01/18/21

Laboratory #1 - Forward & Inverse Kinematics.

Prep:

- [x] 1. Determine DH-Parameters.

i	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	d	q_1
2	ℓ	0	0	q_2
3	ℓ	0	0	q_3

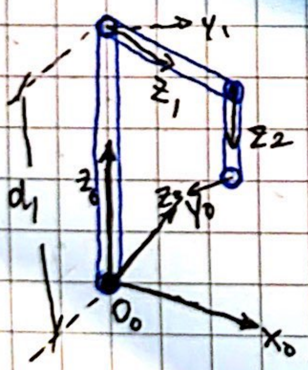
- [x] 2. Write MATLAB script to solve for H_3^0 (solve forward kinematics problem).
- [x] 3. Solve inverse kinematics for $O_3 = [p_x, p_y, p_z]^T$
- [x] 4. Write the MATLAB function "p = for_kin(q)" to solve the forward kinematics problem from step 2.

→ test: for_kin([0.5, 0.5, -0.5])
= [0.21973, 0.12004, 0.23193]

SAME

- [x] 5. " But for "inv_kin(q)"

You have this Robot...



★ Define the DH-parameters.

$a_i = |\overrightarrow{b_i O_i}|$ @ x_i

$b_i = x_i \cap z_{i-1}$

$\theta_i = \measuredangle x_{i-1} \to x_i$, @ z_i

$\alpha_i = \measuredangle z_{i-1} \to z_i$, @ x_i

$d_i = |\overrightarrow{O_{i-1} b_i}|$ @ z_i

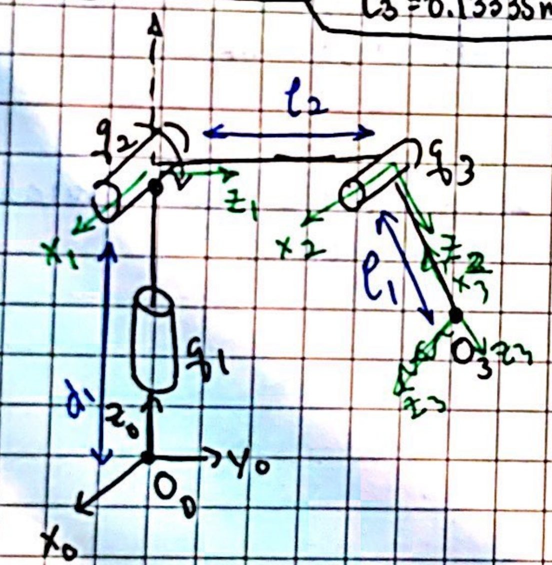
I just want you to know any something fast testing mail after the Election listen who Jimmy does.

ok... lets define a transformation matrix H_n^{n-1} wrt. the DH-Parameters

$$H_n^{n-1} = \begin{bmatrix} \cos(\theta_n) & -\sin(\theta_n)\cos(\alpha_n) & S(\theta_n)S(\alpha_n) & a_i C(\theta_n) \\ S(\theta n) & C(\theta_n)C(\alpha_n) & -C(\theta_n)S(\alpha_n) & a_i S(\theta_n) \\ 0 & S(\alpha_n) & C(\alpha_n) & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

w/ the general expression of H_n^{n-1}, lets go throug each joint, and fine

Parameter Values:
$d_1 = 0.168m$, $l_2 = 0.13335m$
$l_3 = 0.13335m$



Joint # 1

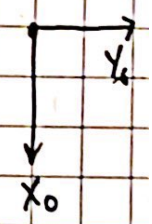
$b_1 = [0, 0, d_1]^T$

$a_1 = \cancel{d_1}\ 0$

$\theta_1 = q_1$

$\alpha_1 = q_2$

$d_1 = d_1 = 0.168$



Joint #2

$a_2 = 0$,

$\theta_2 = 0$,

$\alpha_2 = \cancel{\ }\ q_2$,

$d_2 = l_2$

Joint #3

$a_3 = 0$

$\theta_3 = 0$

$\alpha_2 = 0$

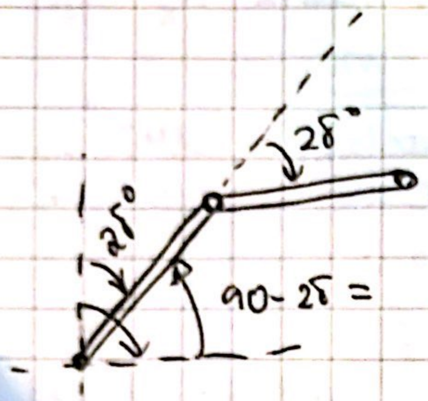
$d_3 = l_3$

all of my α_i are off:

i	a_i	α_i	d_i	θ_i
1	0	q_2	d_1	q_1
2	0	[illegible]	l_2	0
3	0	0	l_3	0

i	a_i	α_i	d_i	θ_i
1	0	0	d_1	q_1
2	0	q_2	l_2	0
3	0	q_3	l_3	0

my something) — fast testing mail after the Election



→ potential issue:

The robot arm does NOT use the upright as the base of reference!

Jun 18, 2021

Remaining issues:

→ The MATLAB function doesn't return expected values, but the values don't seem to be all that off.

$\frac{2\pi r}{4}$ $\frac{\pi}{2}$

Final DH-Parameters:

i	a_i	α_i	d_i	θ_i
1	0	q_2	d_1	q_1
2	0	q_3	l_2	0
3	0	0	l_3	0

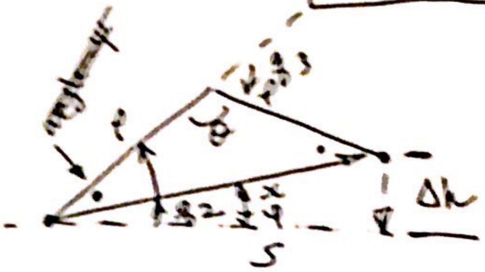
→ could the lab manual be incorrect?

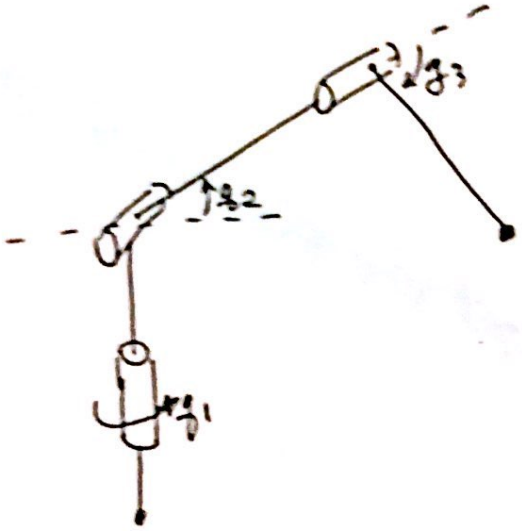
i	a_i	α_i	d_i	θ_i
1.	0	q_2	d_1	q_1
2.	0	q_3	l_2	0
3.	0	0	l_3	0

i	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	d	q_1
2	l	0	0	q_2
3	l	0	0	q_3

→ final solution (as used in the report.)

FORWARD KINEMATICS





$$q_3 = \pi - \theta$$

$$\theta = \pi - q_3$$

$$x = \sqrt{2\ell^2 - 2\ell^2 \cos(\pi - q_3)}$$

$$\cancel{\psi = \cos\left(\ell^2 + \ell^2 - x^2\right)}$$

$$\alpha_1 = \cos^{-1}\left(\frac{\ell^2 - \ell^2 - x^2}{2\ell x}\right)$$

$$= \cos^{-1}\left(\frac{x^2}{2\ell x}\right) = \cos^{-1}\left(\frac{x}{2\ell}\right)$$

$$\psi = q_2 - \overbrace{\cos^{-1}\left(\frac{x}{2\ell}\right)}^{\alpha_1}$$

$$\Delta h = x \sin\left(\overbrace{q_2 - \cos^{-1}\left(\frac{x}{2\ell}\right)}^{\psi}\right)$$

3 * $\underline{P_z = d + \Delta h}$

$$s = x \cos\left(\overbrace{q_2 - \cos^{-1}\left(\frac{x}{2\ell}\right)}^{\psi}\right)$$

1 * $\underline{P_x = s\cos(q_1)}$

2 * $\underline{P_y = s \cdot \sin(q_1)}$

* The robot is strange, why does the second arm (q_2) set it's zero point @ 90° from z_0. whould it not be easier to resolve q_2 wrt/ to z_0?

$\boxed{\therefore P_x, P_y \;\&\; P_z \text{ found wrt/ to } q_1, q_2 \;\&\; q_3}$

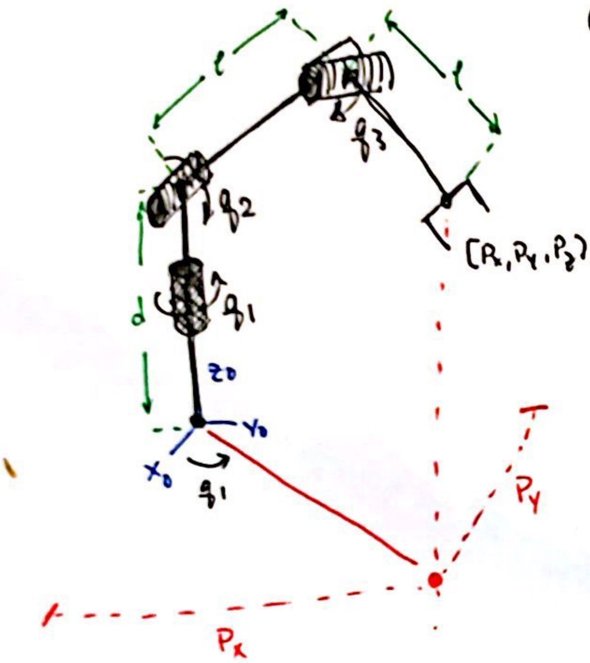
INVERSE KINEMATICS EQUATION

$l = 0.13335m$

$d = 0.168$ m

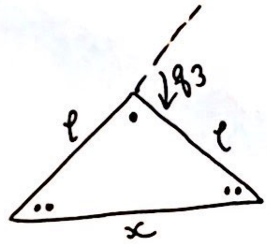
Unkowns, q_1, q_2, q_3

Given, P_x, P_y ; P_z



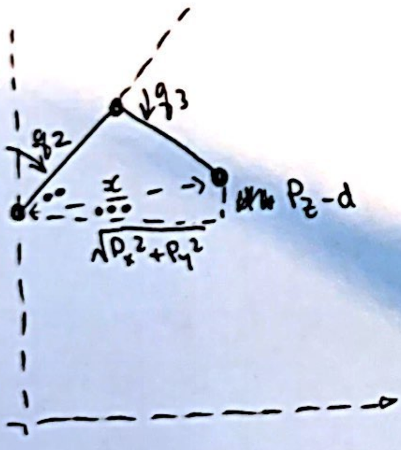
$$q_1 = \tan^{-1}\left(\frac{P_y}{P_x}\right)$$

$$\pi - q_3 = -\cos^{-1}\left(\frac{x^2 - 2l^2}{2l^2}\right)$$



$$q_3 = \pi + \cos^{-1}\left(\frac{x^2 - 2l^2}{2l^2}\right)$$

$$q_3 \overset{?}{=} \pi - \cos^{-1}\left(\frac{2l^2 - x^2}{2l^2}\right)$$



$$x^2 = P_x^2 + P_y^2 + (P_z - d)^2$$

$$x = \sqrt{P_x^2 + P_y^2 + (P_z - d)^2}$$

$$\therefore q_3 = \pi - \cos^{-1}\left(\frac{2l^2 - P_x^2 - P_y^2 - (P_z - d)^2}{2l^2}\right)$$

$$q_2 = \frac{\pi}{2} - \underset{\alpha_1}{\cdot\cdot} - \underset{\alpha_2}{\cdot\cdot\cdot}$$

$$\alpha_1 = -\cos^{-1}\left(\frac{l^2 - l^2 - x^2}{2lx}\right) = \cos^{-1}\left(\frac{x}{2l}\right)$$

$$\alpha_2 = \tan^{-1}\left(\frac{P_z - d}{\sqrt{P_x^2 - P_y^2}}\right)$$