LAB # 2 p.1

Gravity Compensation:

1. Determine total potential energy U of the robot

$$U_i = \sum_{i=1}^{n} U_i$$

$$U_i = -m_i g_v^T P_{c_i}^0 \qquad g_v = \begin{bmatrix} 0 \\ 0 \\ -9.81 \end{bmatrix}$$

2. Determin G_i

$$G_i = \frac{\partial U}{\partial q_i} \quad ; \quad \underline{\underline{Q_i}} = G_i$$

torque applied to joint i

3. Write MATLAB function $G = grav(q)$ which computes G_i, and reterns the results as a vector

$$G = [G1, G2, G3]^T$$

The input to the function is

$$q = [q_1, q_2, q_3]^T$$

$$\text{joint angles} \longrightarrow \boxed{G = grav(q)} \longrightarrow \text{joint torque's}$$

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \qquad \begin{bmatrix} G1 \\ G2 \\ G3 \end{bmatrix}$$

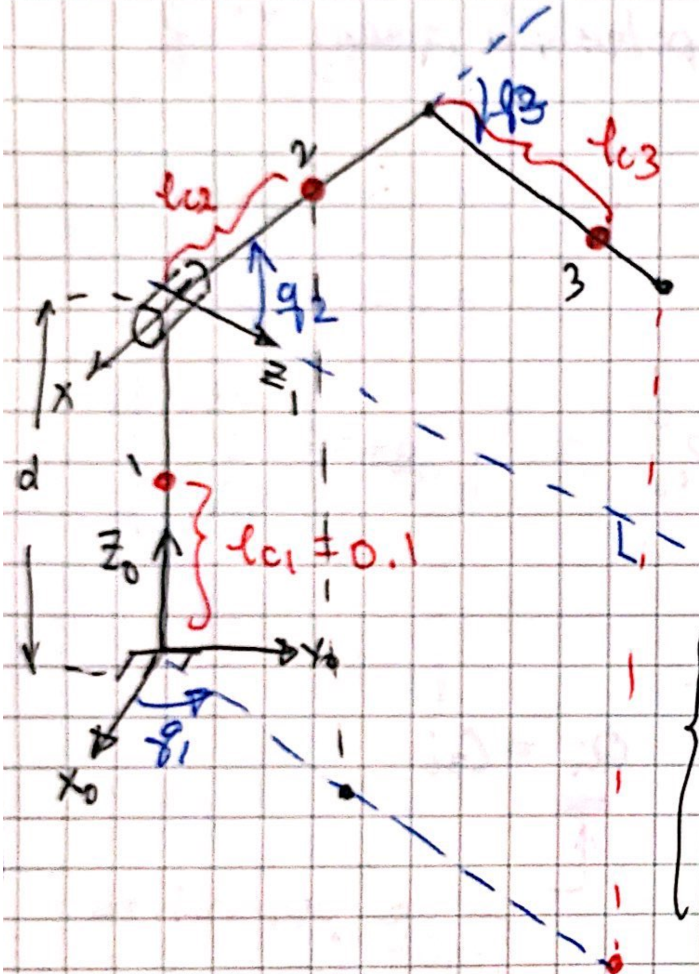
$$U_i = -m_{ci}\, g_v^T\, P^0_{cei}$$

$\rightarrow l_{c2} = 0.5(l_2)$
$= 0.5(0.1335)$
$= 0.066675\,m$

$\rightarrow l_{c3} = 0.9(l_3)$
$= 0.9(0.1335)$
$= 0.120015\,m$

$m_2 = 0.035\ Kg$

$m_3 = 0.1\ Kg$



$$\begin{cases} p_{2z} = d + l_{c2}\sin(q_2) \\ p_{2x} = l_{c2}\cdot\cos(q_2)\cos(q_1) \\ p_{2y} = l_{c2}\cdot\cos(q_2)\sin(q_1) \end{cases}$$

INSERT

For $i = 1$

$\cancel{U_1}$

$U_1 = 0$

For $i = 2$

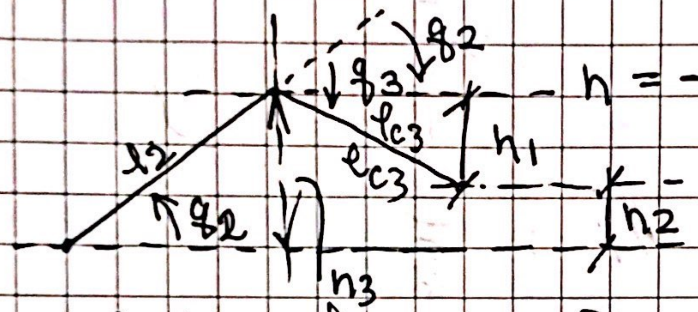
$$U_2 = \overbrace{-0.035}^{m_1} \times \overbrace{\begin{bmatrix} 0 & 0 & -9.81 \end{bmatrix}}^{g_v^T} \times \underbrace{\begin{bmatrix} P_{2x} \\ P_{2y} \\ P_{2z} \end{bmatrix}}_{P^0_{cei}}$$

$$\boxed{U_2 = 0.34335\left(d + l_{c2}\sin(q_2)\right)}$$

For $i = 3$.

$$U_3 = \underbrace{-0.1}_{-m_3} \times \underbrace{\begin{bmatrix} 0 & 0 & -9.81 \end{bmatrix}}_{g_v^T} \times \underbrace{\begin{bmatrix} p_{c3x} \\ p_{c3y} \\ p_{c3z} \end{bmatrix}}_{{}^{0}p_{c_3}}$$

* $p_{c3z} = d + l_2 \sin(q_2) - l_{c3} \sin(q_3 - q_2)$



$h = -l_{c3} \cdot \sin(q_3 - q_2)$

$h_2 = h_3 - h_1$

* projection of center of gravity of arm 3 on the x-y plane...

$$Abs_1 = l_1 \cos(q_2) + l_{c3} \sin(q_3 - q_2)$$

* $p_{c3x} = \left[l_1 \cos(q_2) + l_{c3} \sin(q_3 - q_2) \right] \cos(q_1)$
* $p_{c3y} = \left[l_1 \cos(q_2) + l_{c3} \sin(q_3 - q_2) \right] \sin(q_1)$

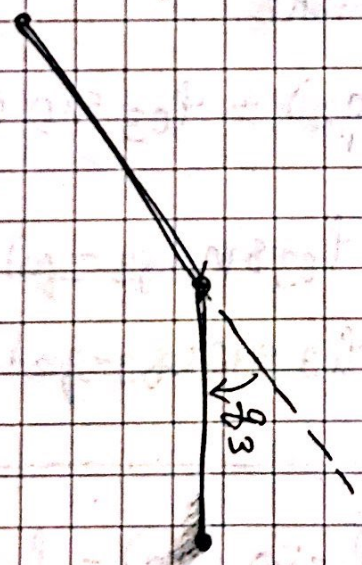
$$\boxed{U_3 = \underbrace{0.981}_{(g \times m_3)} \left(d + l_2 \sin(q_2) - l_{c3} \sin(q_3 - q_2) \right)}$$

$\boxed{\therefore U_{1,2,3} \text{ are all found!}}$:)

$$U = \sum_{i=1}^{3} U_i$$

$$= \underbrace{0.34335}_{m_2 g}\left(d + \ell_{c2}\sin(q_2)\right) + \overbrace{0.981}^{m_3 g}\left(d + \ell_2 \sin(q_2) \ldots - \ell_{3c}\sin(q_3 - q_2)\right)$$

$$G_i = \frac{\partial U}{\partial q_i}$$



$$g\left[(d + \ell_{c2}\sin(q_2))m_2 + (d + \ell_2\sin(q_2) - \ell_{3c}\sin(q_3 - q_2))m_3\right]$$

$$\text{U} = m_2 g d + m_2 g \overset{l_{c2}}{} \sin(q_2) + m_3 g d + m_3 g l_2 \sin(q_2) - m_3 g l_{3c} \sin(q_3 - q_2)$$

$m_2 l_{c2} \sin(q_2)$ $l_2 \sin(q_2) - l_{3c} \sin(q_3 - q_2)$

↳ For $i = 1$, $G_1 = \partial U / \partial q_1$

$g m_2 l_{c2} \cos(q_2) + m_3 l_2 g \cos(q_2) + m_3 g l_{3c} \cos(q_3 - q_2)$

→ $G_1 = \emptyset$

↳ For $i = 2$

$$G_2 = \partial / \partial q_2 \left(\cancel{m_2 g d}^{\,0} + \cancel{m_2 g \sin(q_2)}^{\,m_2 g \cos(q_2)} + \cancel{m_3 g d}^{\,0} + \cancel{m_3 g l_2 \sin(q_2)}^{\,m_3 g l_2 \cos(q_2)} - \cancel{m_3 g l_{3c} \sin(q_3 - q_2)}^{\,-m_3 g l_{c3} \cos(q_3 - q_2)} \right)$$

→ $G_2 = m_2 g \cos(q_2) + m_3 g l_2 \cos(q_2) + m_3 g l_{c3} \cos(q_3 - q_2)$

↳ For $i = 3$

$$G_3 = \partial / \partial q_3 \left(\cancel{m_2 g d}^{\,0} + \cancel{m_2 g \sin(q_2)}^{\,0} + \cancel{m_3 g d}^{\,0} + \cancel{m_3 g l_2 \sin(q_2)}^{\,0} - \cancel{m_3 g l_{3c} \sin(q_3 - q_2)}^{\,m_3 g \cos(q_3 - q_2)} \right)$$

→ $G_3 = -m_3 g \cos(q_3 - q_2)$

and $Q_i = G_i$

2) $\boxed{\therefore G_i \text{ and } Q_i \text{ are found,}}$