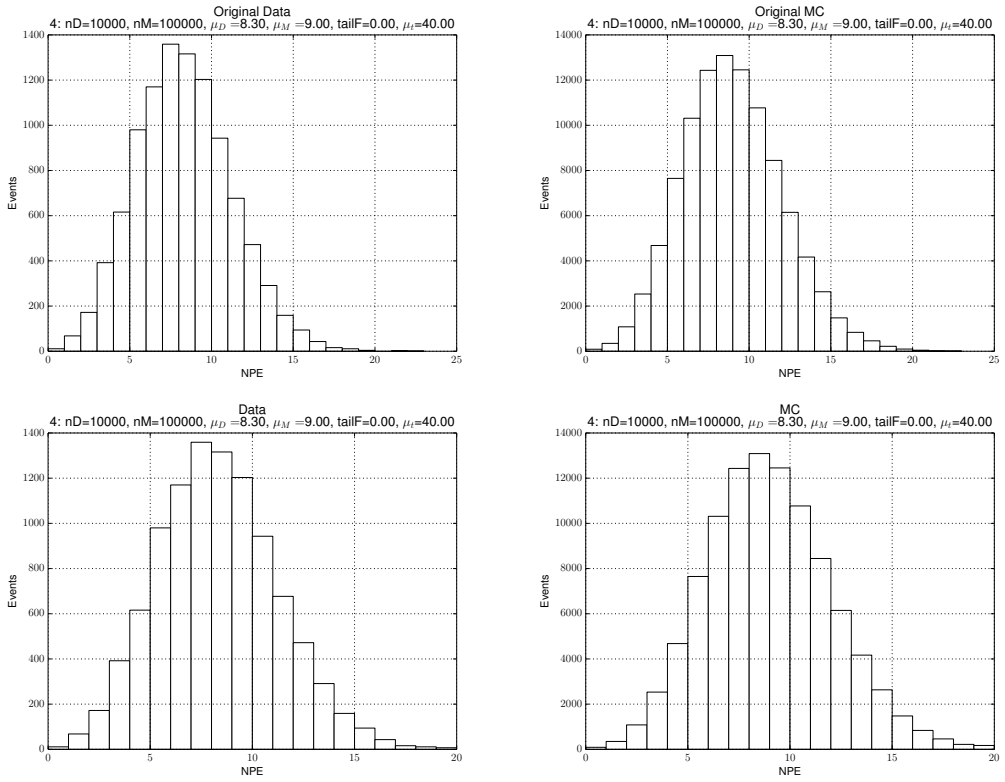

Figure 10: NPE histograms for data and MC for configuration 04. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

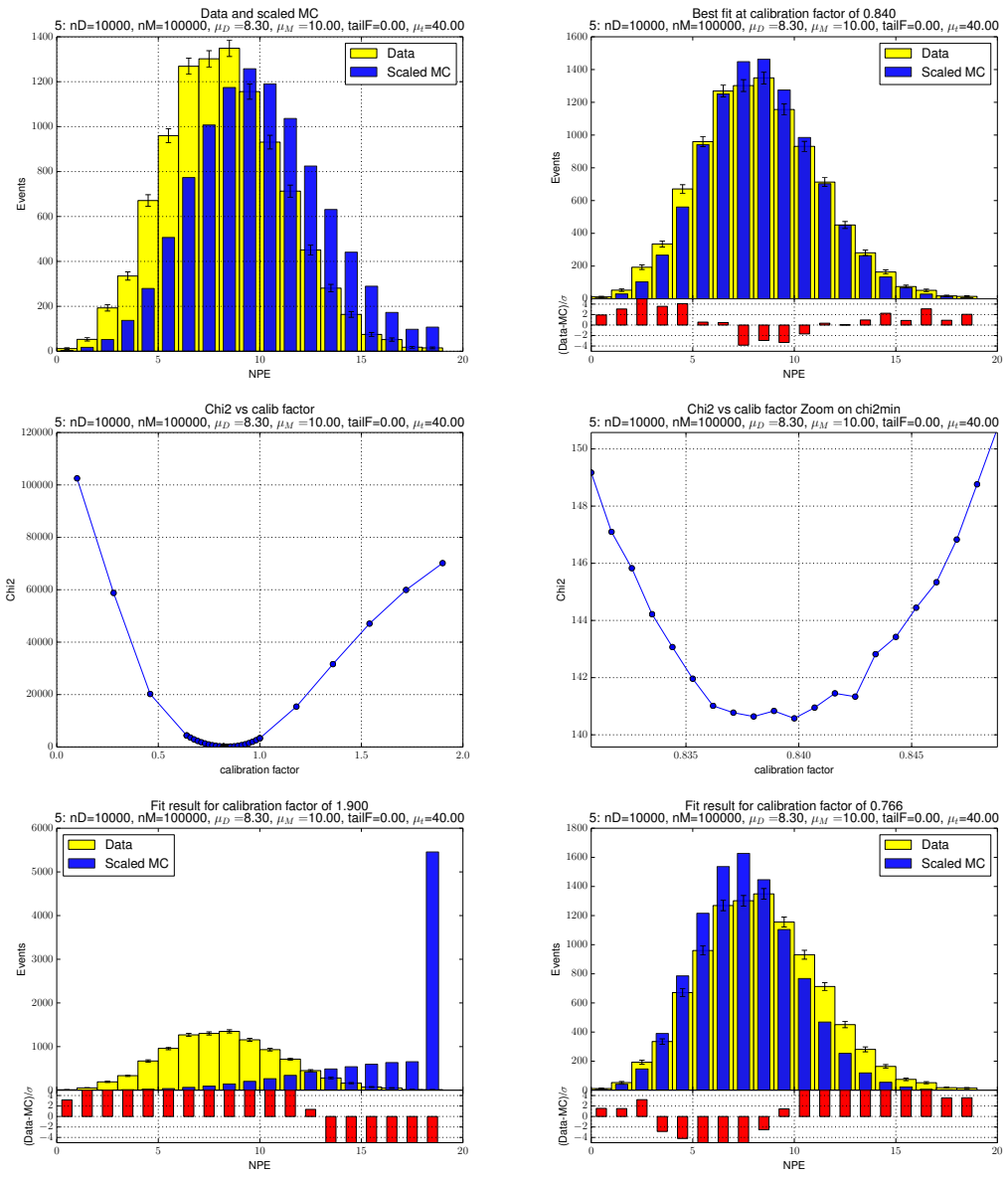

Figure 11: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 05. Data compared to MC scaled by two randomly chosen calibration factors.

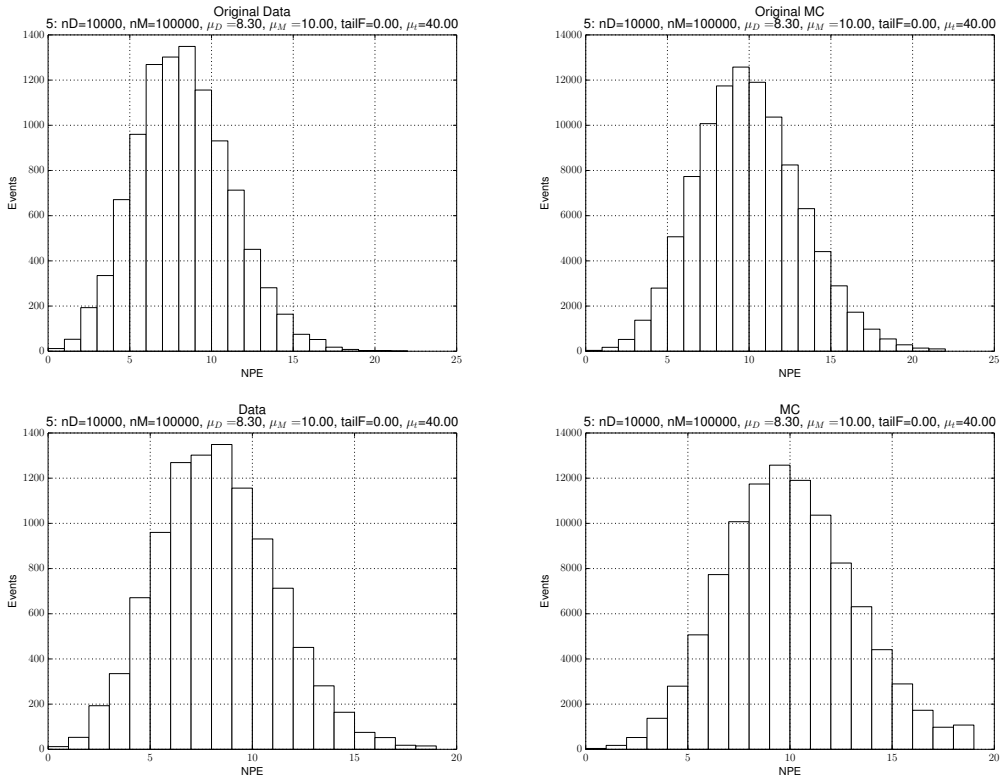

Figure 12: NPE histograms for data and MC for configuration 05. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

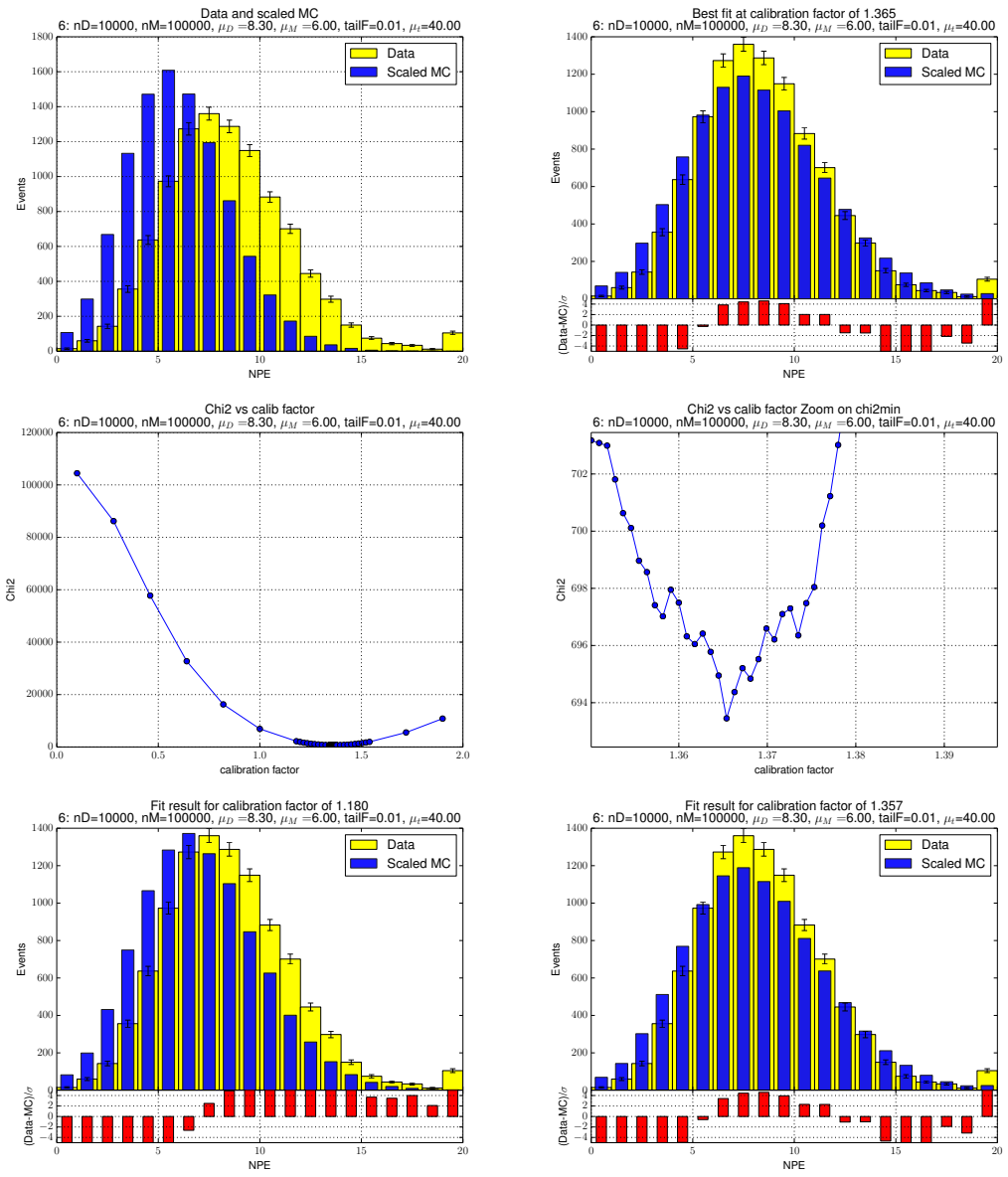

Figure 13: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 06. Data compared to MC scaled by two randomly chosen calibration factors.

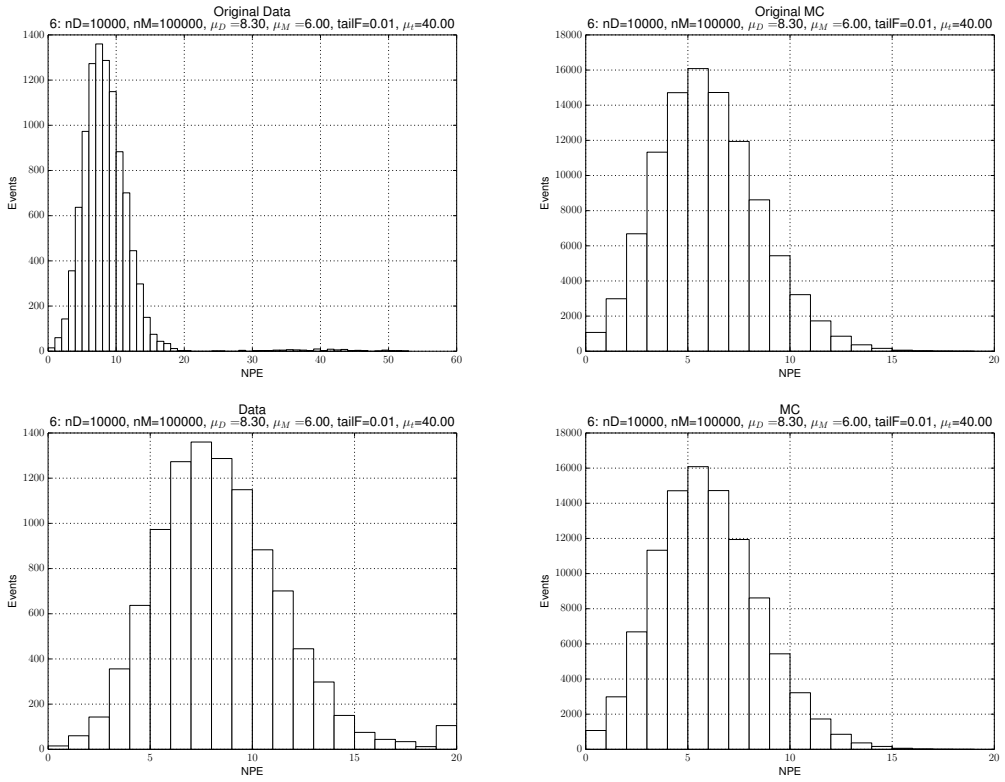

Figure 14: NPE histograms for data and MC for configuration 06. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

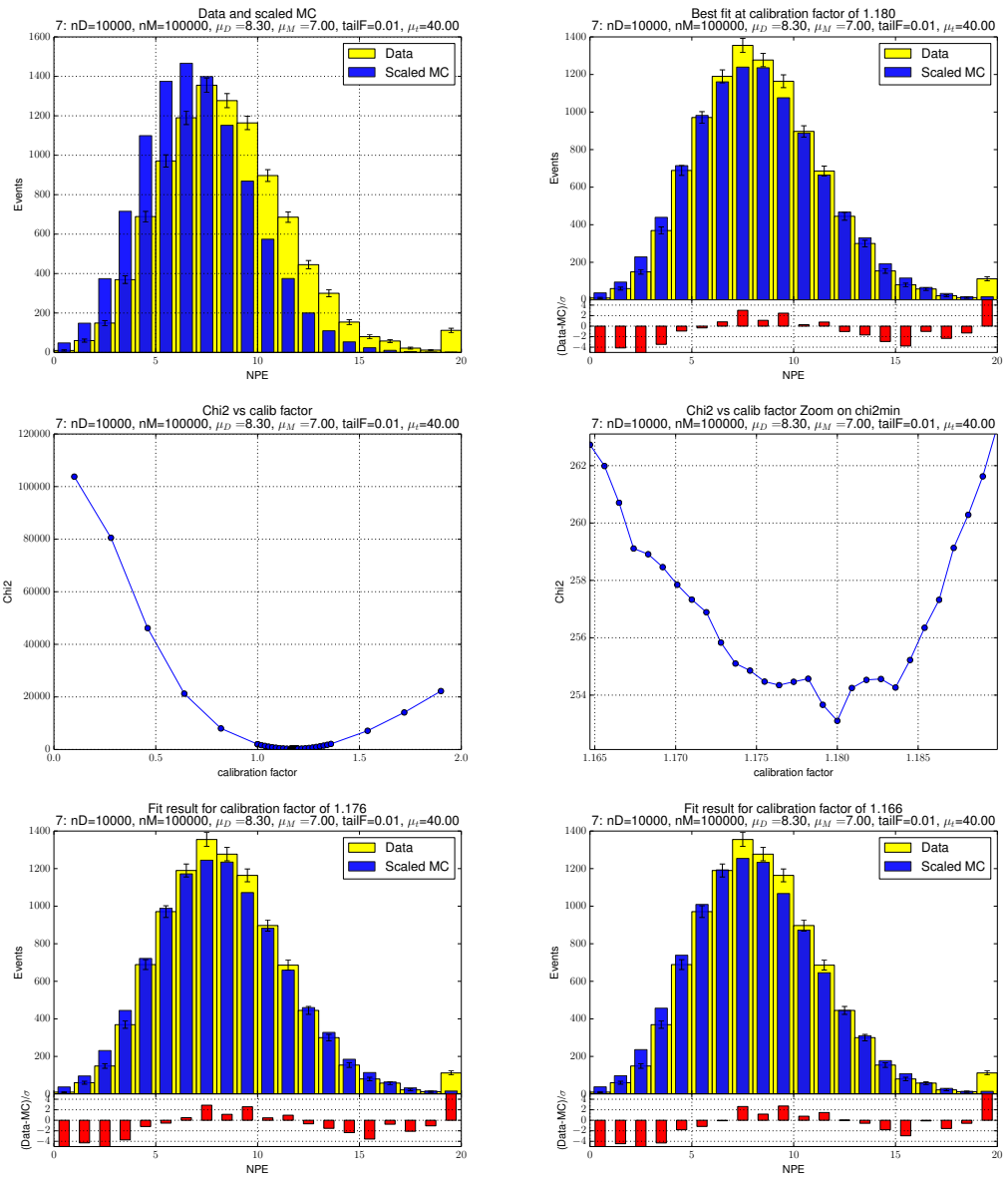

Figure 15: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 07. Data compared to MC scaled by two randomly chosen calibration factors.

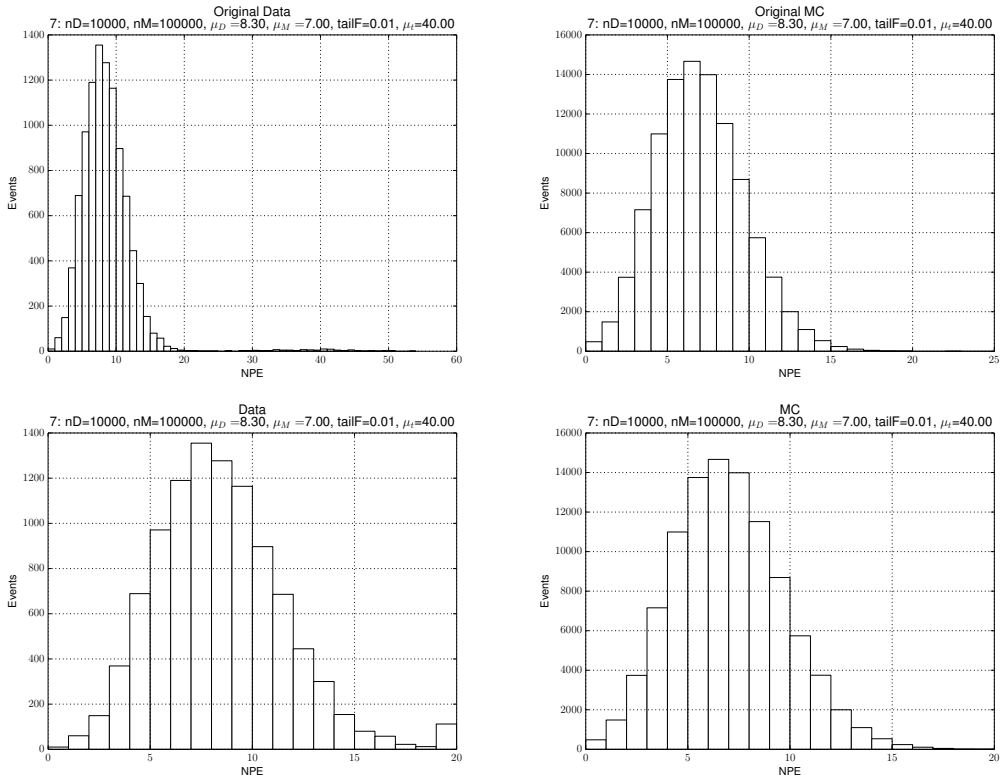

Figure 16: NPE histograms for data and MC for configuration 07. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

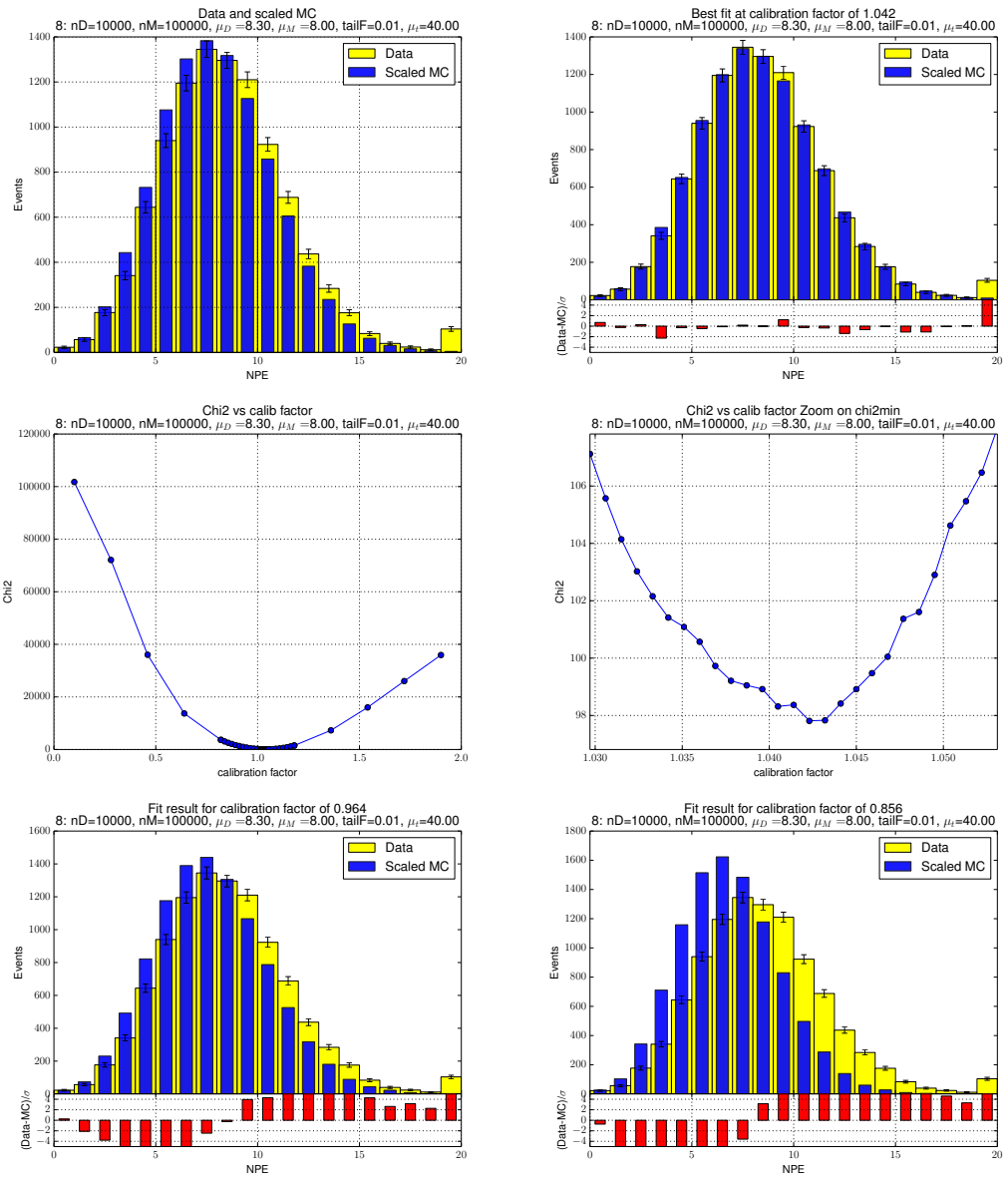

Figure 17: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 08. Data compared to MC scaled by two randomly chosen calibration factors.

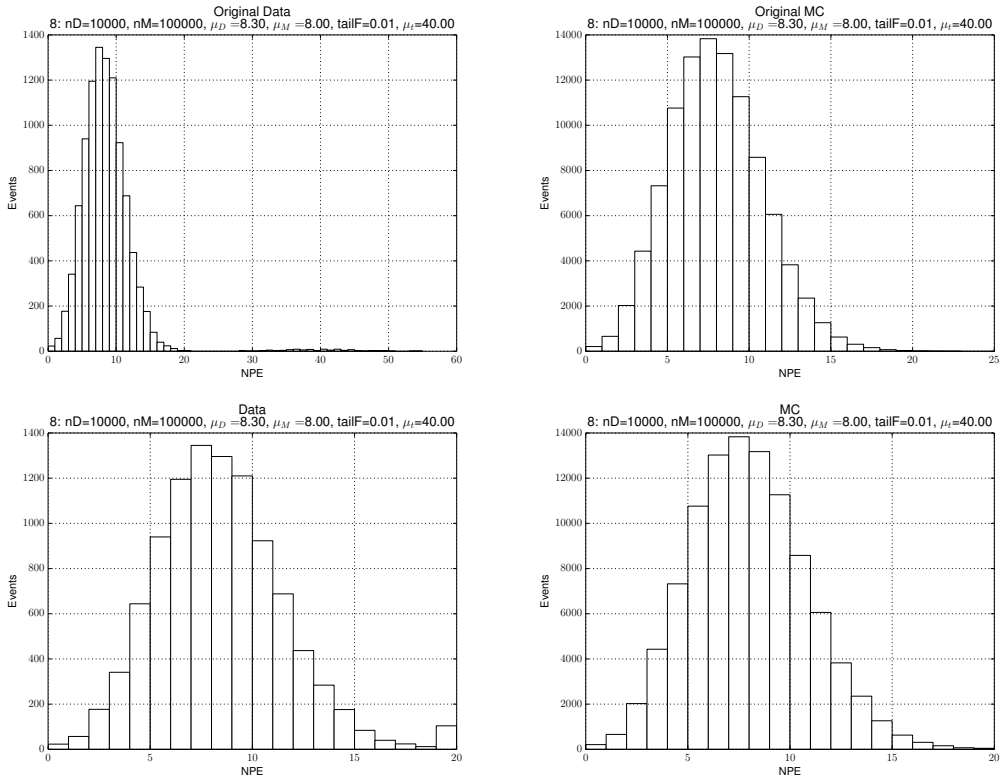

Figure 18: NPE histograms for data and MC for configuration 08. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

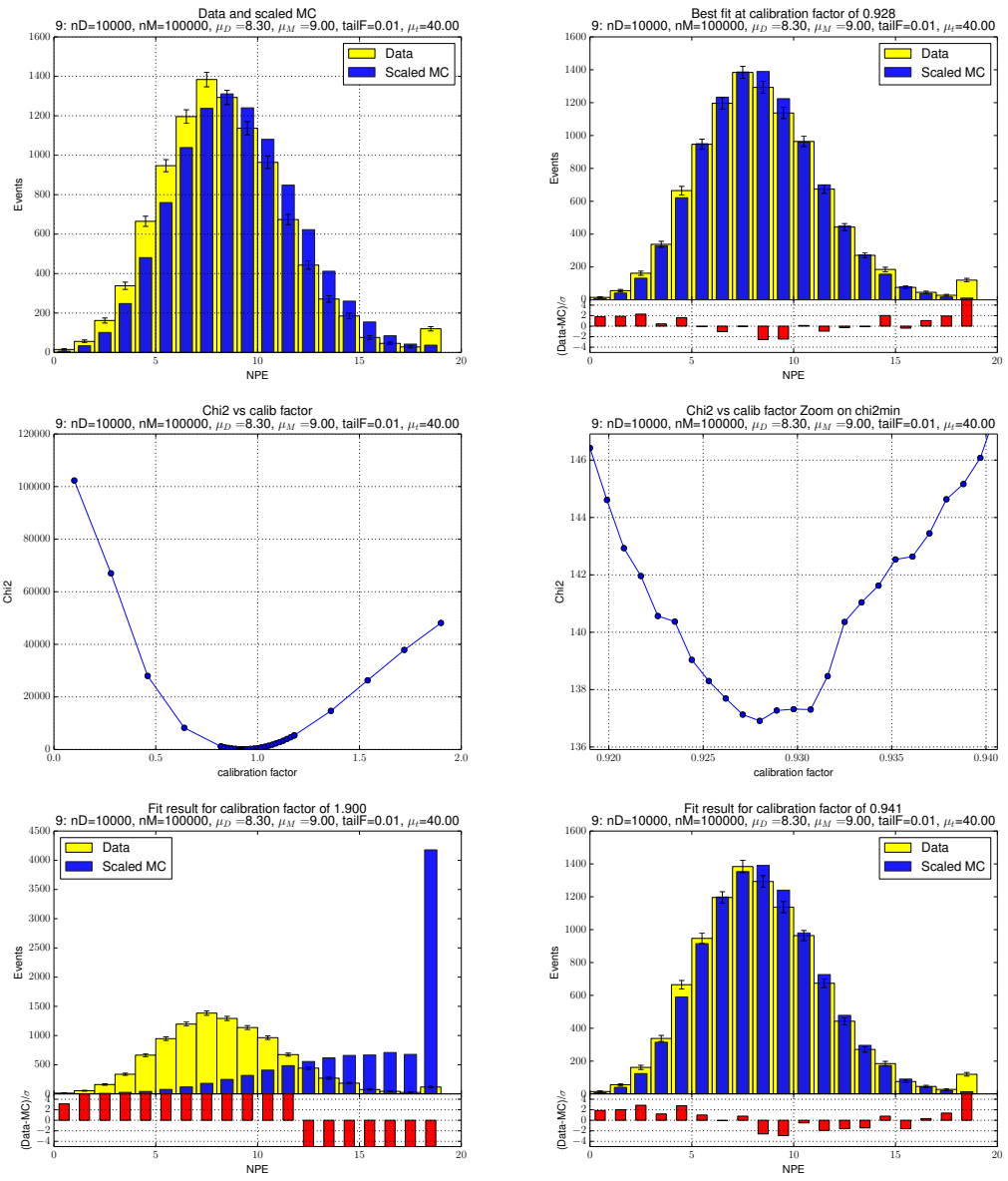

Figure 19: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 09. Data compared to MC scaled by two randomly chosen calibration factors.

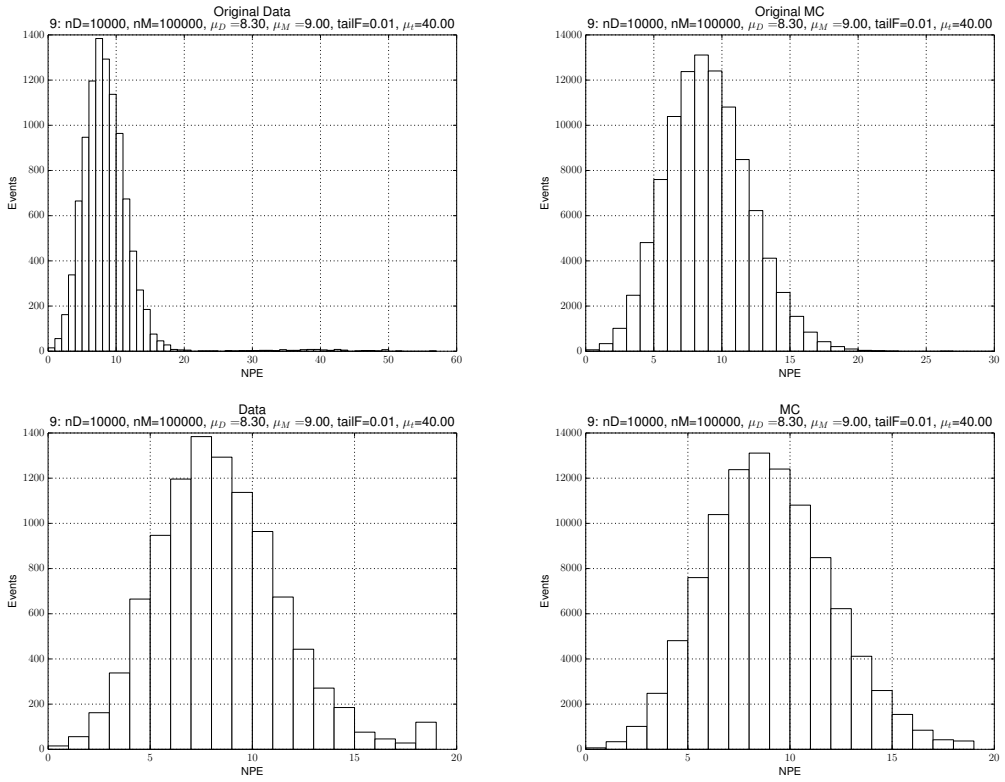

Figure 20: NPE histograms for data and MC for configuration 09. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

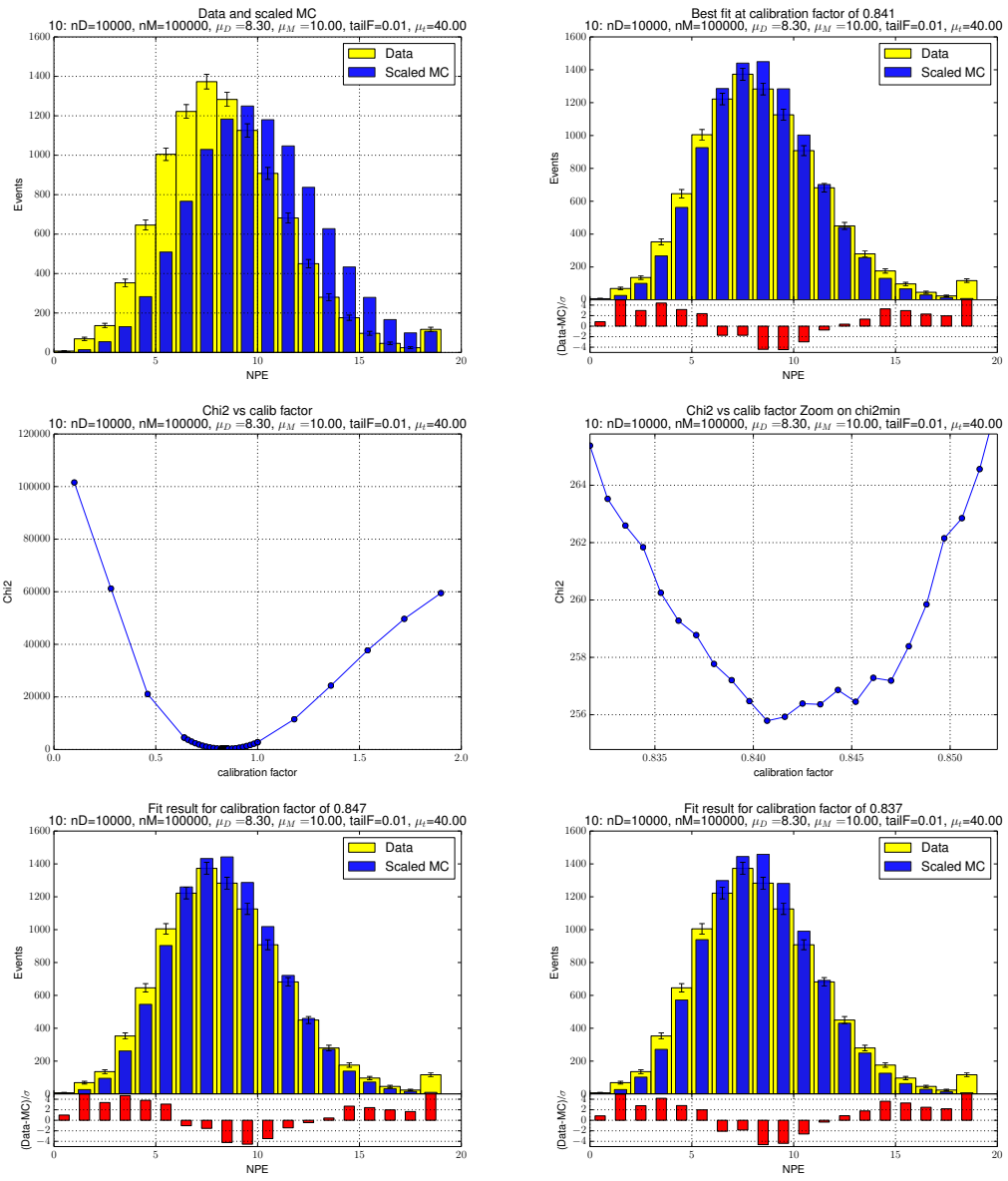

Figure 21: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 10. Data compared to MC scaled by two randomly chosen calibration factors.

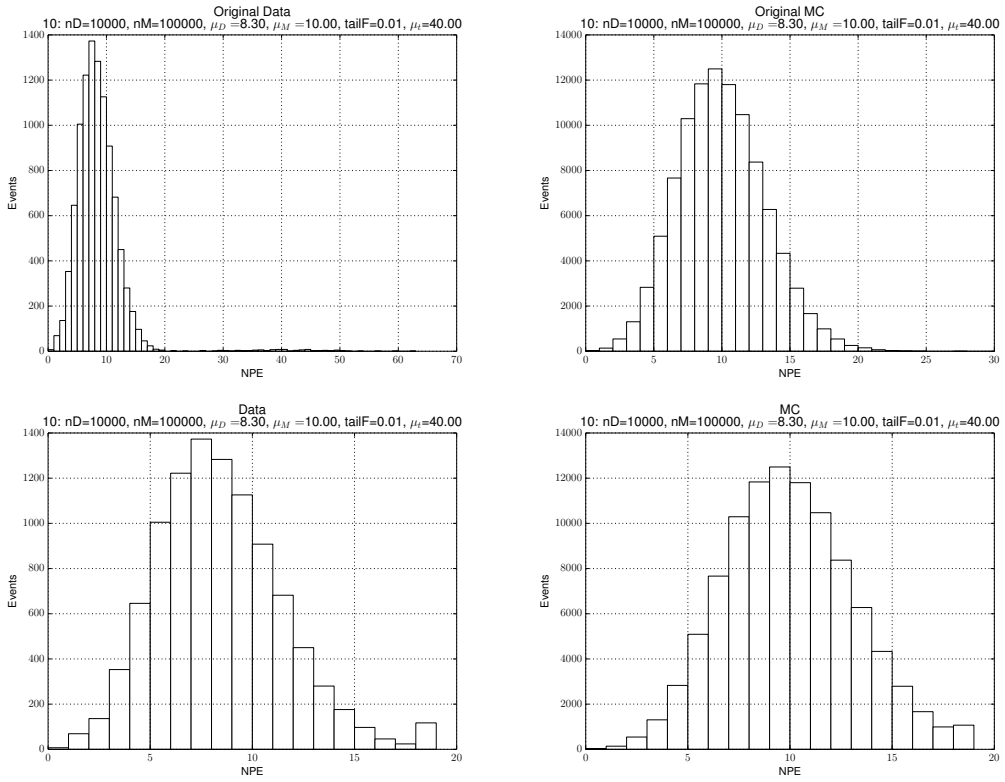

Figure 22: NPE histograms for data and MC for configuration 10. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

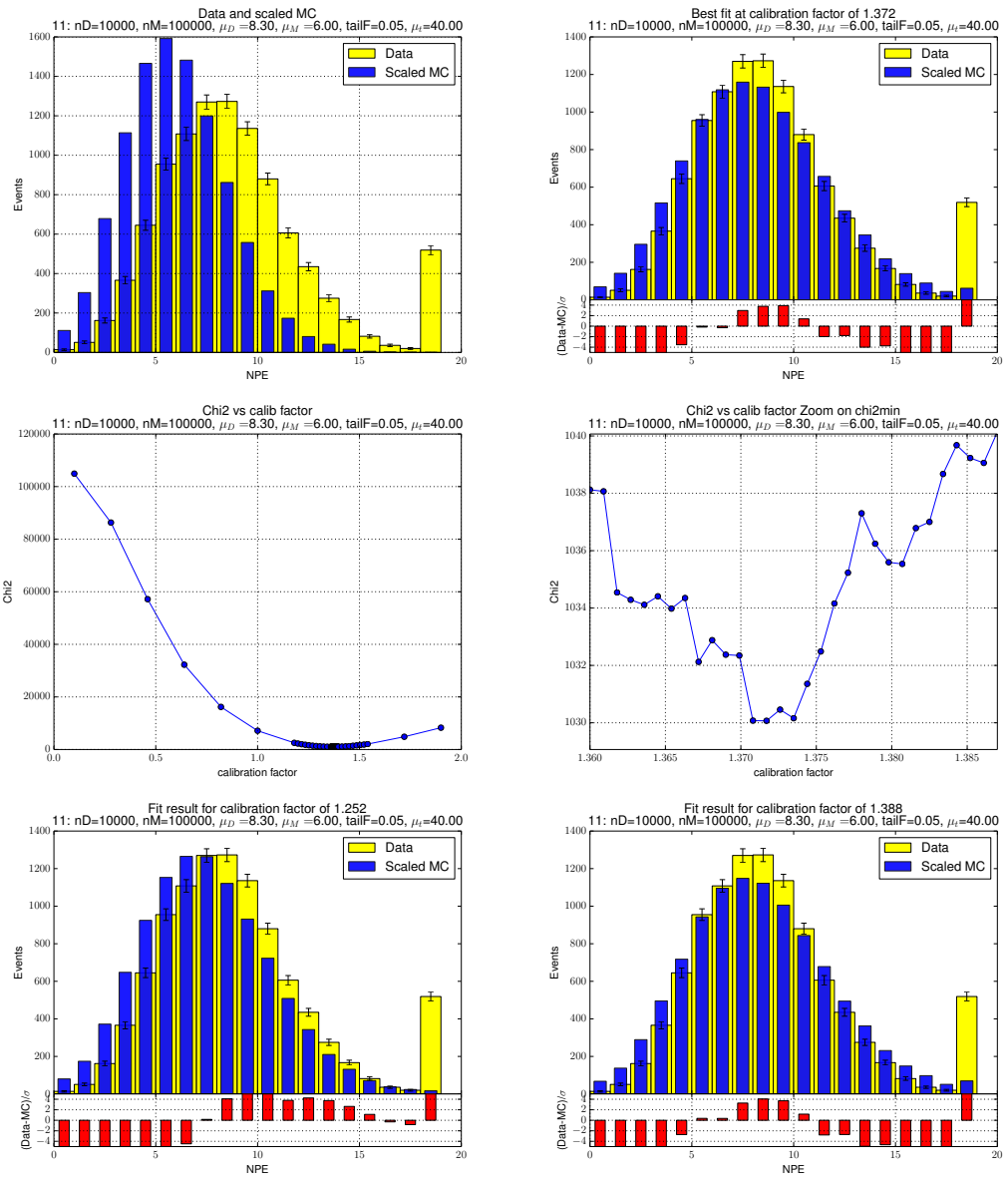

Figure 23: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 11. Data compared to MC scaled by two randomly chosen calibration factors.

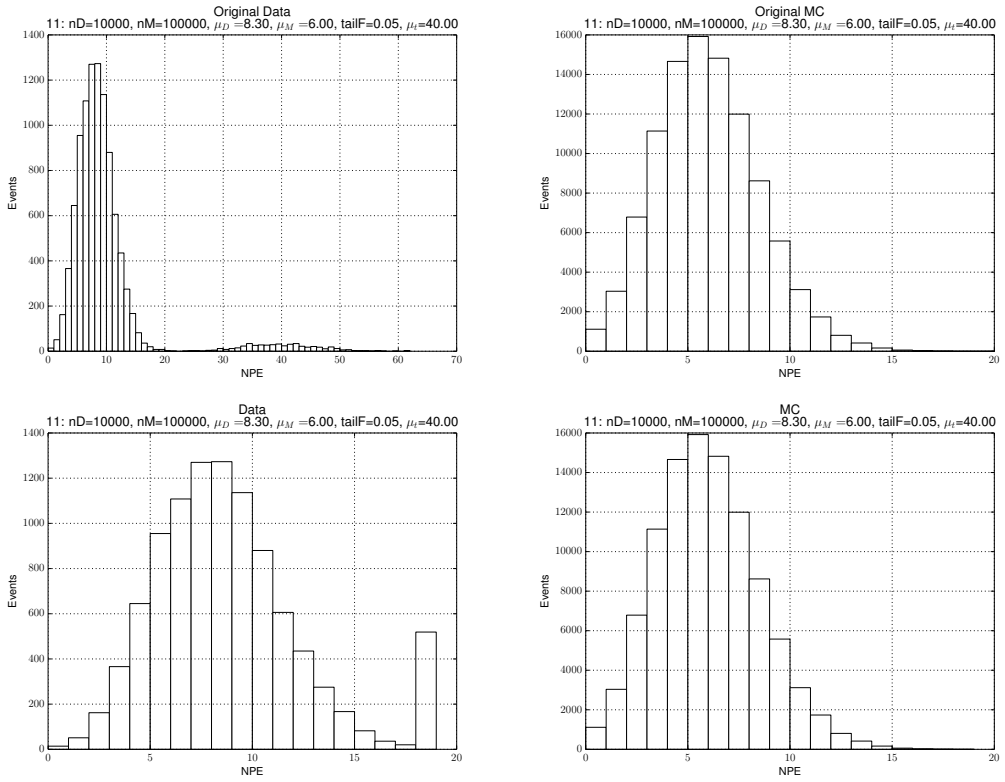

Figure 24: NPE histograms for data and MC for configuration 11. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

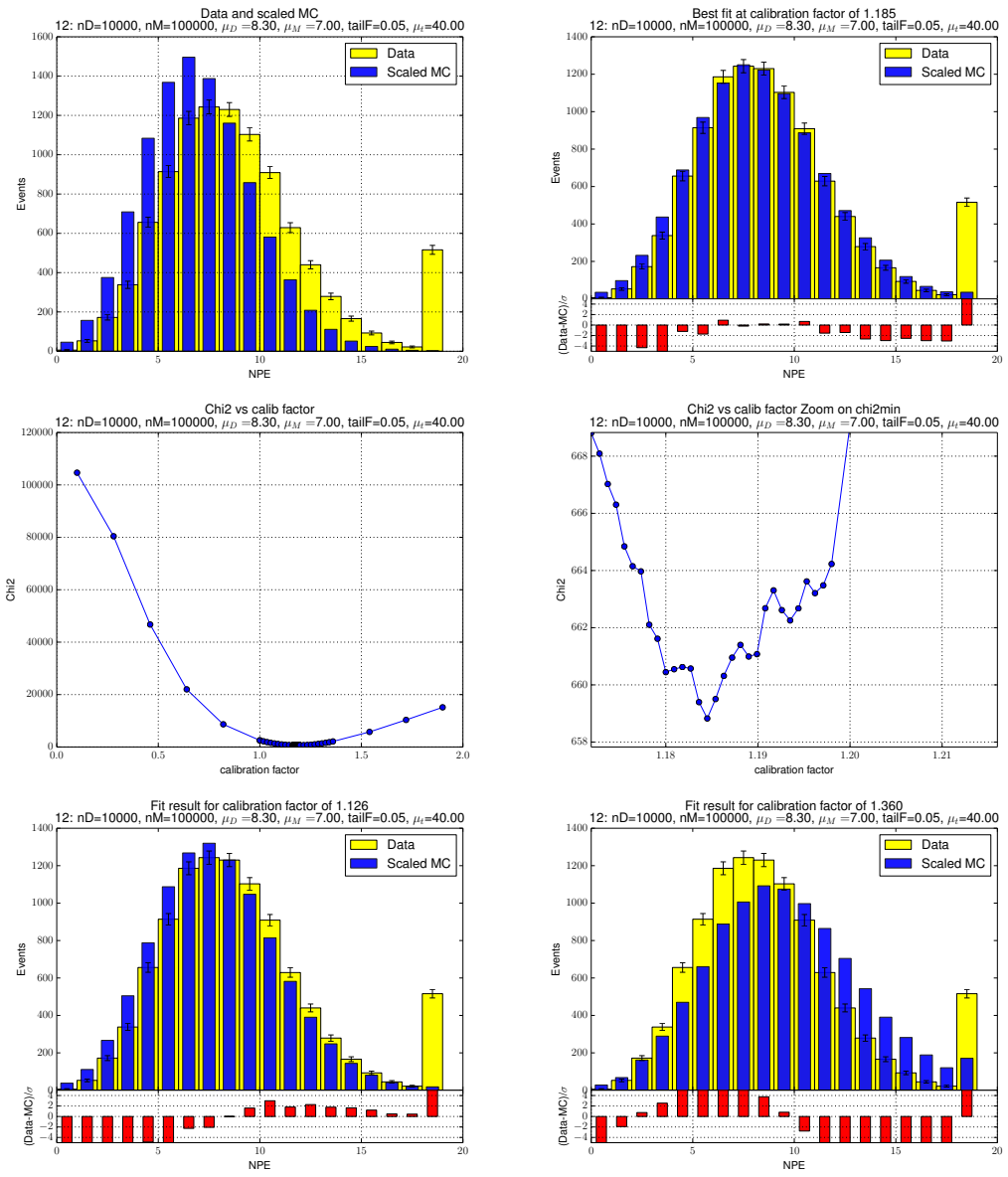

Figure 25: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 12. Data compared to MC scaled by two randomly chosen calibration factors.

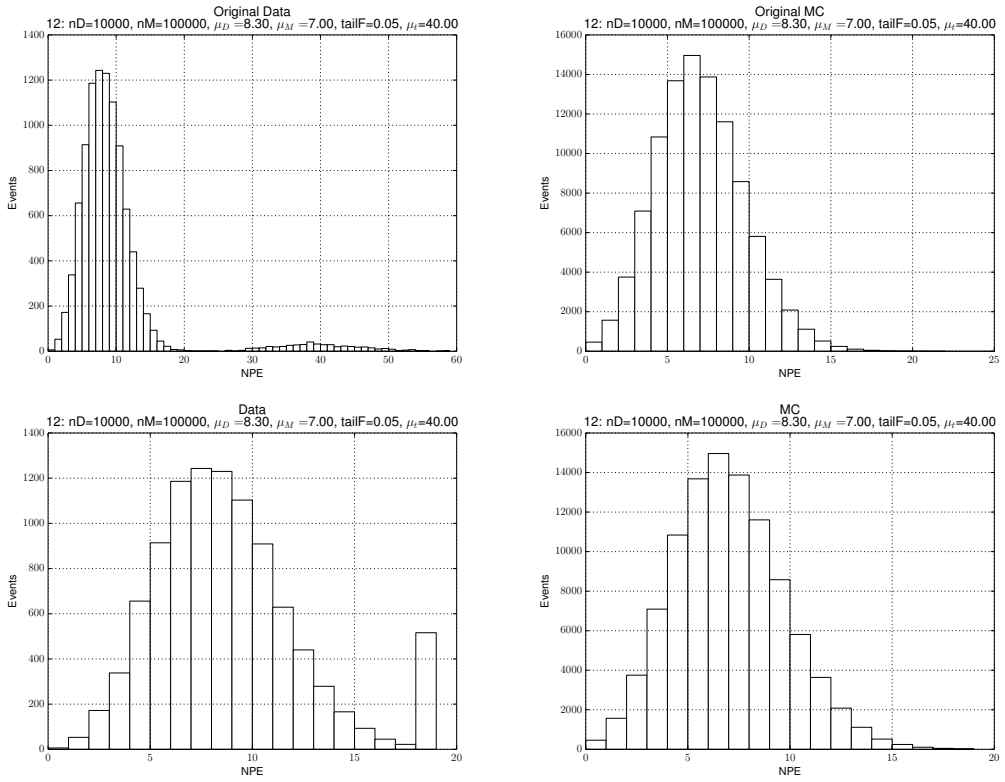

Figure 26: NPE histograms for data and MC for configuration 12. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

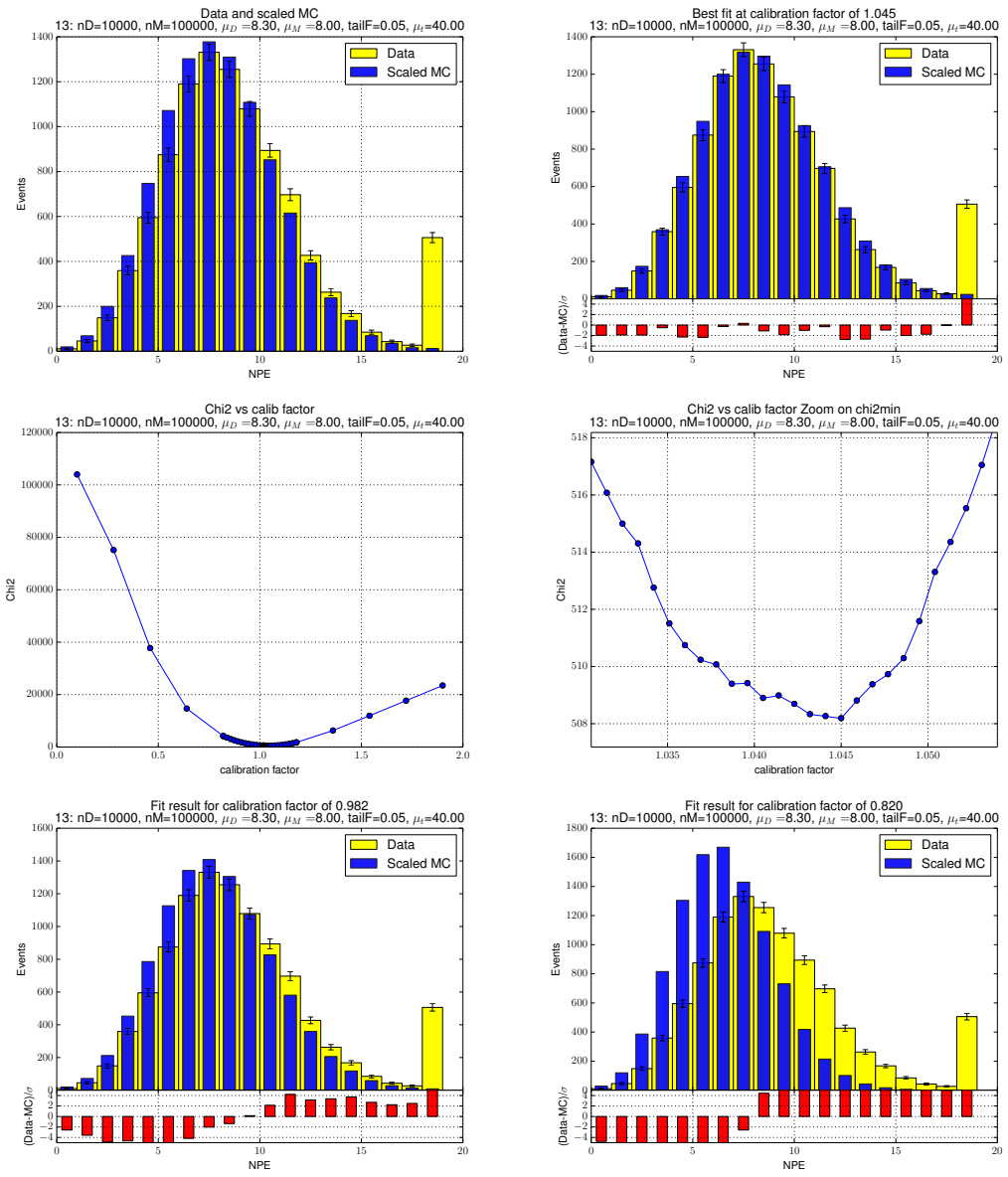

Figure 27: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 13. Data compared to MC scaled by two randomly chosen calibration factors.

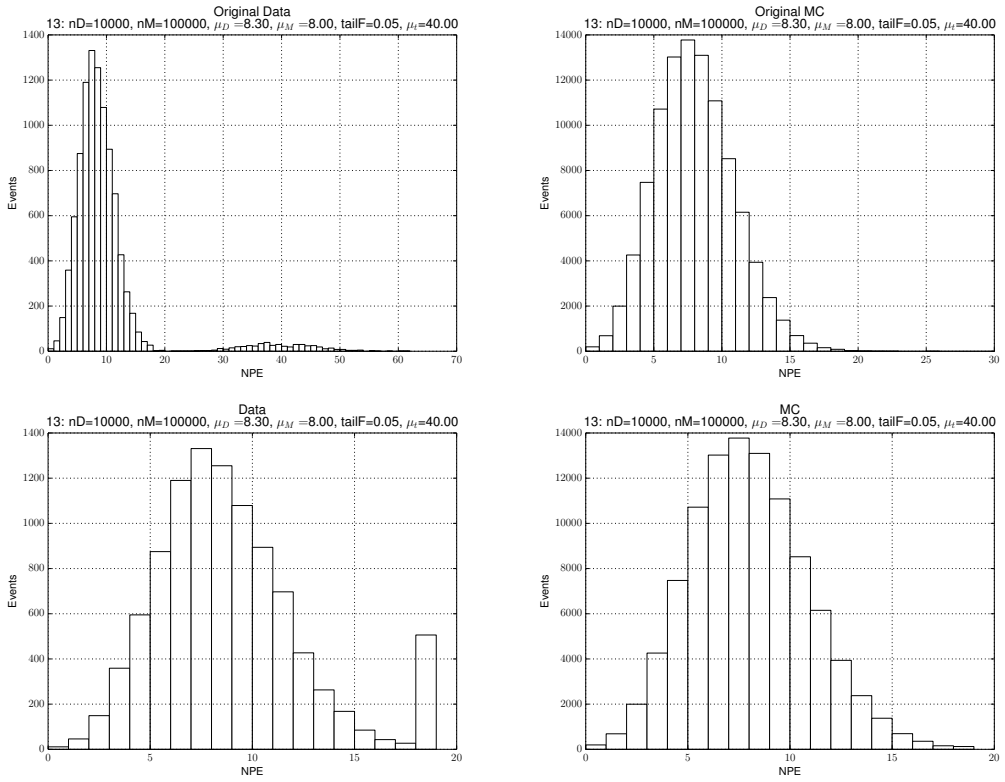

Figure 28: NPE histograms for data and MC for configuration 13. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

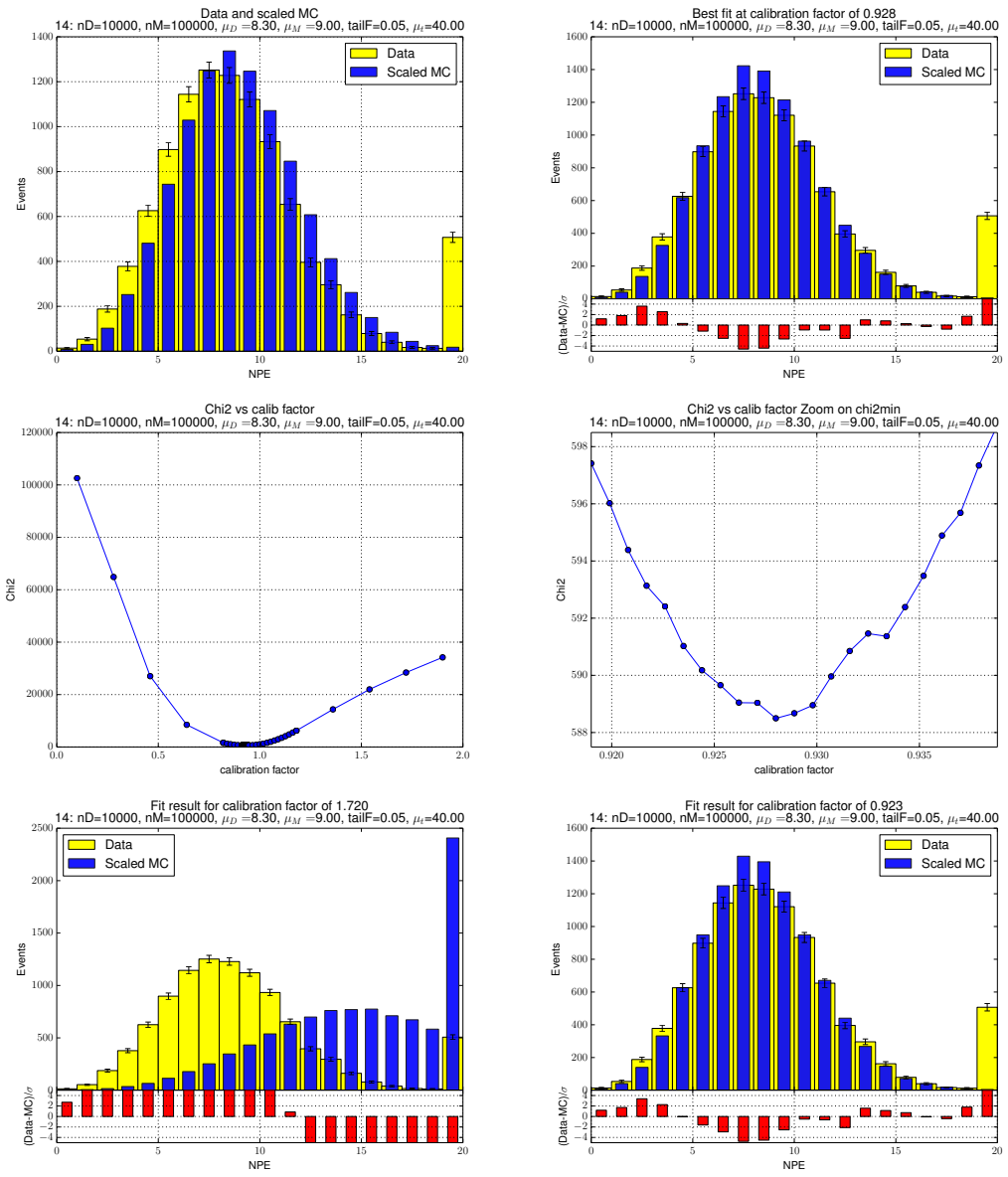

Figure 29: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 14. Data compared to MC scaled by two randomly chosen calibration factors.

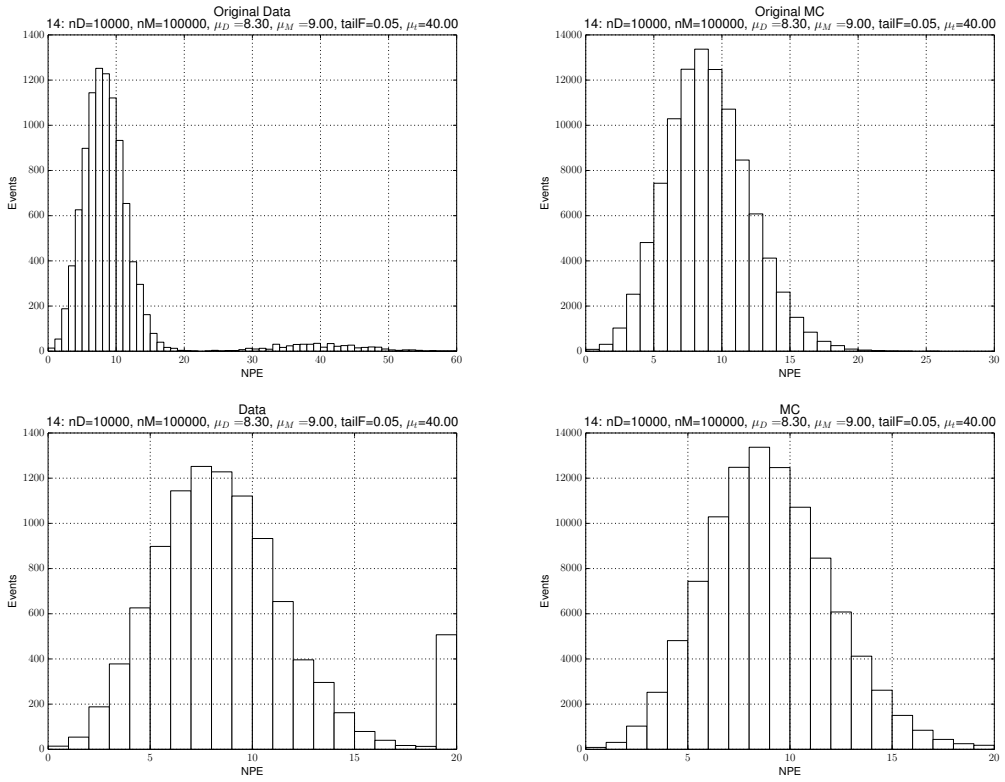

Figure 30: NPE histograms for data and MC for configuration 14. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

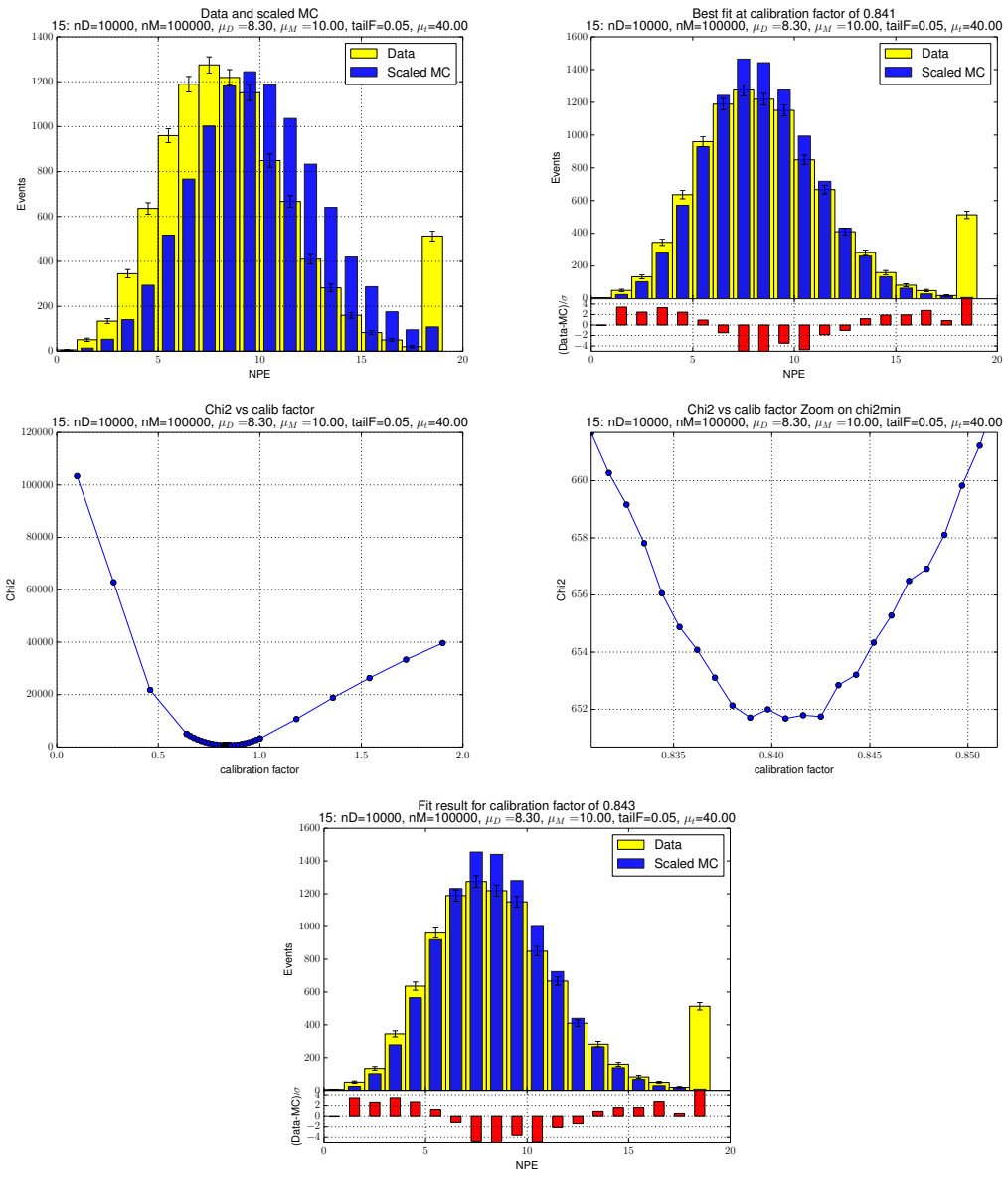

Figure 31: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 15. Data compared to MC scaled by two randomly chosen calibration factors.

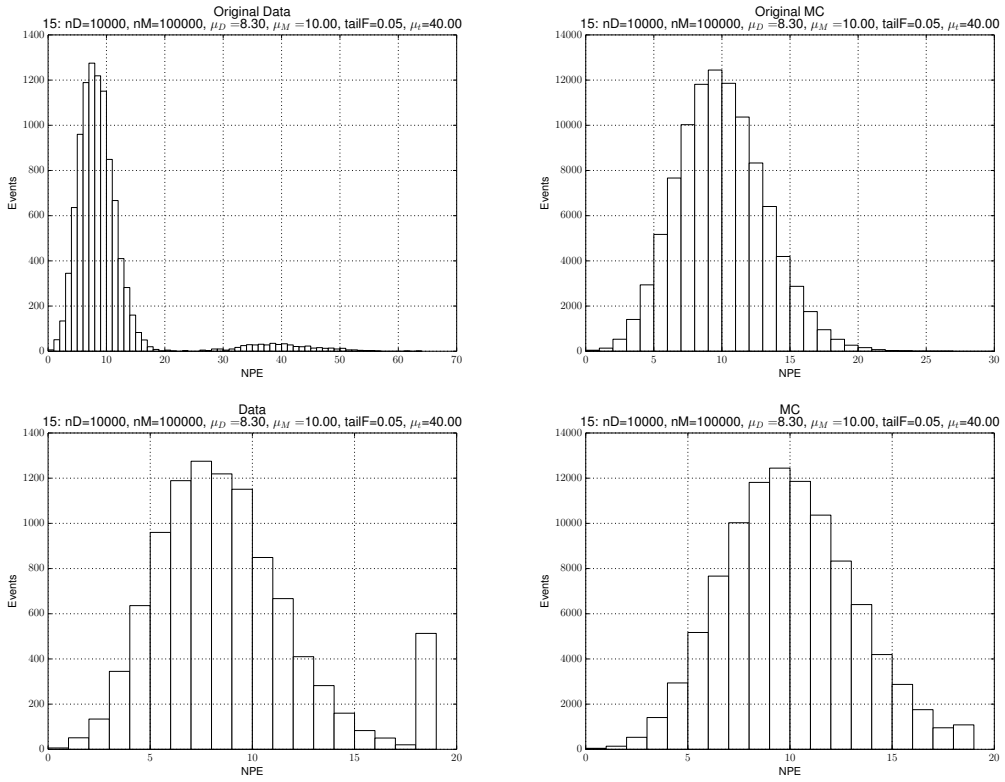

Figure 32: NPE histograms for data and MC for configuration 15. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

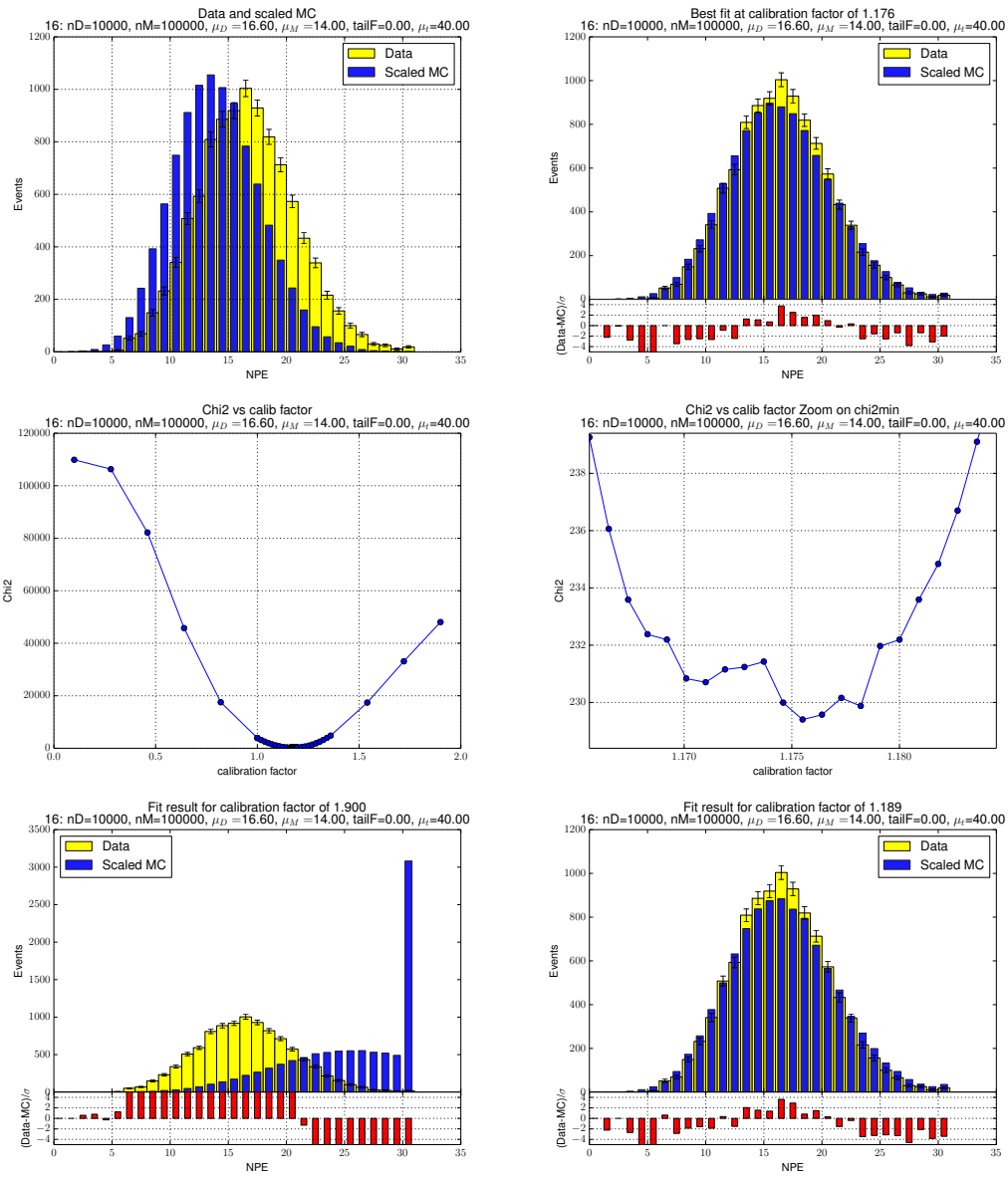

Figure 33: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 16. Data compared to MC scaled by two randomly chosen calibration factors.

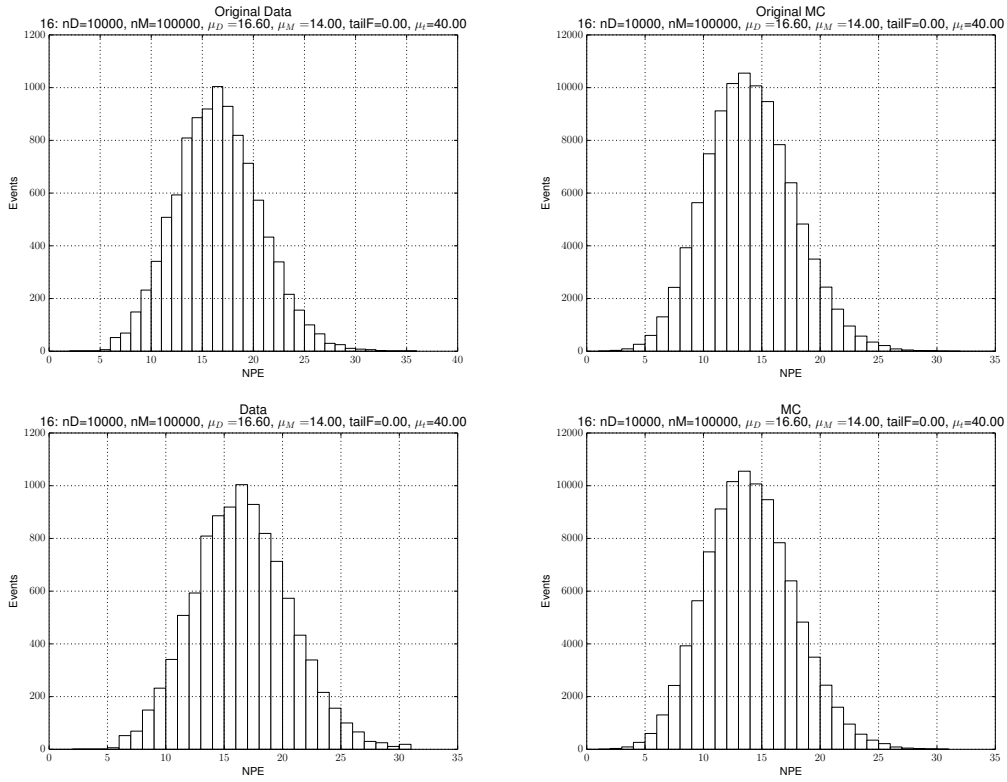

Figure 34: NPE histograms for data and MC for configuration 16. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

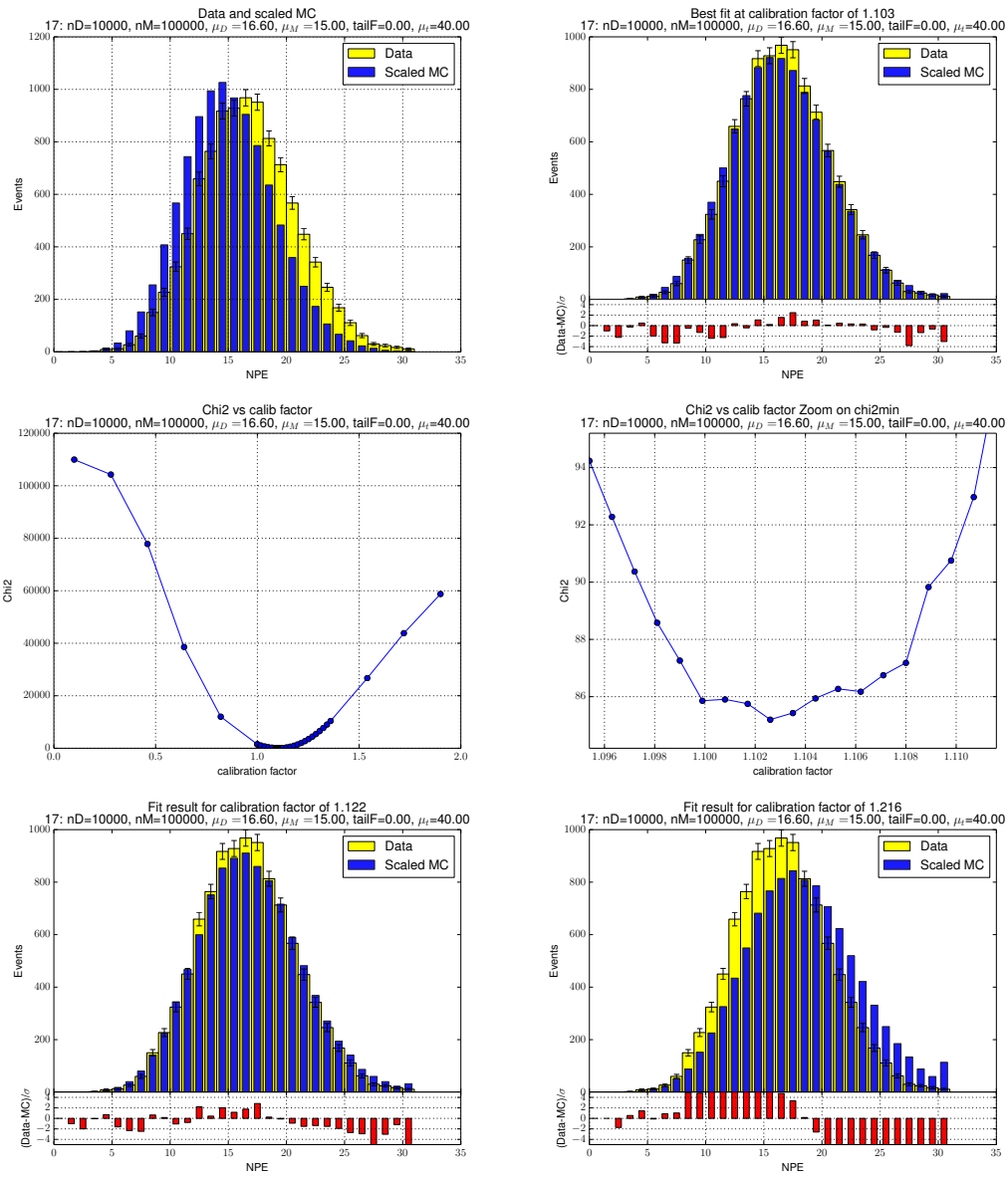

Figure 35: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 17. Data compared to MC scaled by two randomly chosen calibration factors.

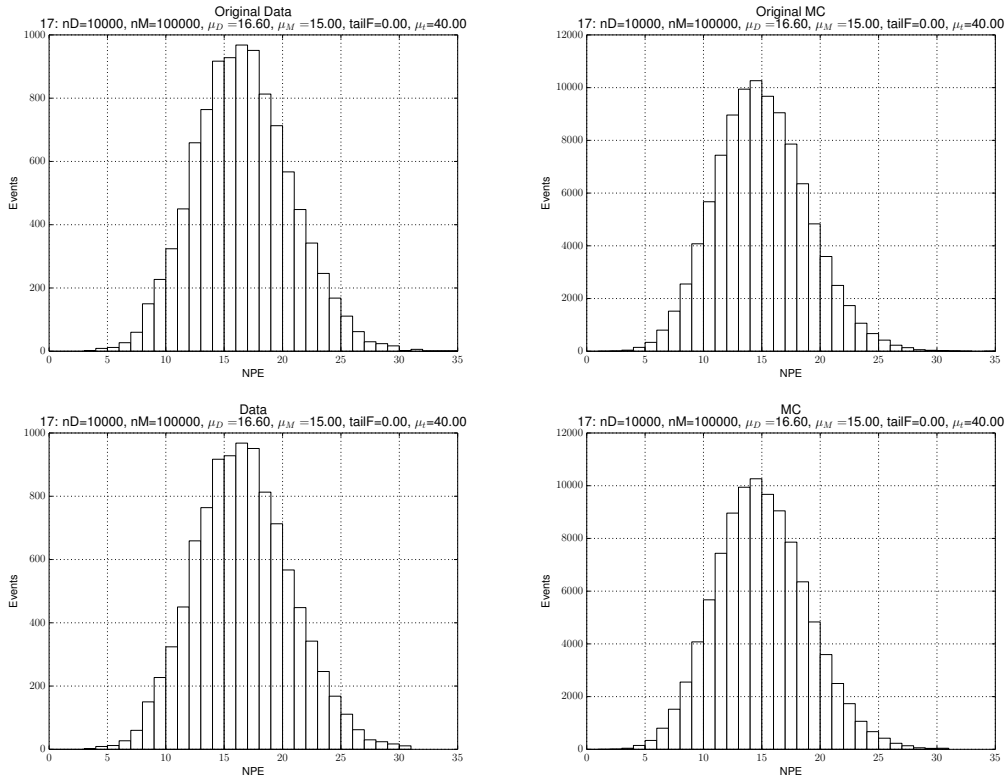

Figure 36: NPE histograms for data and MC for configuration 17. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

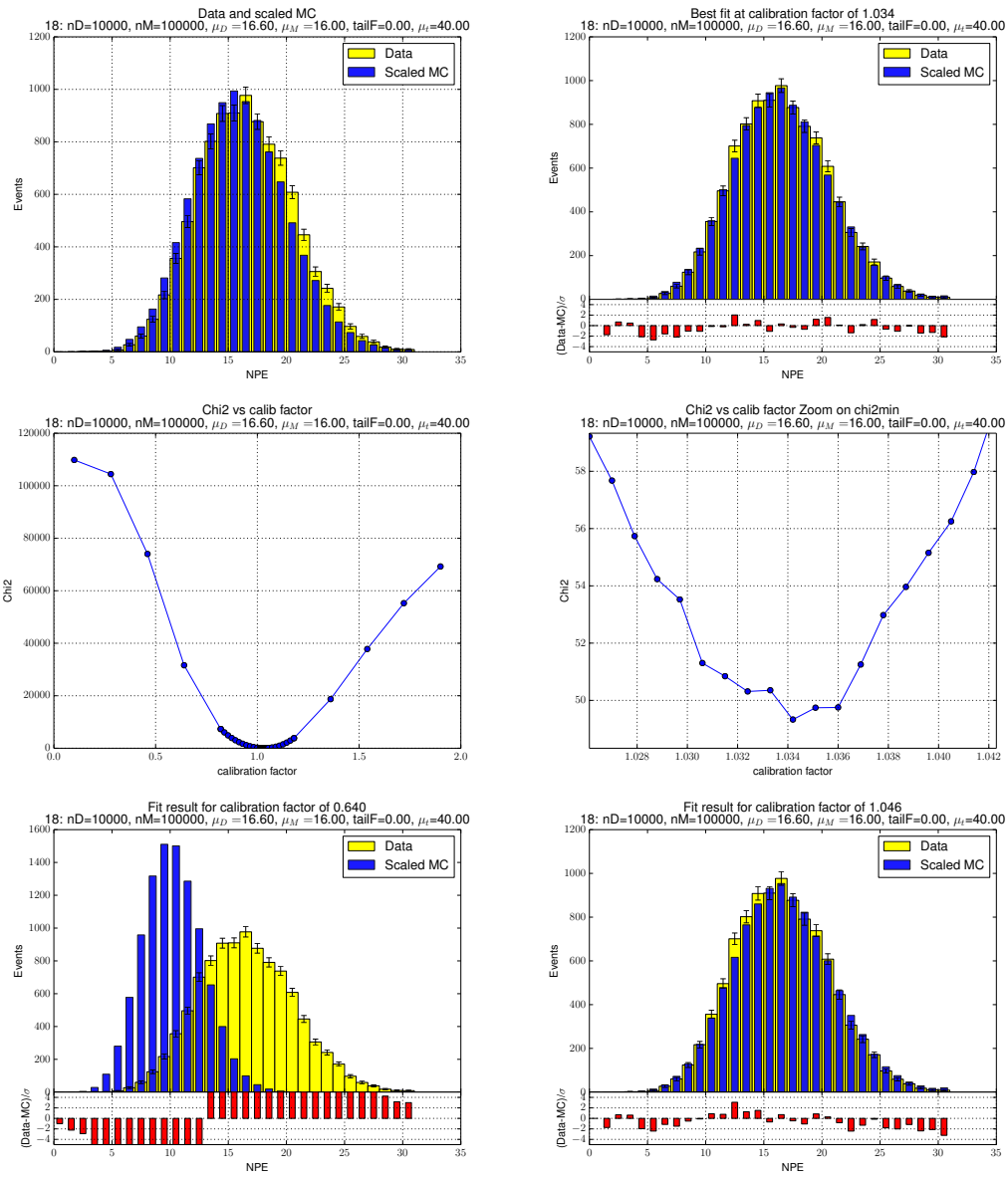

Figure 37: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 18. Data compared to MC scaled by two randomly chosen calibration factors.

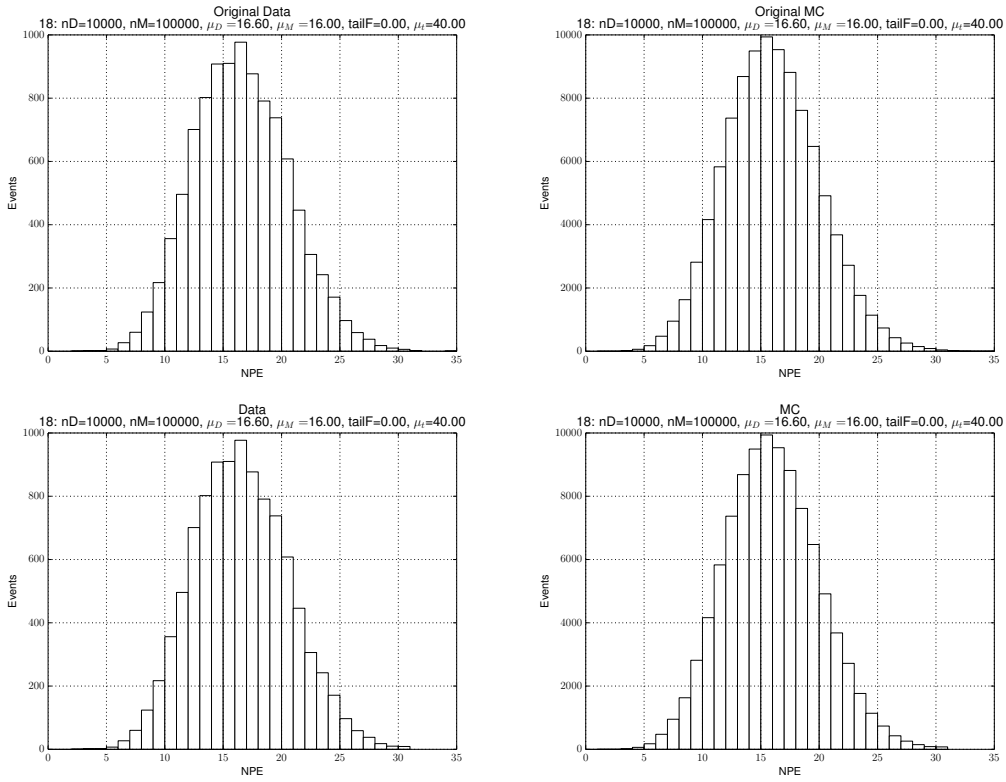

Figure 38: NPE histograms for data and MC for configuration 18. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

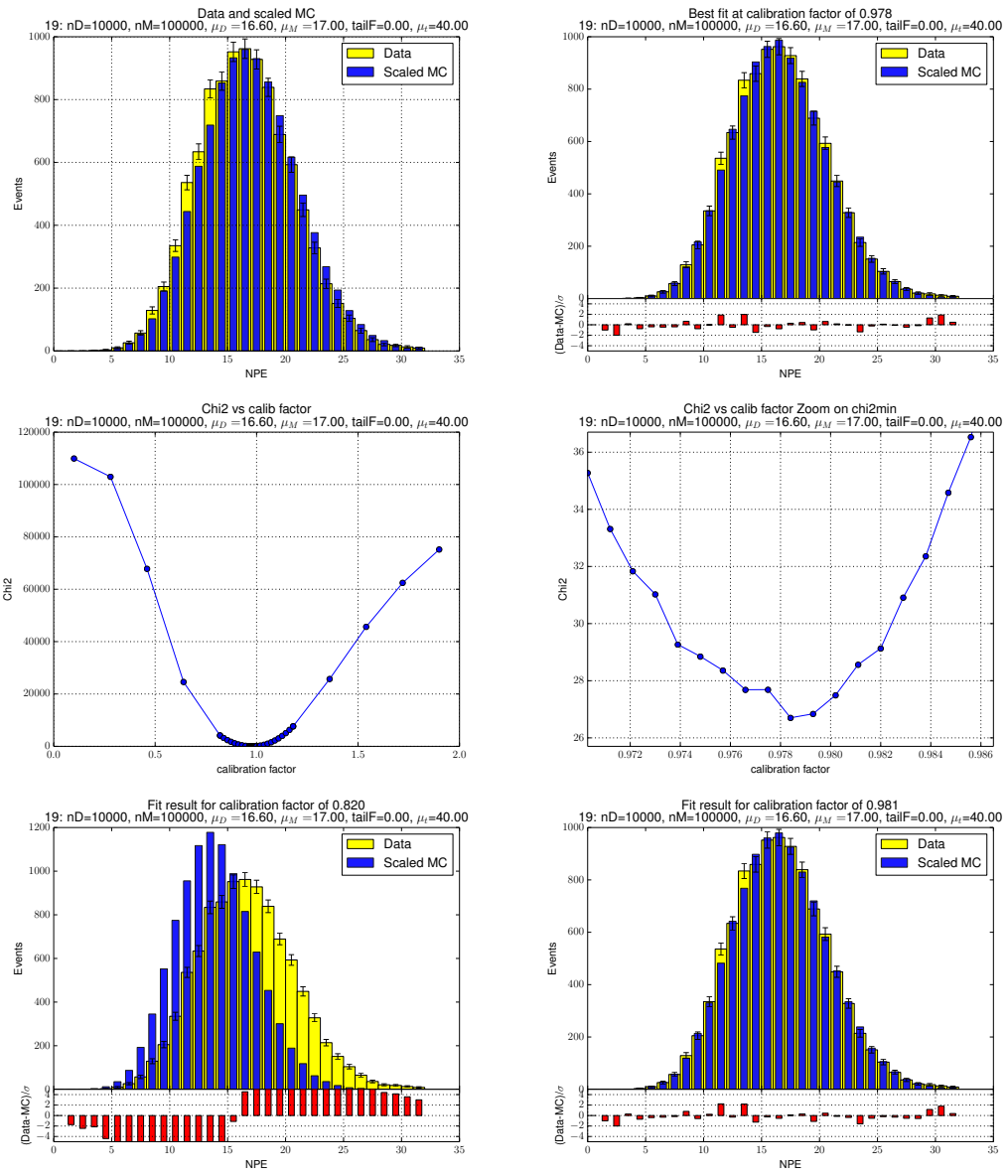

Figure 39: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 19. Data compared to MC scaled by two randomly chosen calibration factors.

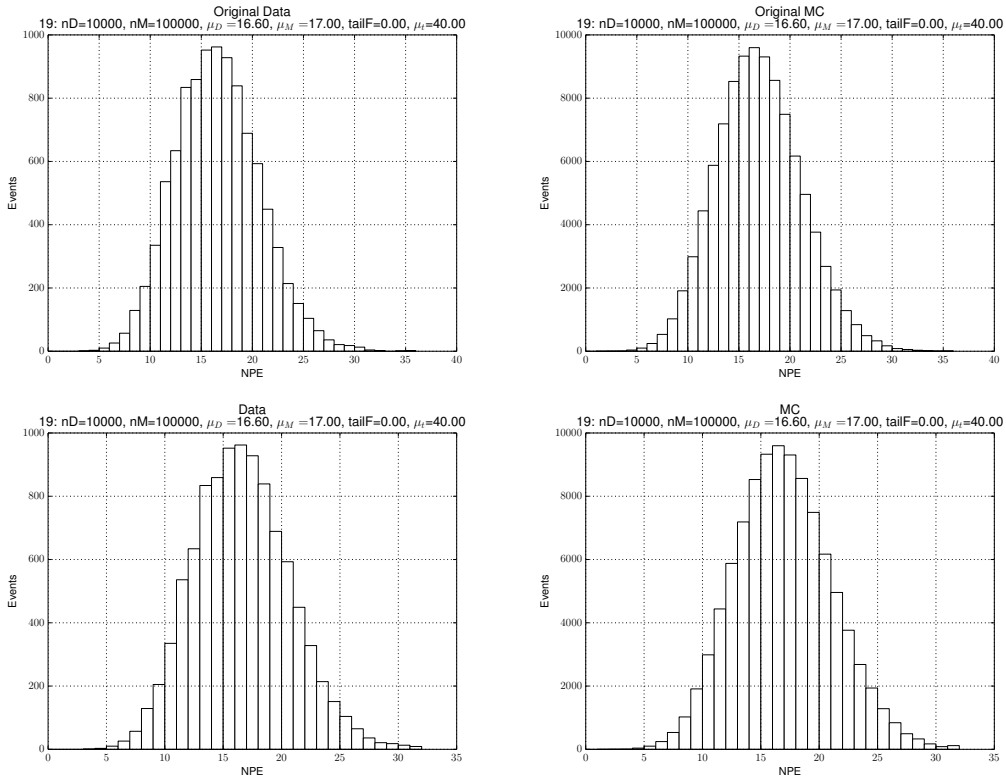

Figure 40: NPE histograms for data and MC for configuration 19. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

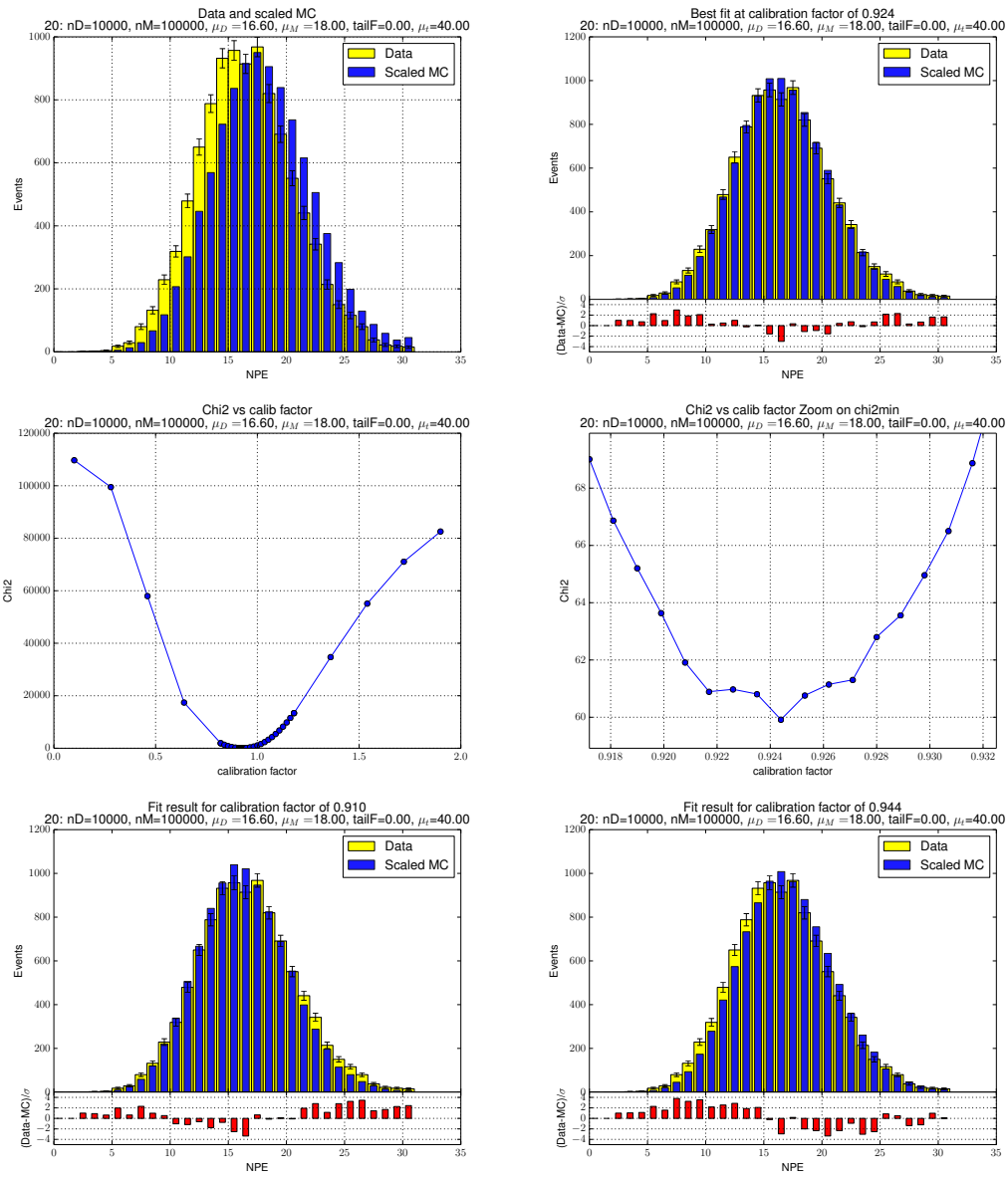

Figure 41: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 20. Data compared to MC scaled by two randomly chosen calibration factors.

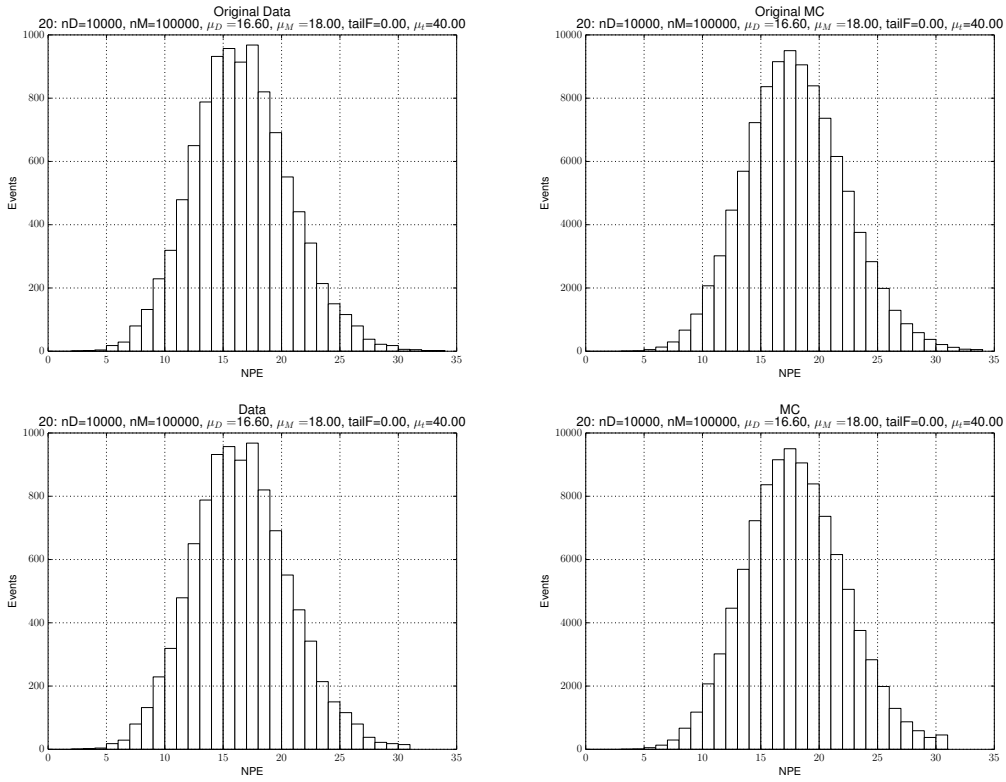

Figure 42: NPE histograms for data and MC for configuration 20. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

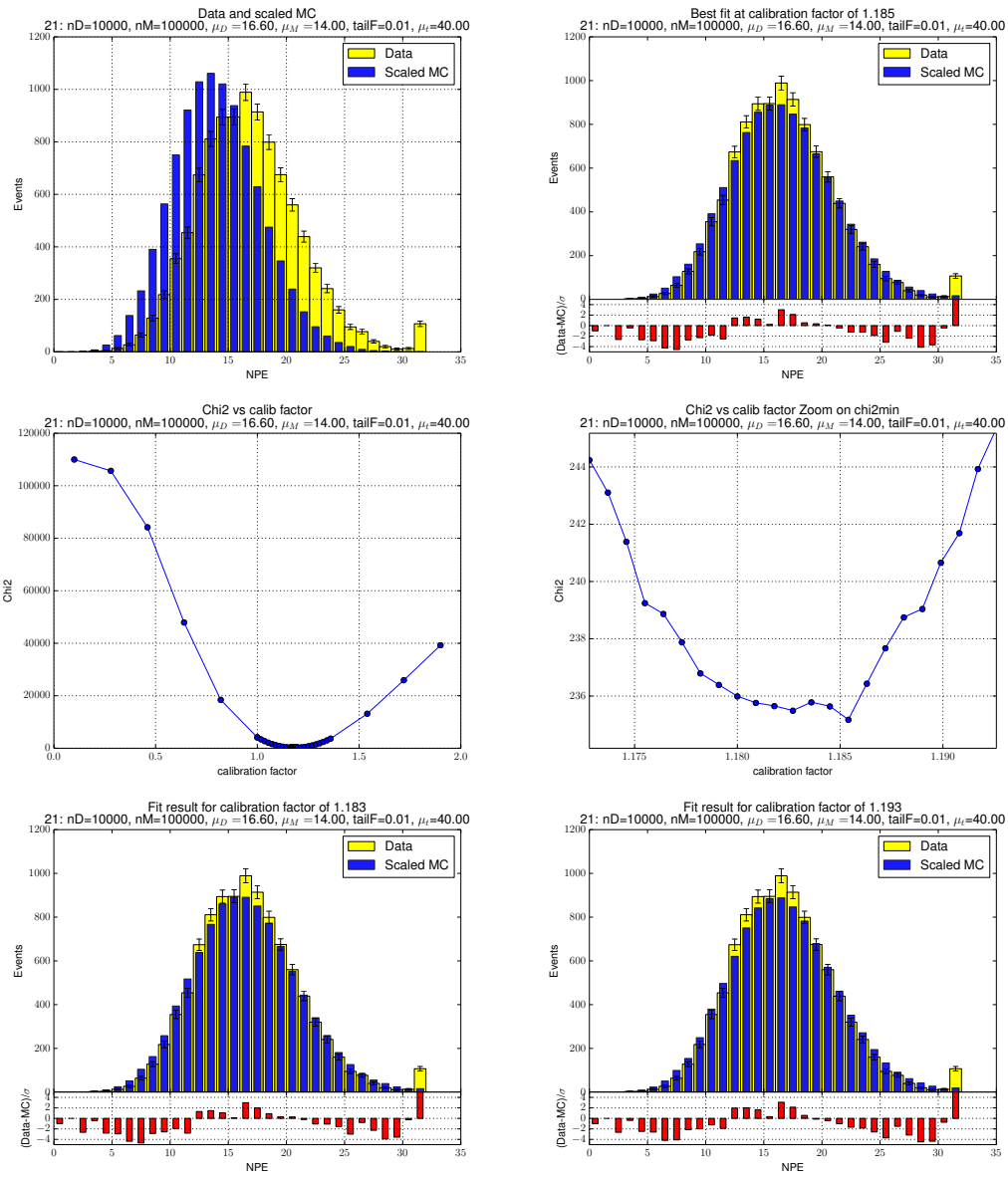

Figure 43: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 21. Data compared to MC scaled by two randomly chosen calibration factors.

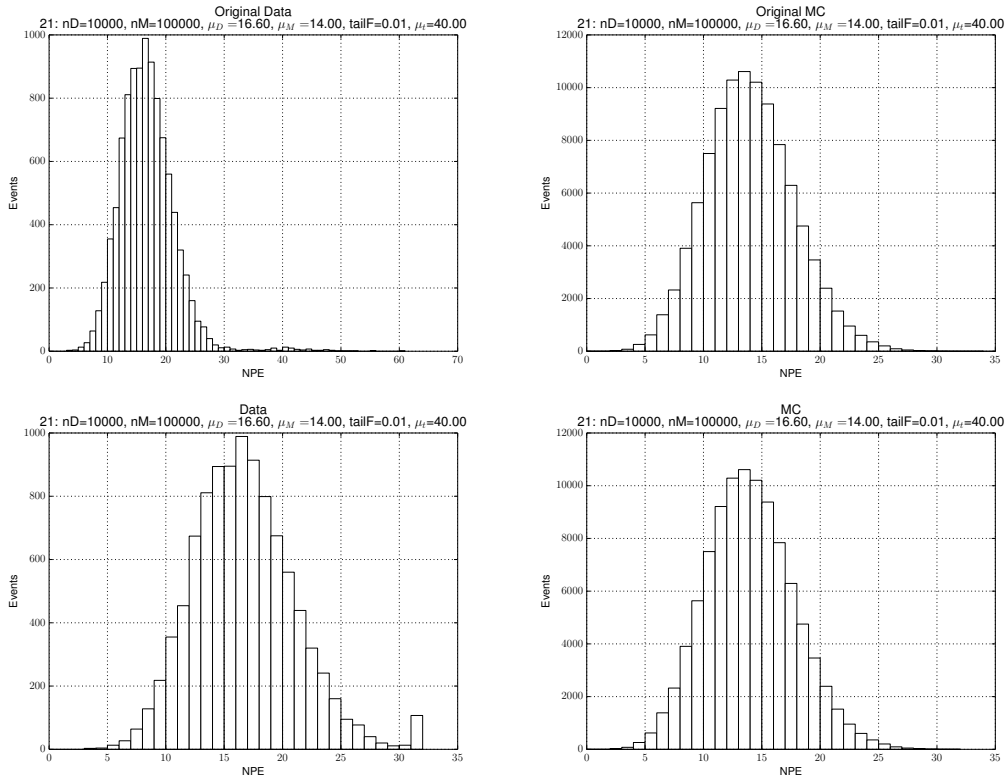

Figure 44: NPE histograms for data and MC for configuration 21. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

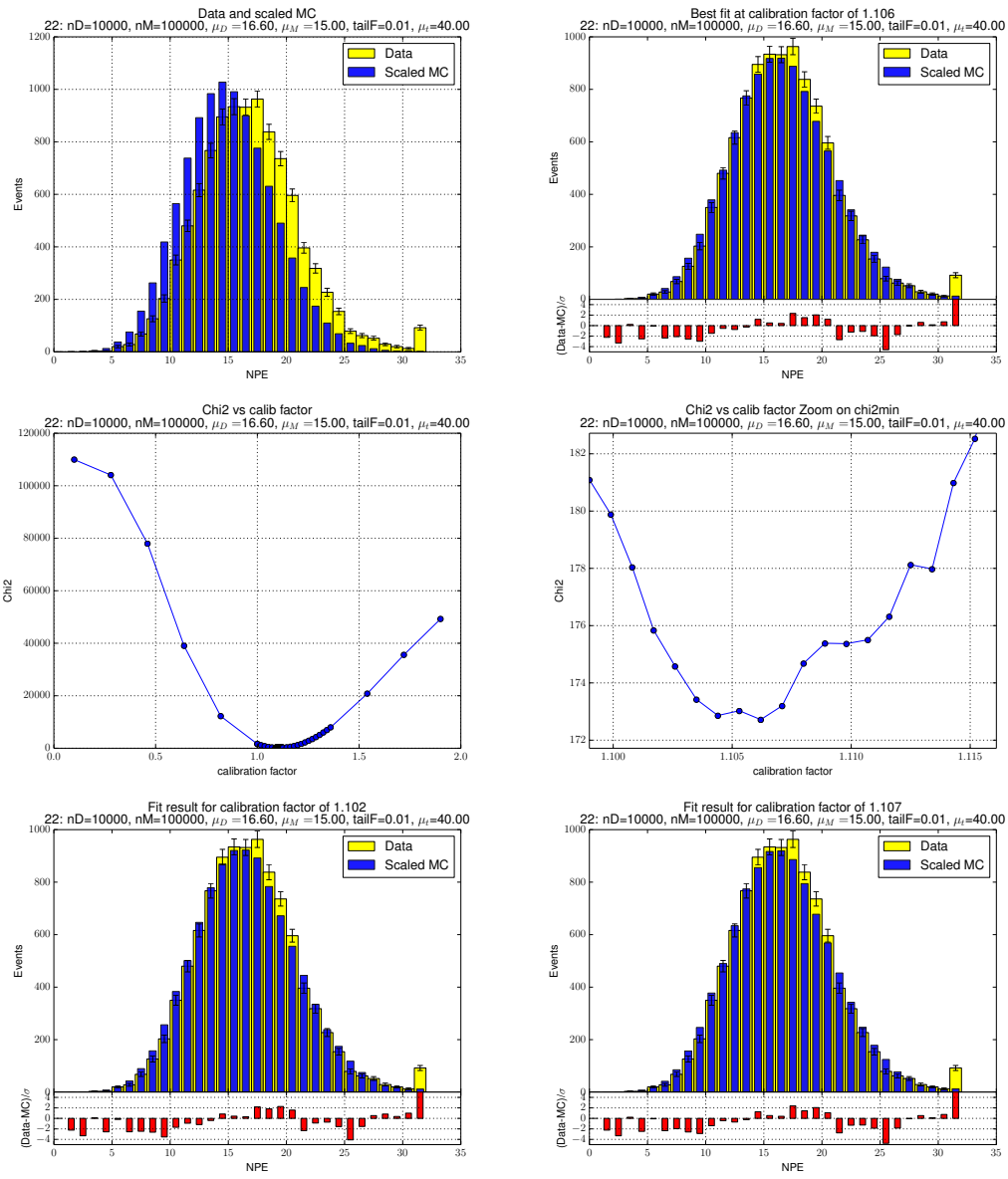

Figure 45: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 22. Data compared to MC scaled by two randomly chosen calibration factors.

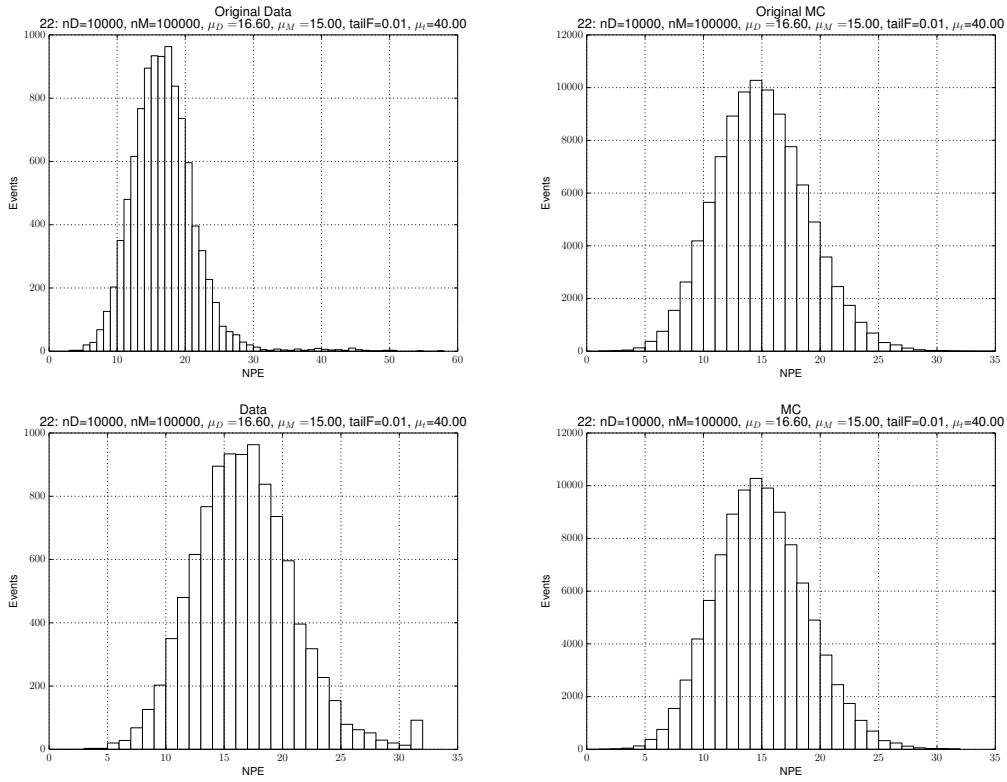

Figure 46: NPE histograms for data and MC for configuration 22. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

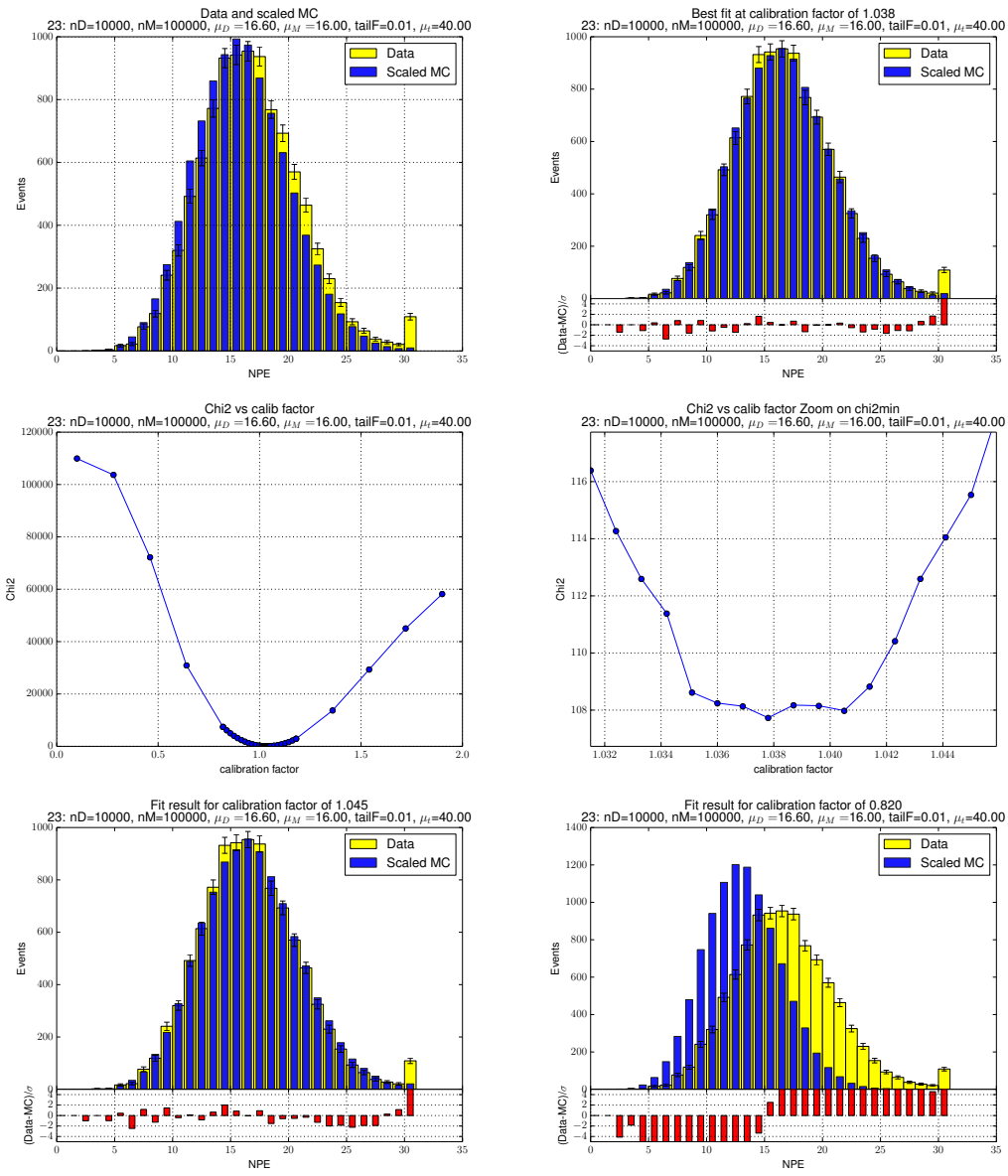

Figure 47: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 23. Data compared to MC scaled by two randomly chosen calibration factors.

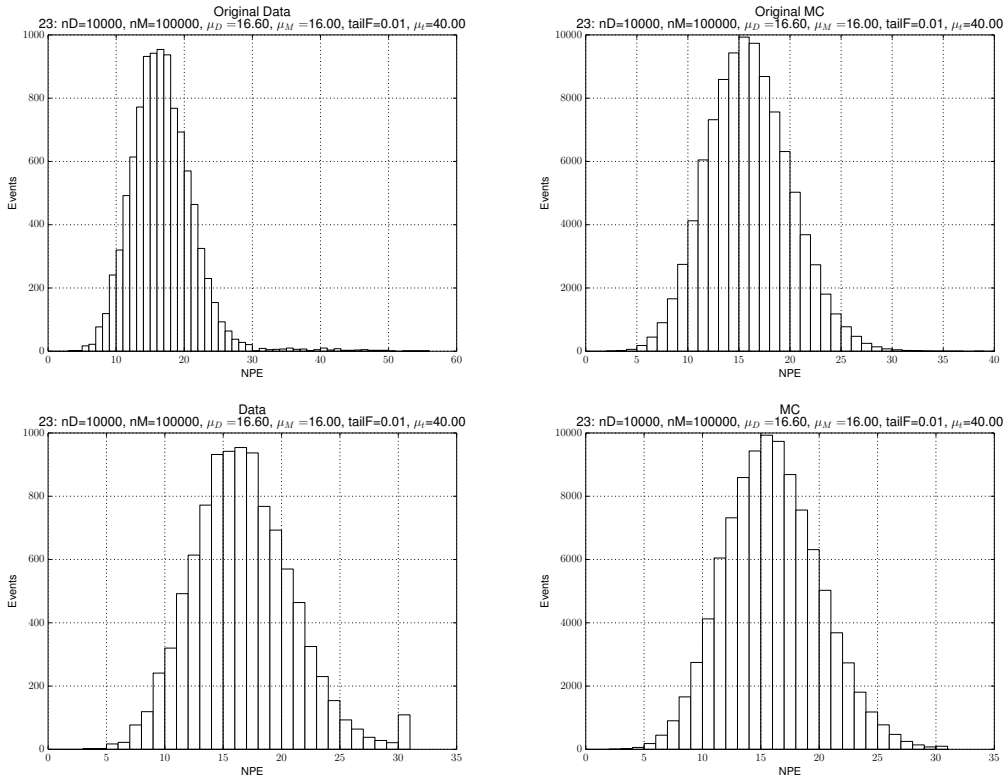

Figure 48: NPE histograms for data and MC for configuration 23. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

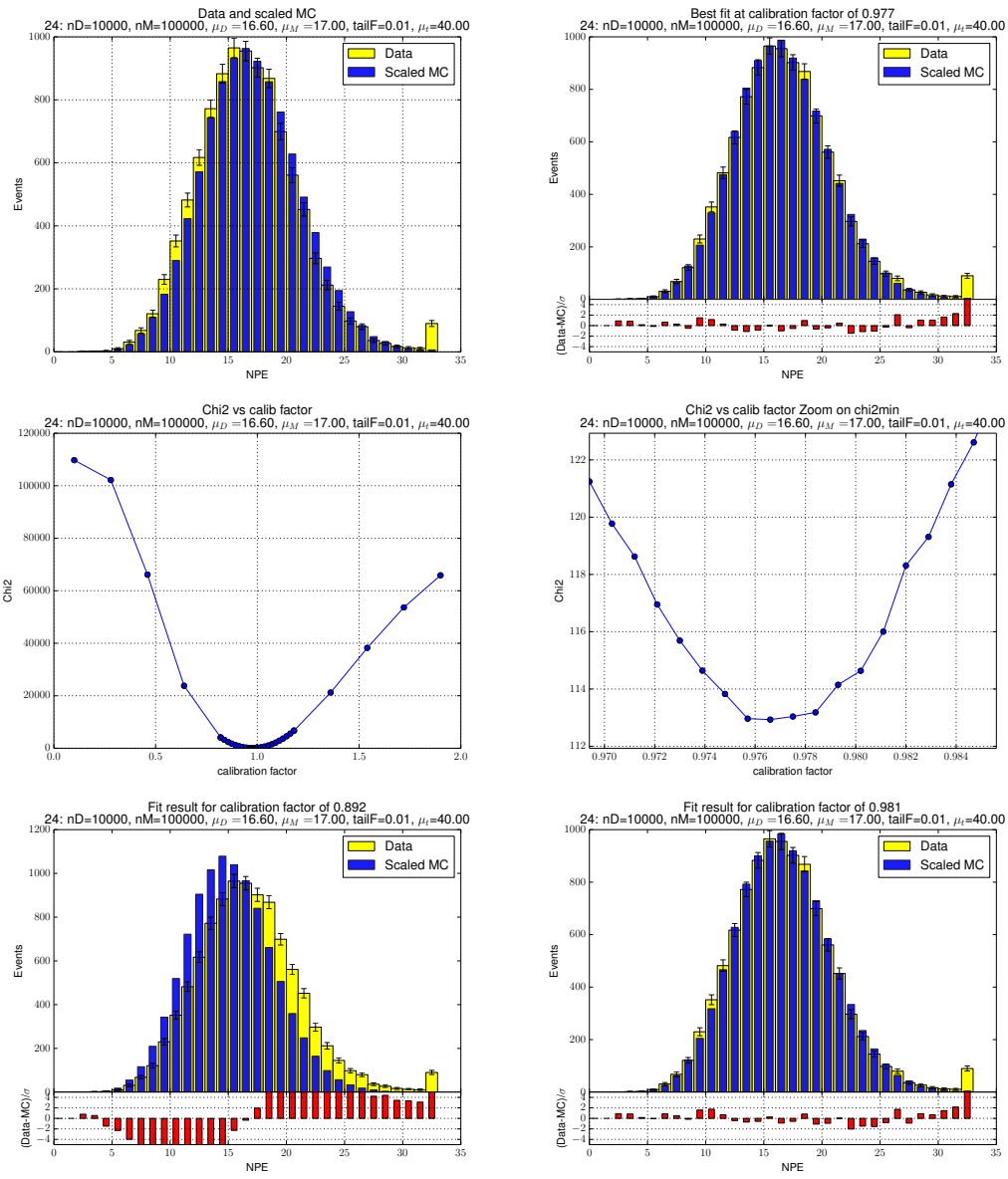

Figure 49: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 24. Data compared to MC scaled by two randomly chosen calibration factors.

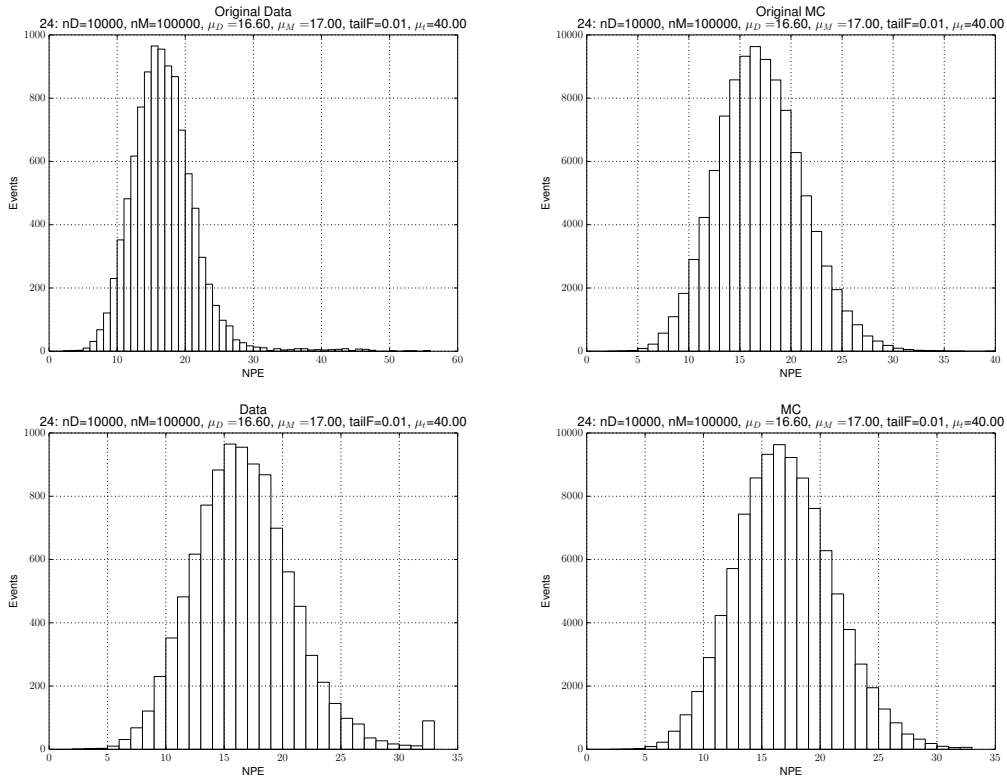

Figure 50: NPE histograms for data and MC for configuration 24. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

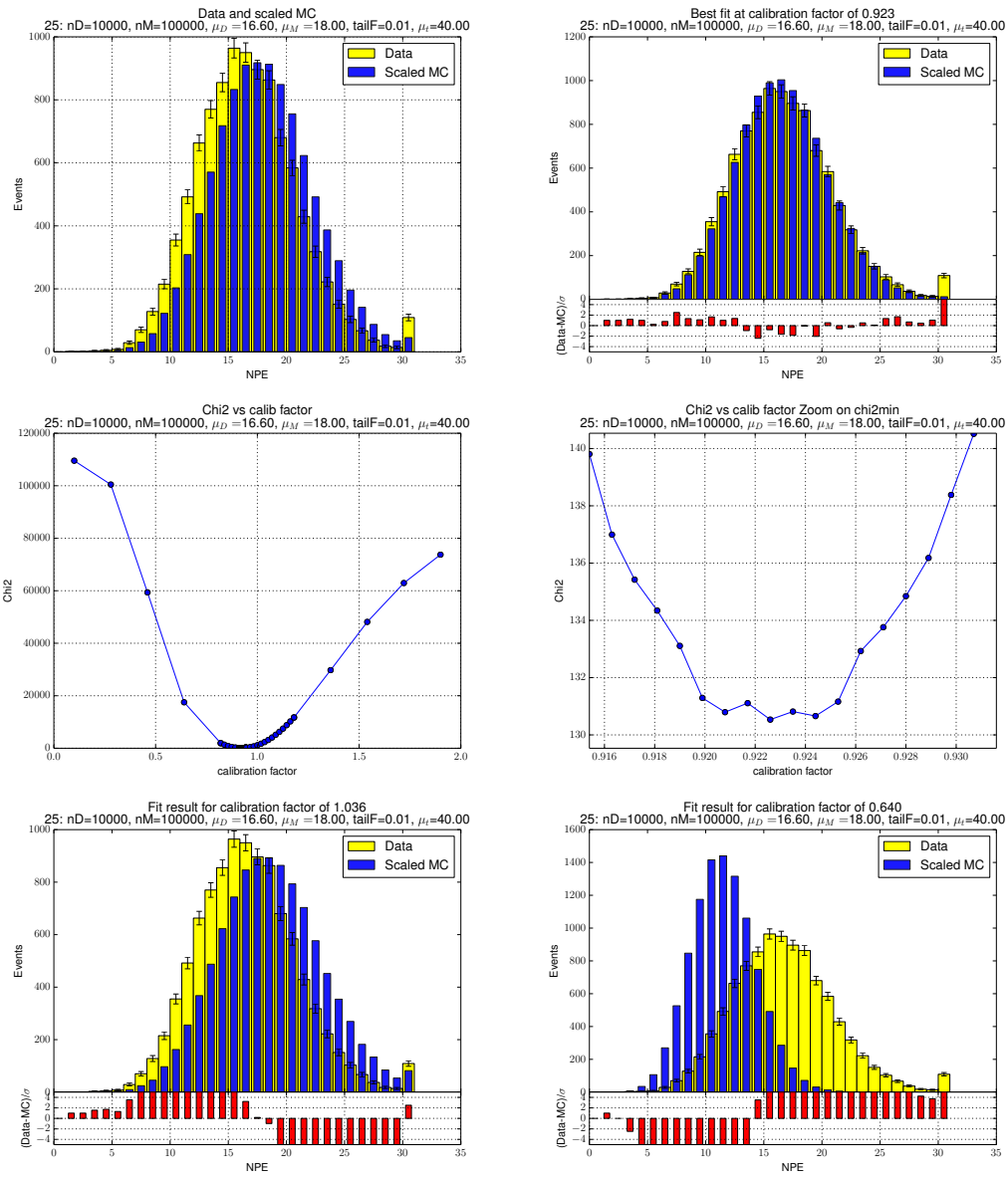

Figure 51: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 25. Data compared to MC scaled by two randomly chosen calibration factors.

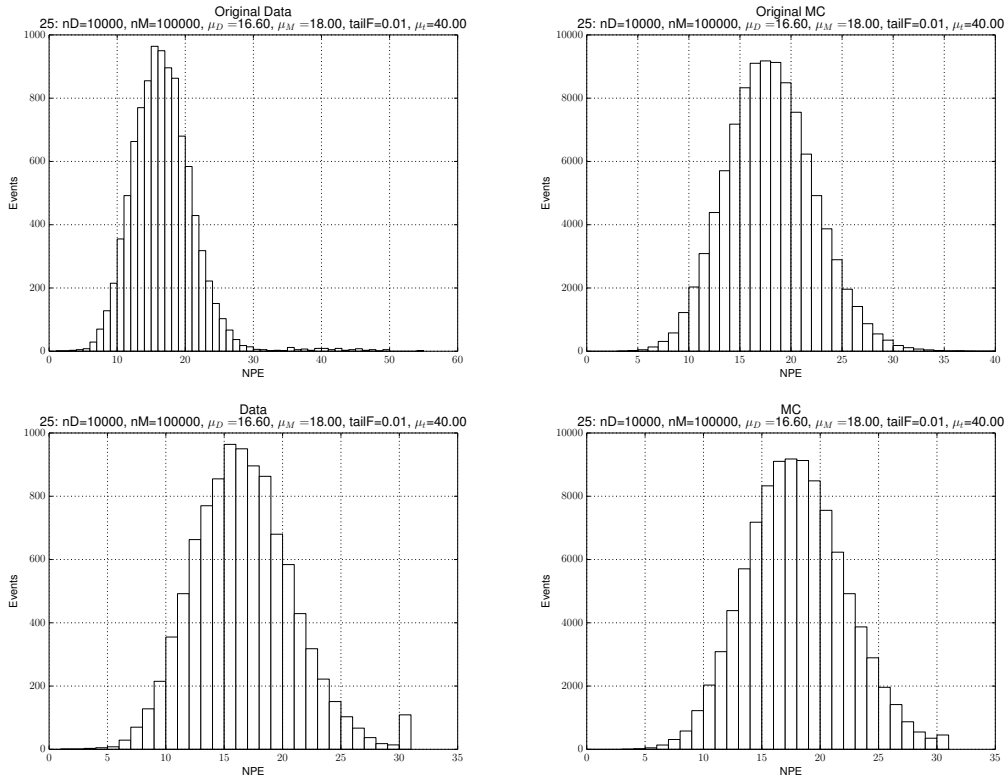

Figure 52: NPE histograms for data and MC for configuration 25. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

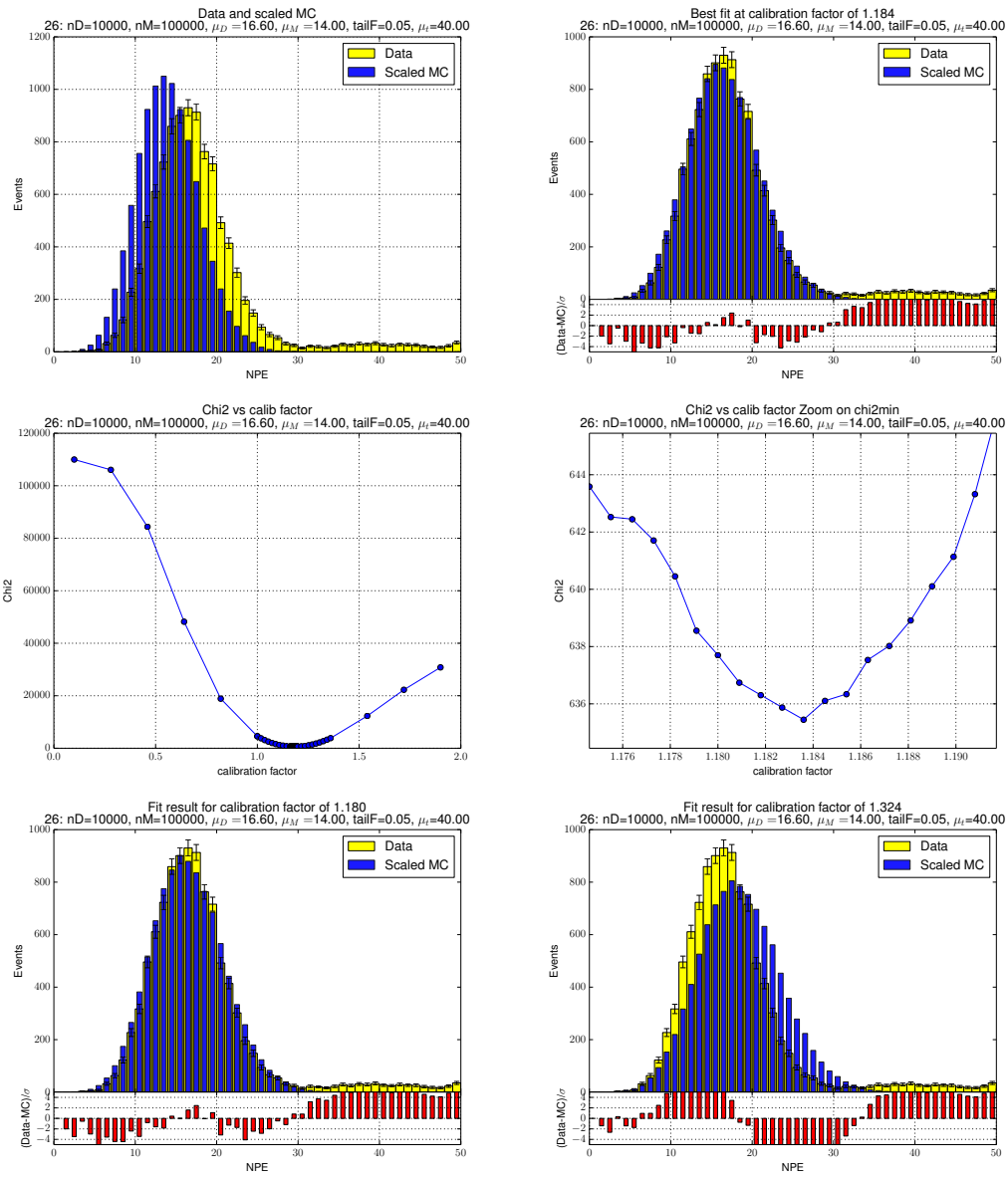

Figure 53: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 26. Data compared to MC scaled by two randomly chosen calibration factors.

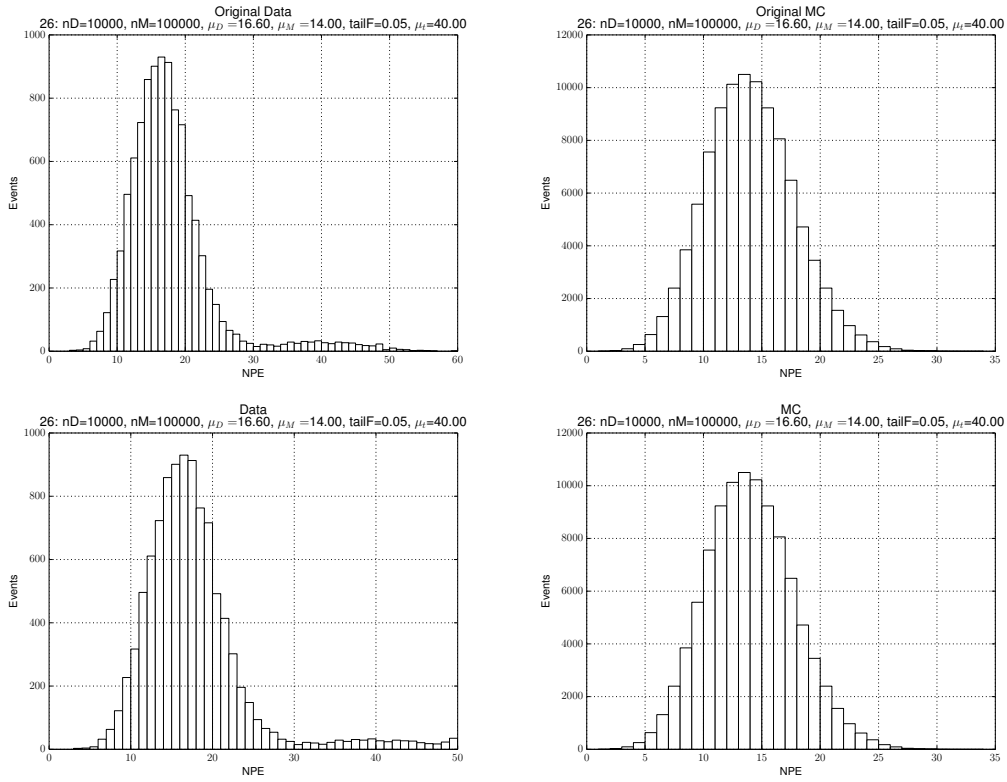

Figure 54: NPE histograms for data and MC for configuration 26. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

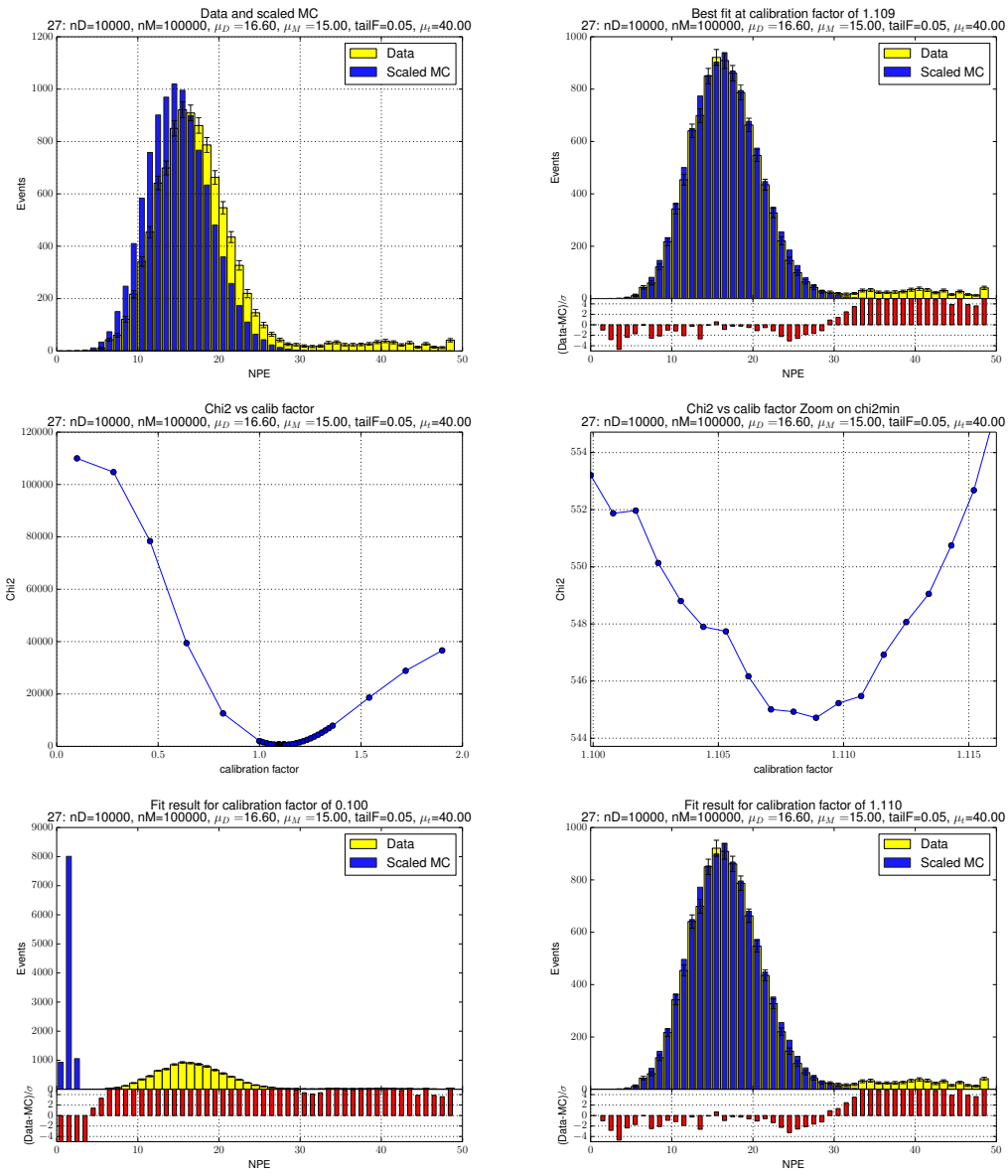

Figure 55: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 27. Data compared to MC scaled by two randomly chosen calibration factors.

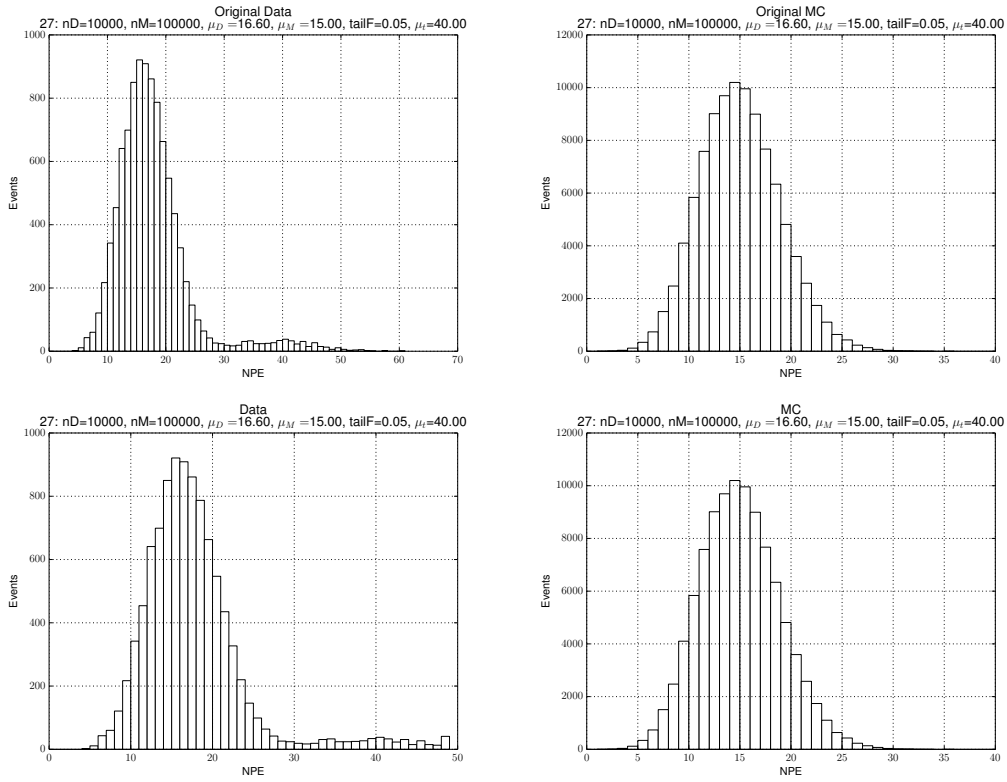

Figure 56: NPE histograms for data and MC for configuration 27. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

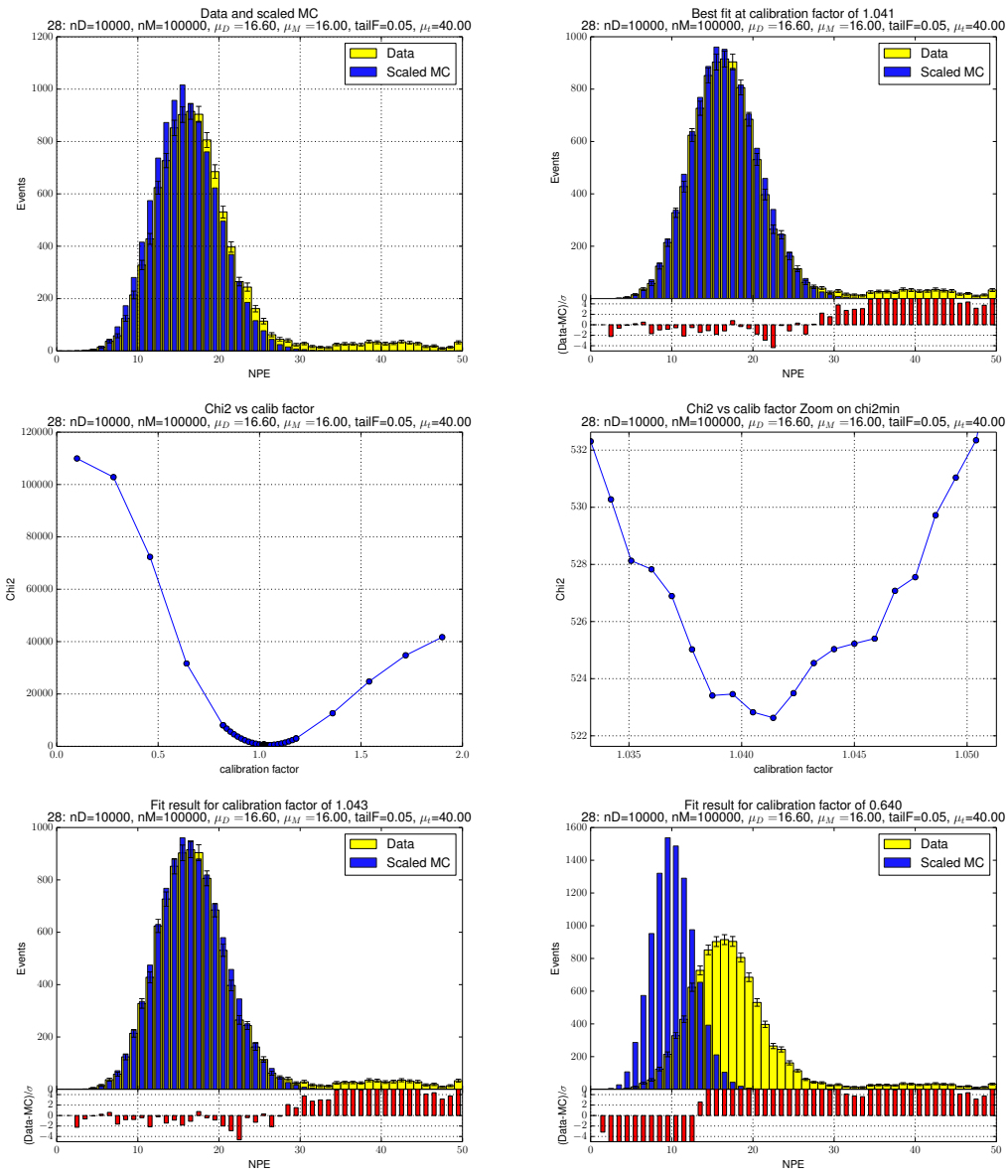

Figure 57: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 28. Data compared to MC scaled by two randomly chosen calibration factors.

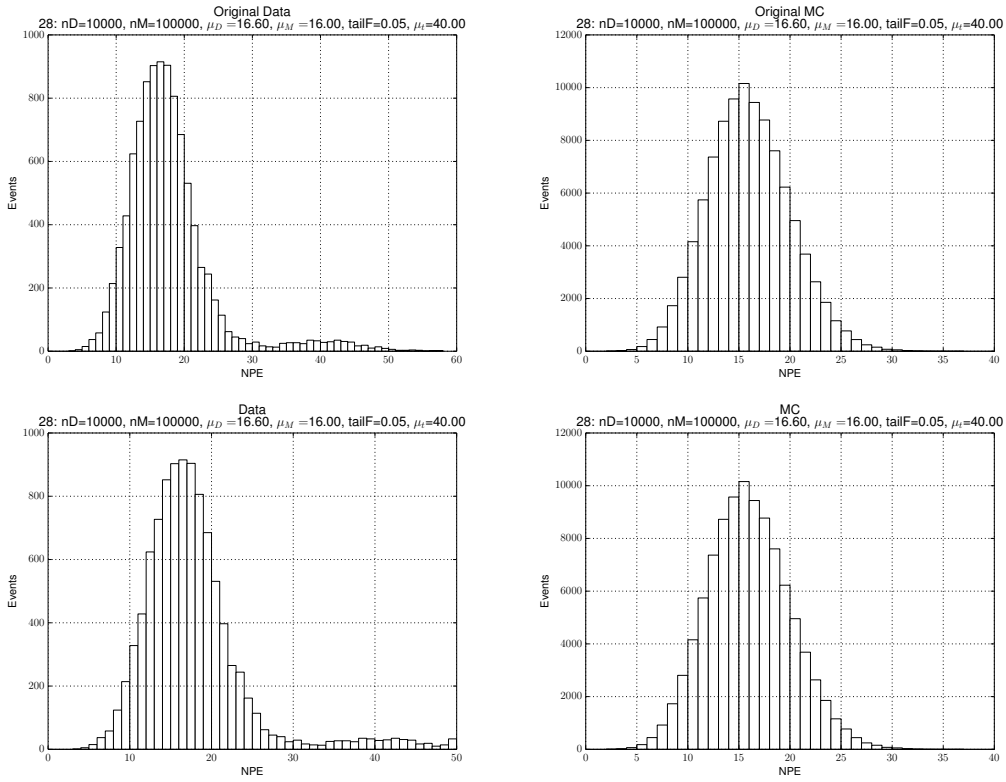

Figure 58: NPE histograms for data and MC for configuration 28. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

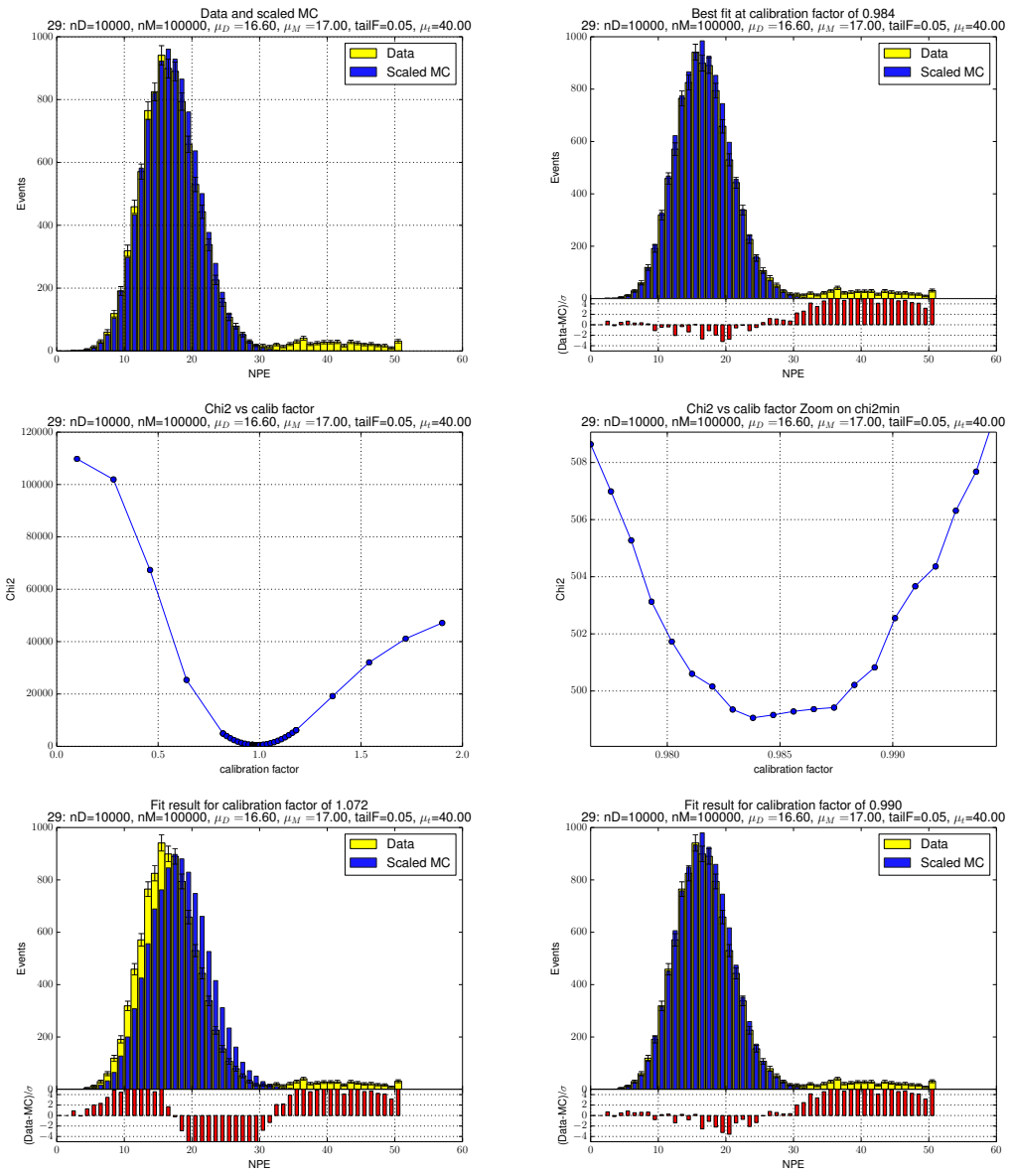

Figure 59: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 29. Data compared to MC scaled by two randomly chosen calibration factors.

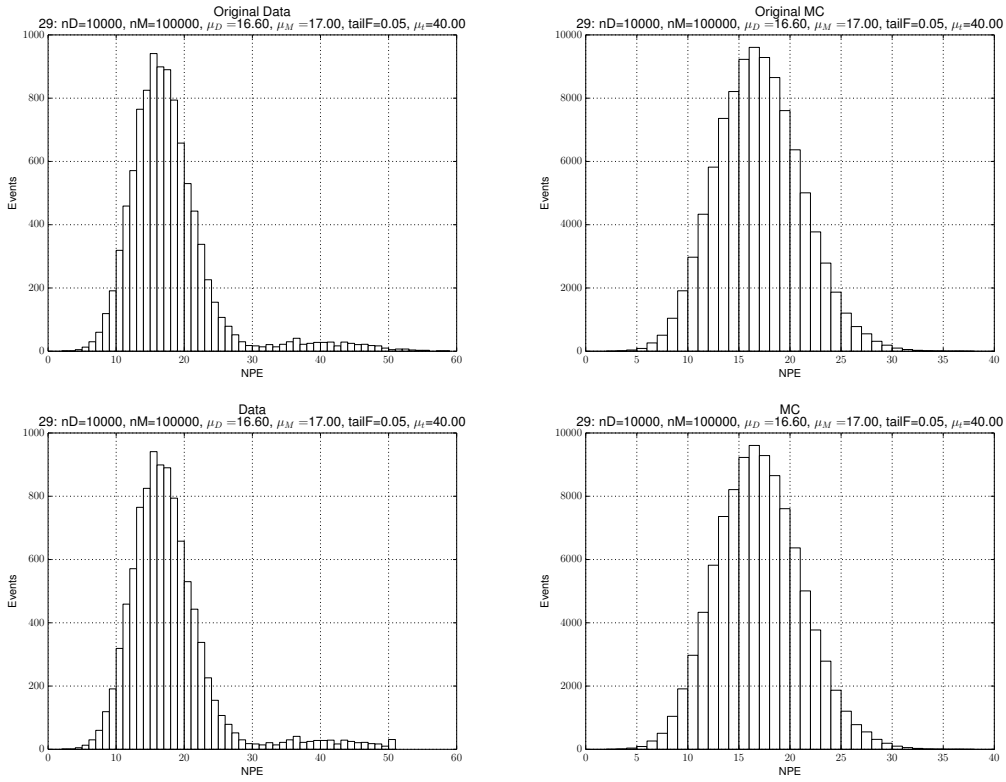

Figure 60: NPE histograms for data and MC for configuration 29. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

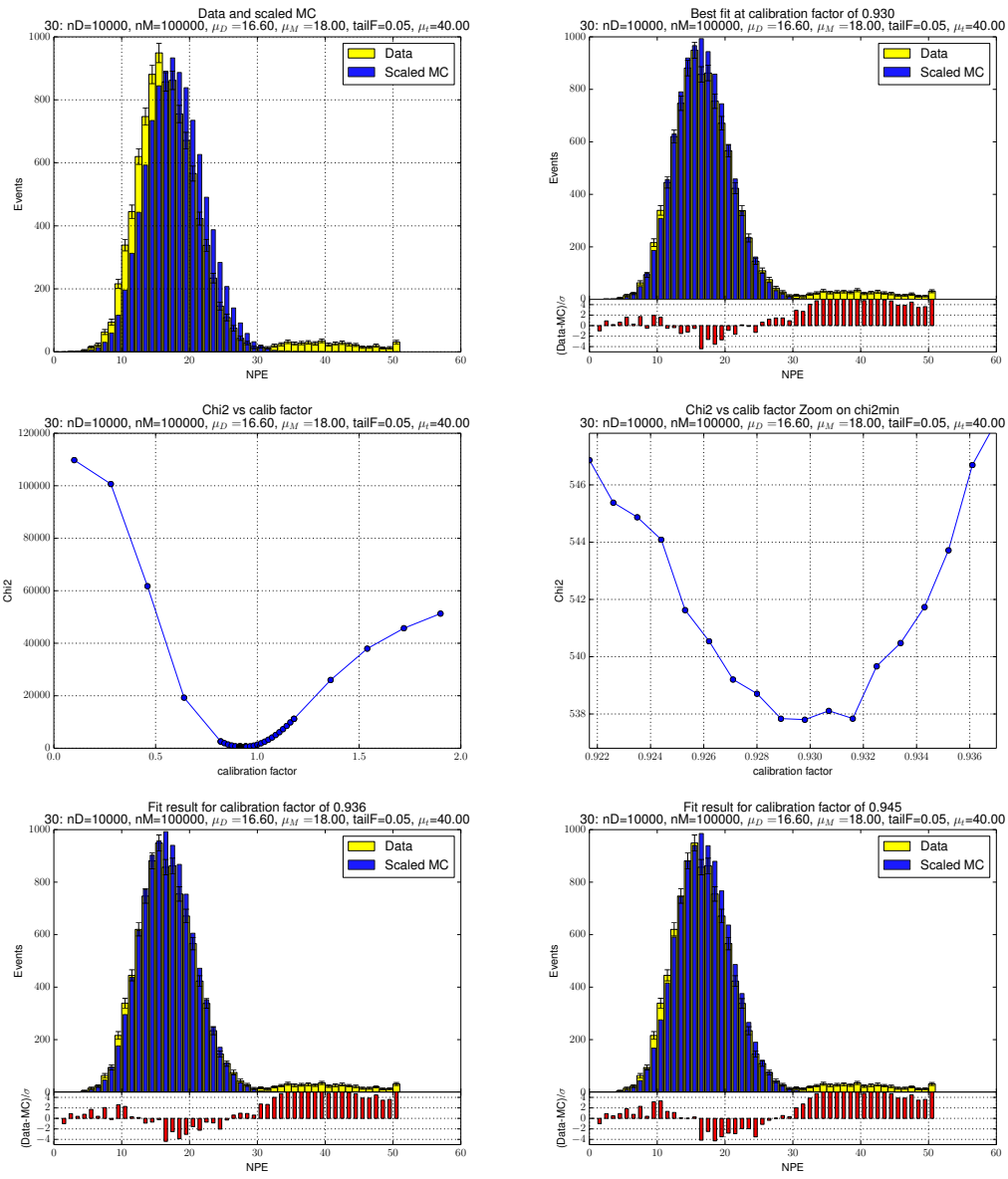

Figure 61: Data compared to nominal MC, MC scaled by the best fit calibration factor, scans of χ^2 over a large range and about the minimum for configuration 30. Data compared to MC scaled by two randomly chosen calibration factors.

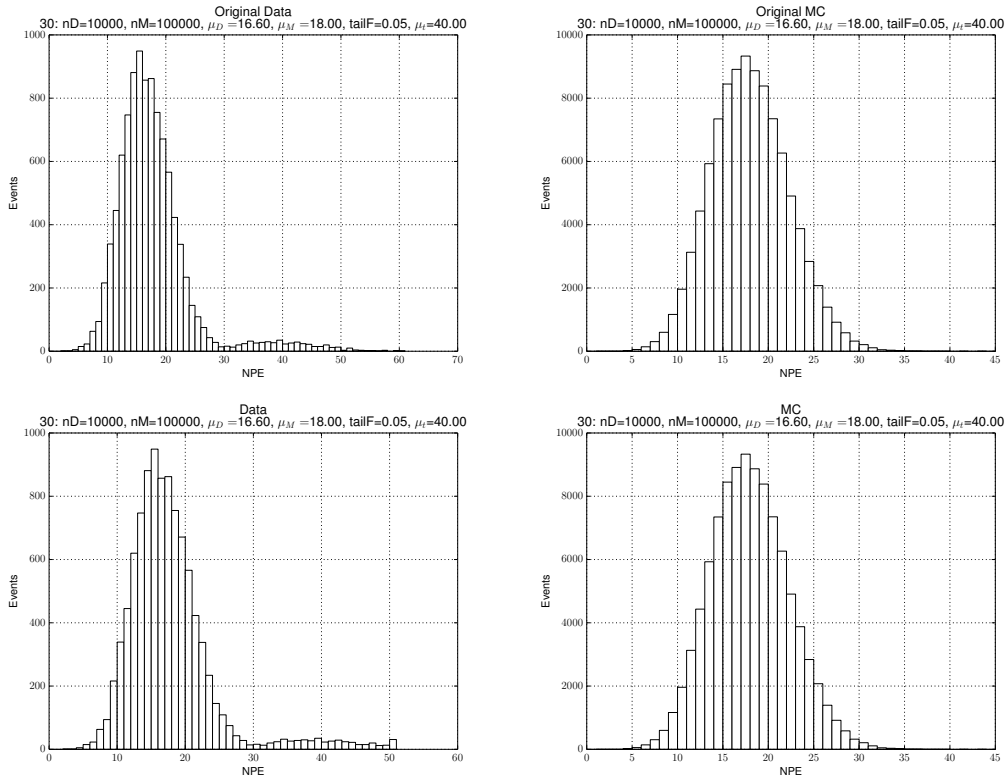

Figure 62: NPE histograms for data and MC for configuration 30. Top are original hists. Bottom are hists after truncation at bin containing less than 10 entries with that bin containing overflows.

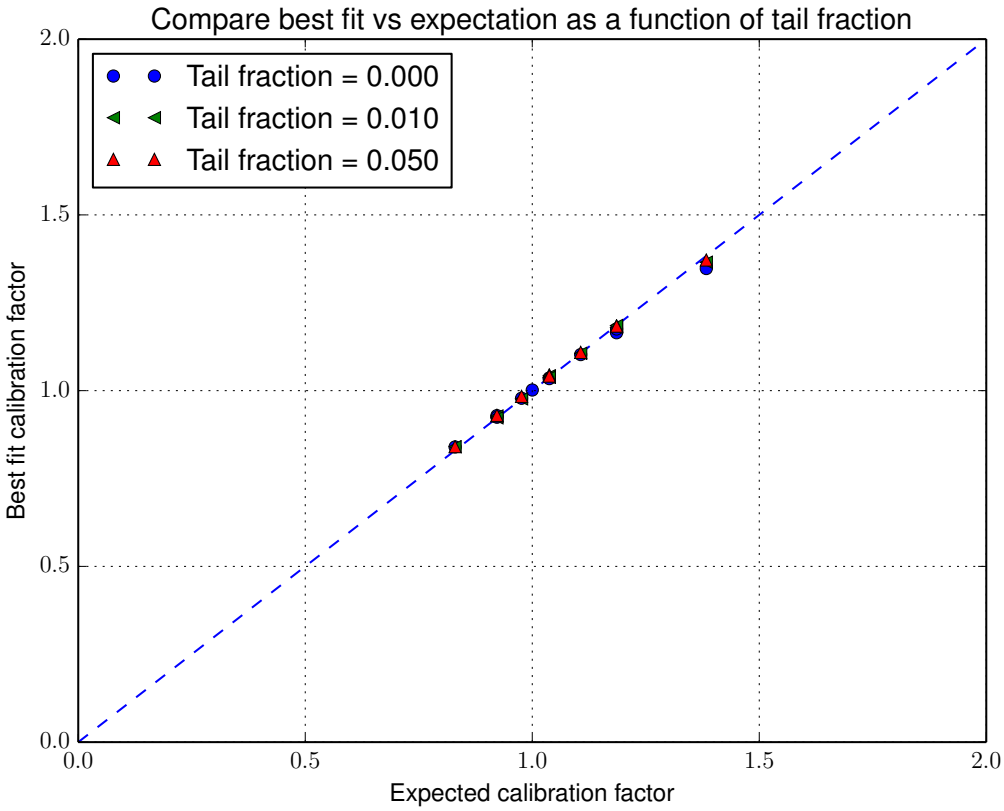

Figure 63: Comparison of best fit with expectation as a function of tail fractions