



Calculus in *Machine Learning*

For Mr. Conlon's BC Calculus class

David Evans

Groton-Dunstable Regional High School

mr. conlon

me

ms. ordemann



My timeline so far...



Graduated in
2013



Bachelors in
Electrical
Engineering

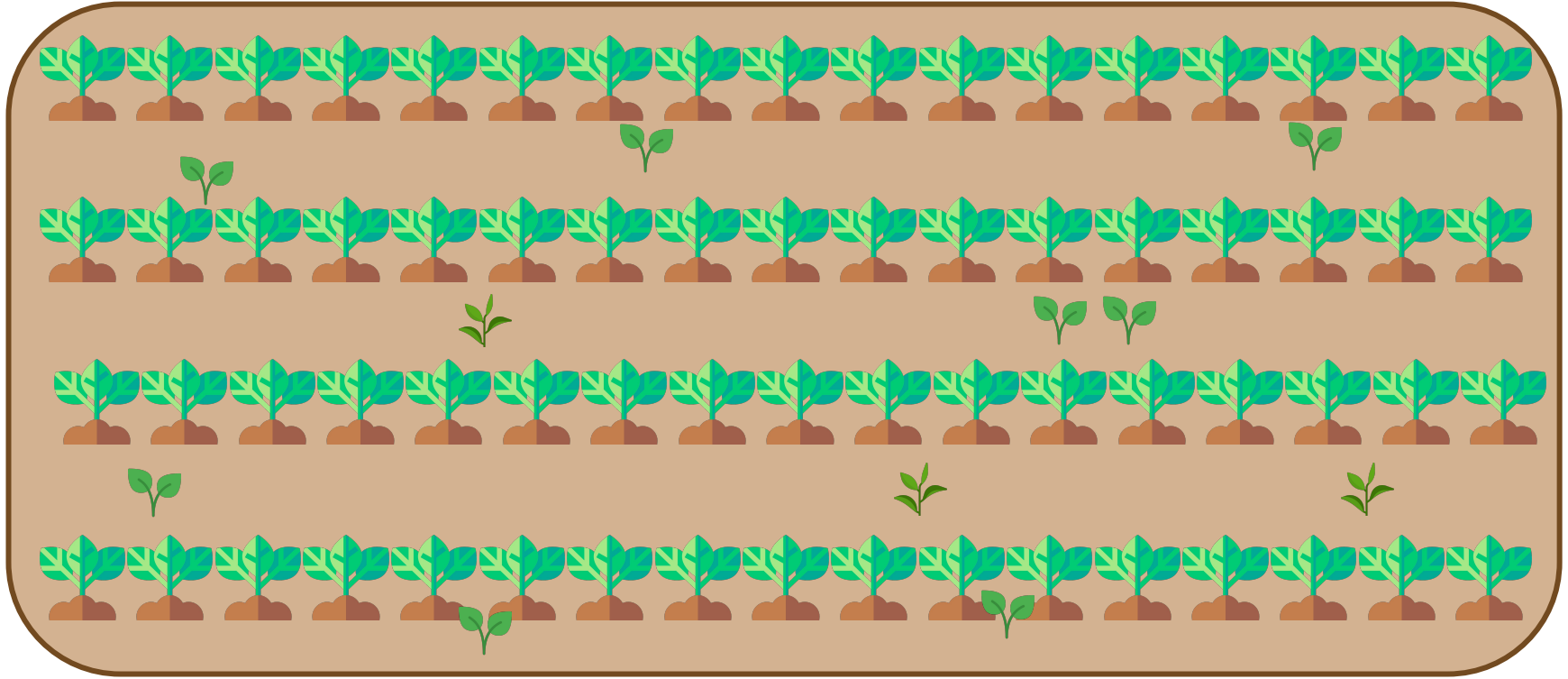


Masters in
Robotics

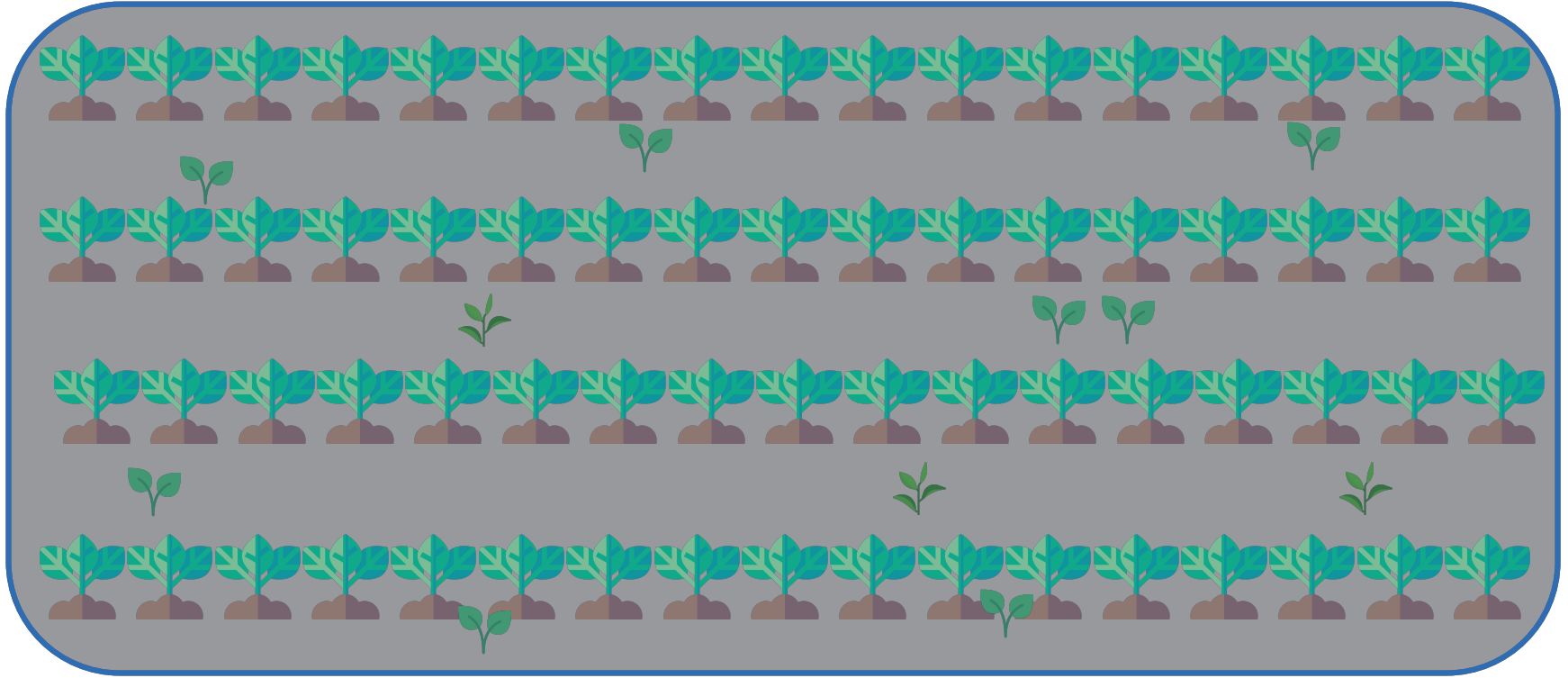


In San Francisco,
California

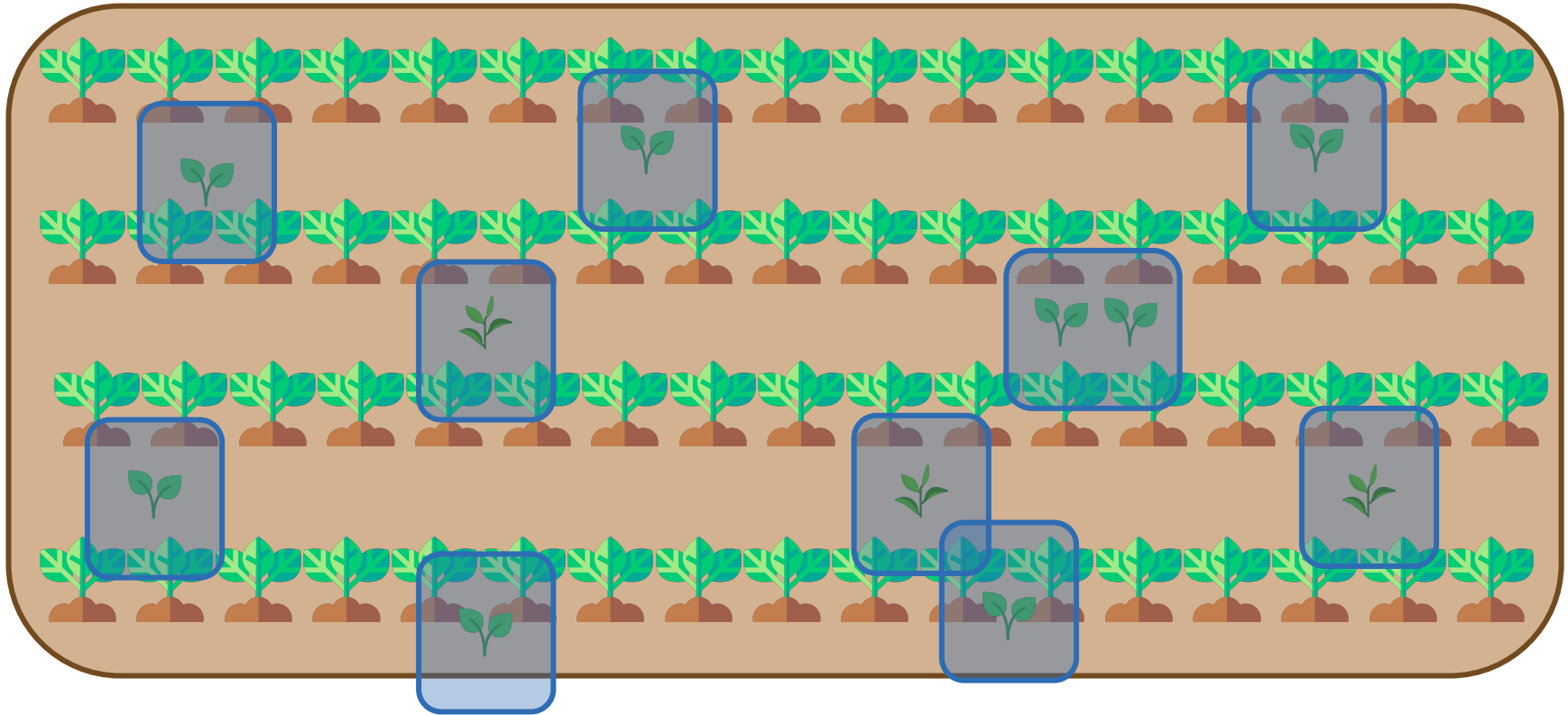
Farmers have to kill weeds in their fields



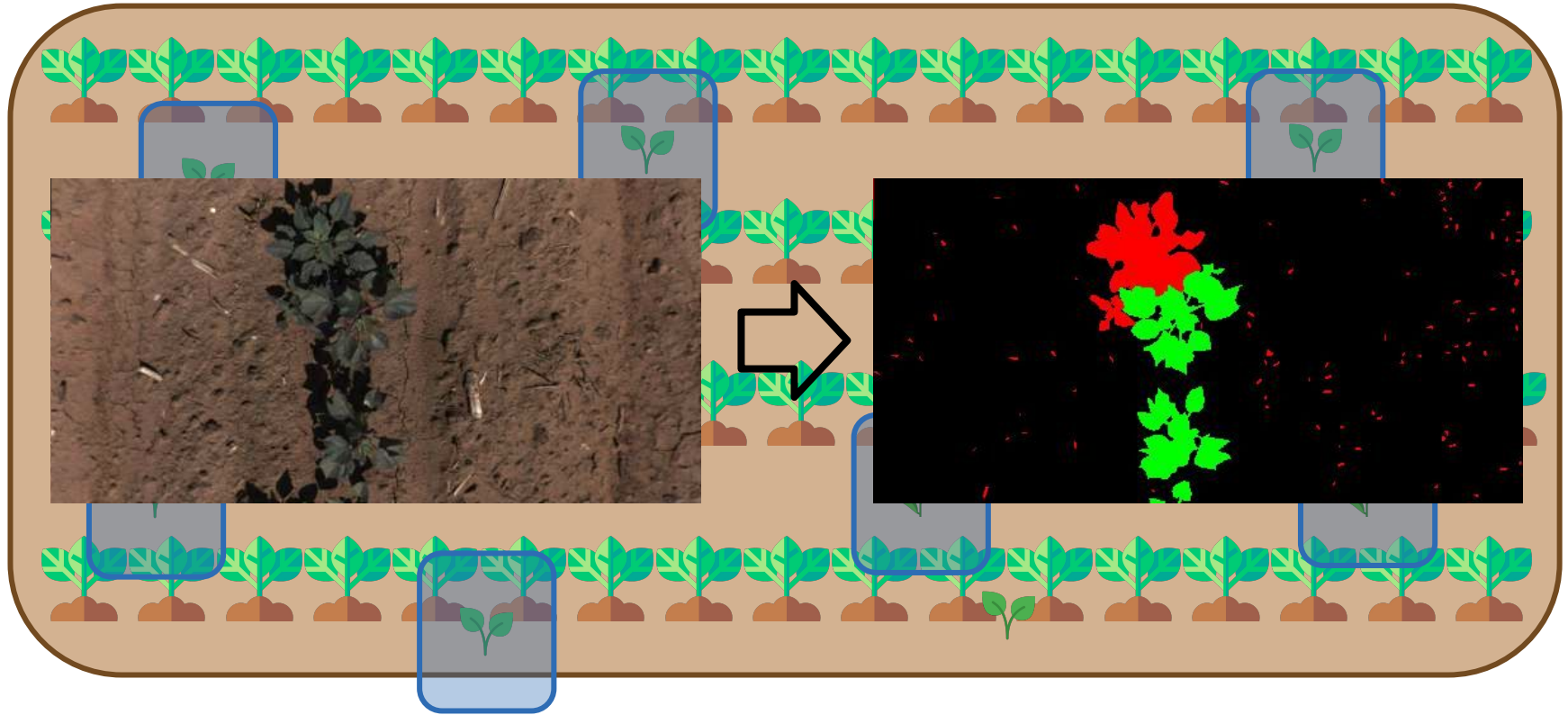
Today they broadcast their field...



What if we just sprayed the weeds?

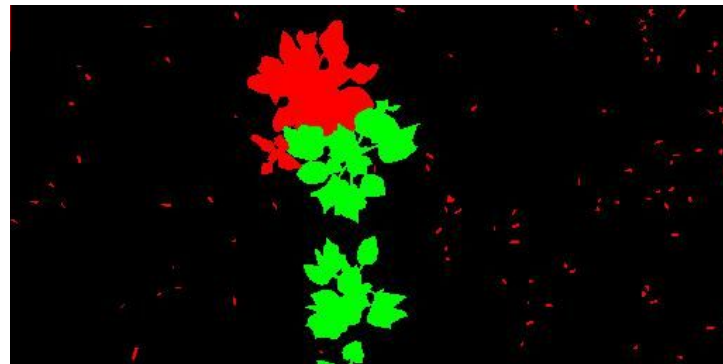


What if we just sprayed the weeds?





How do we do this???



How do we do this???



What is machine learning?

Machine learning: Computers using data(pattern recognition) to map an input to an output.



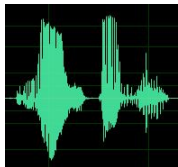
Hot Dog
or
Not Hot Dog

What is machine learning?

Machine learning: Computers using data(pattern recognition) to map an input to an output.



Hot Dog
or
Not Hot Dog



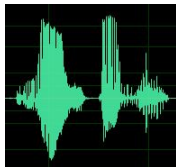
“Alexa, play Mr.
Brightside”

What is machine learning?

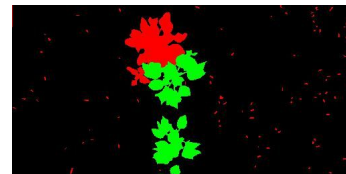
Machine learning: Computers using data(pattern recognition) to map an input to an output.



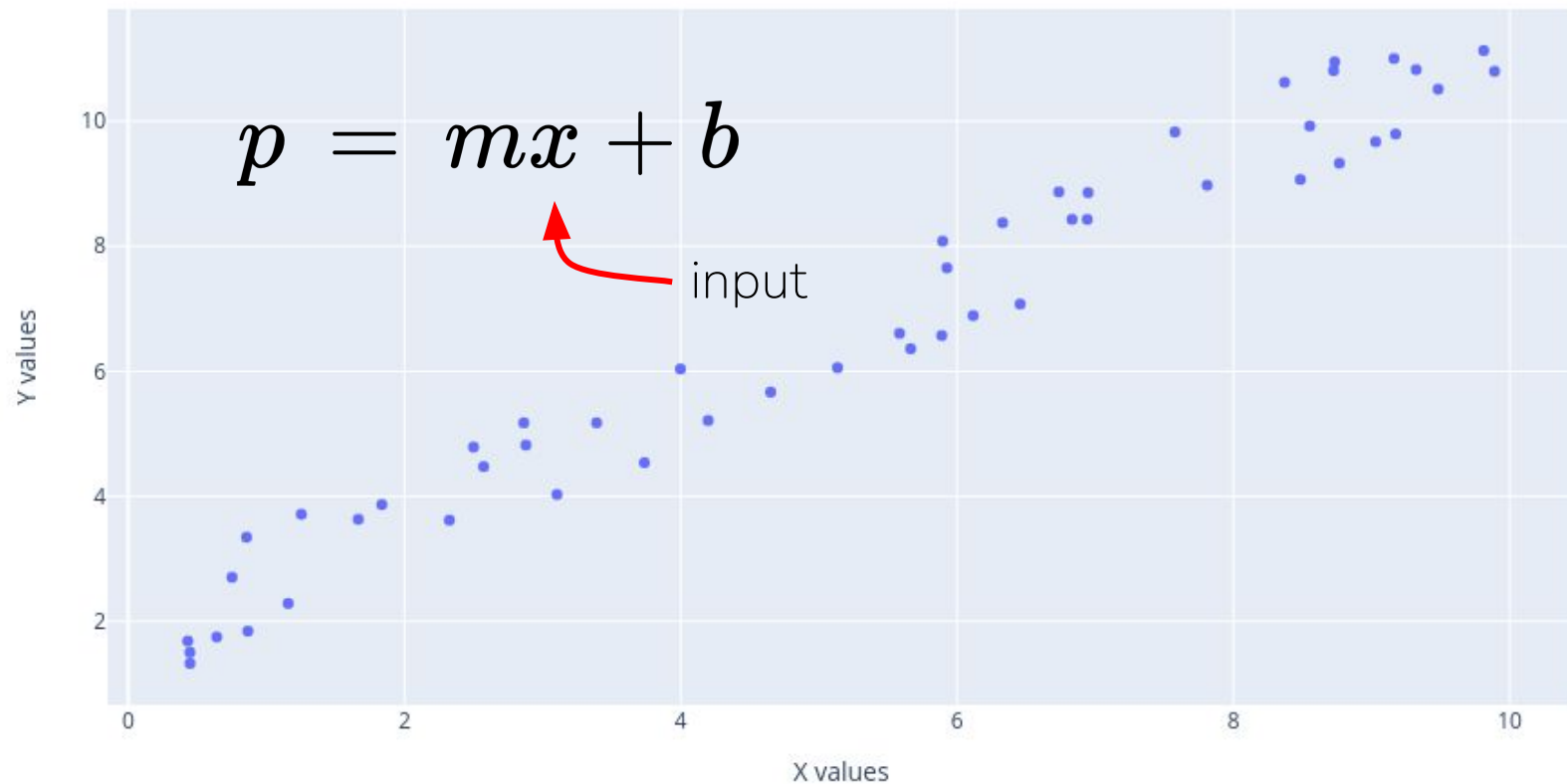
Hot Dog
or
Not Hot Dog



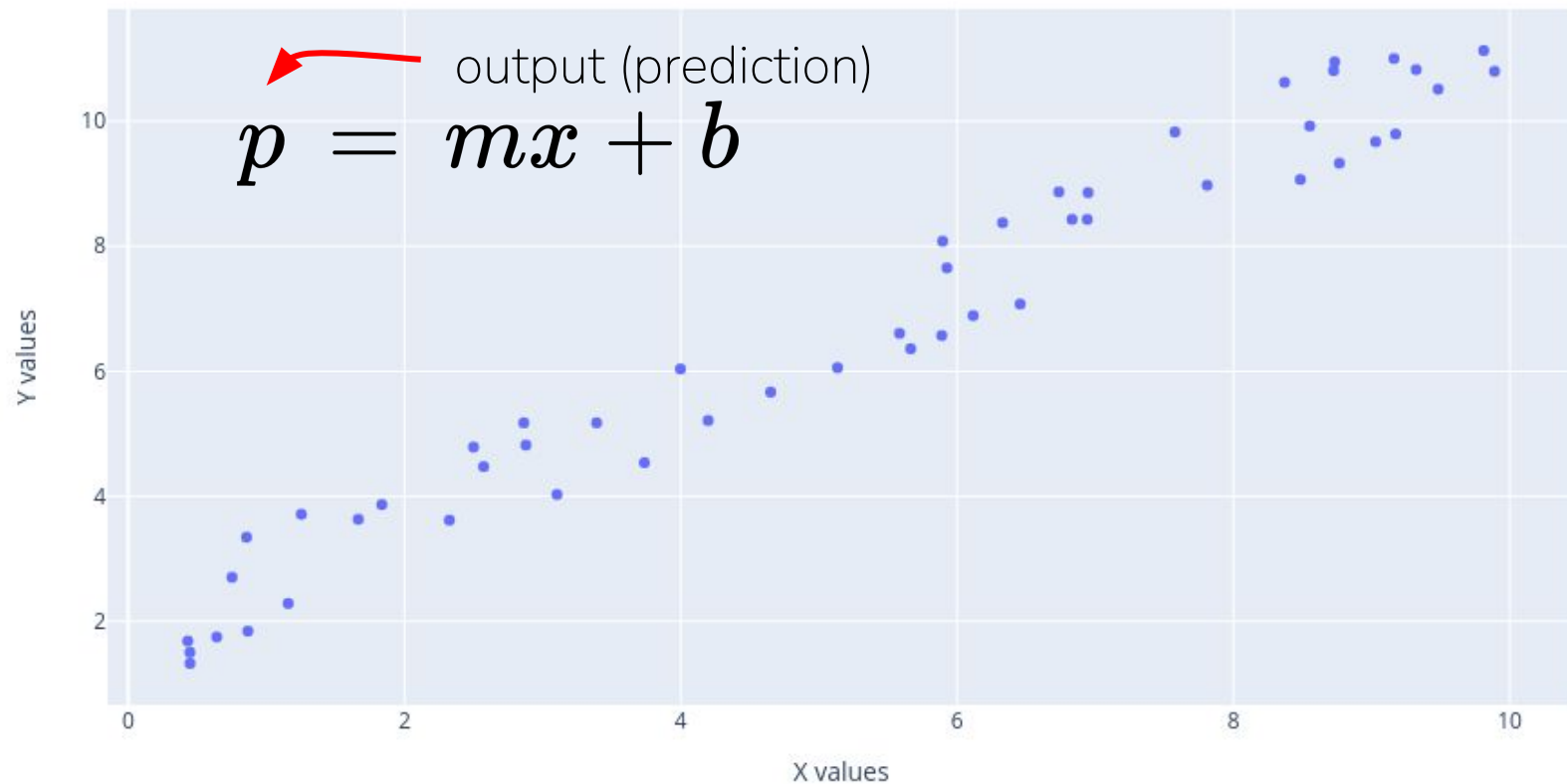
“Alexa, play Mr.
Brightside”



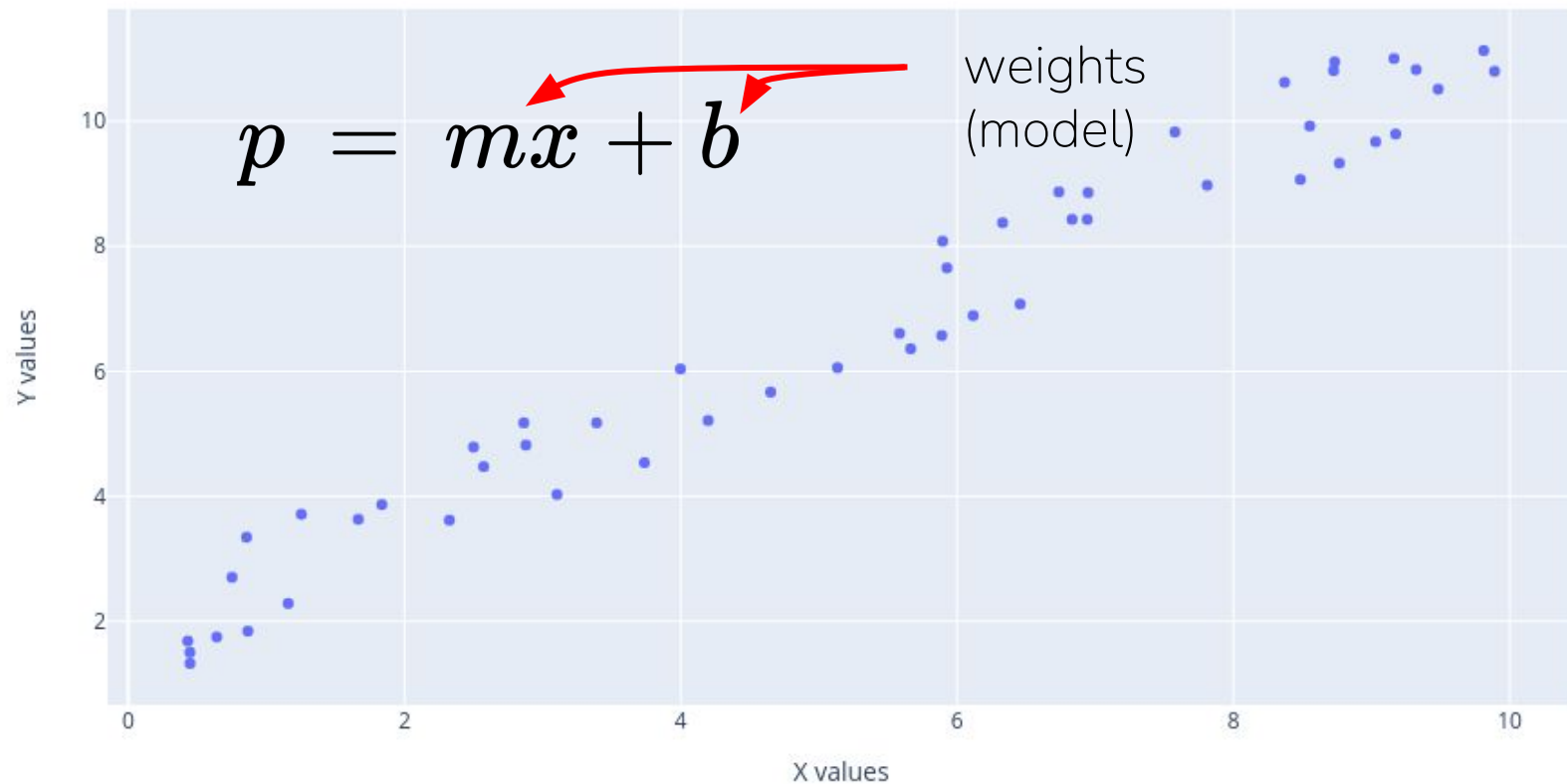
Let's **learn** a simple function



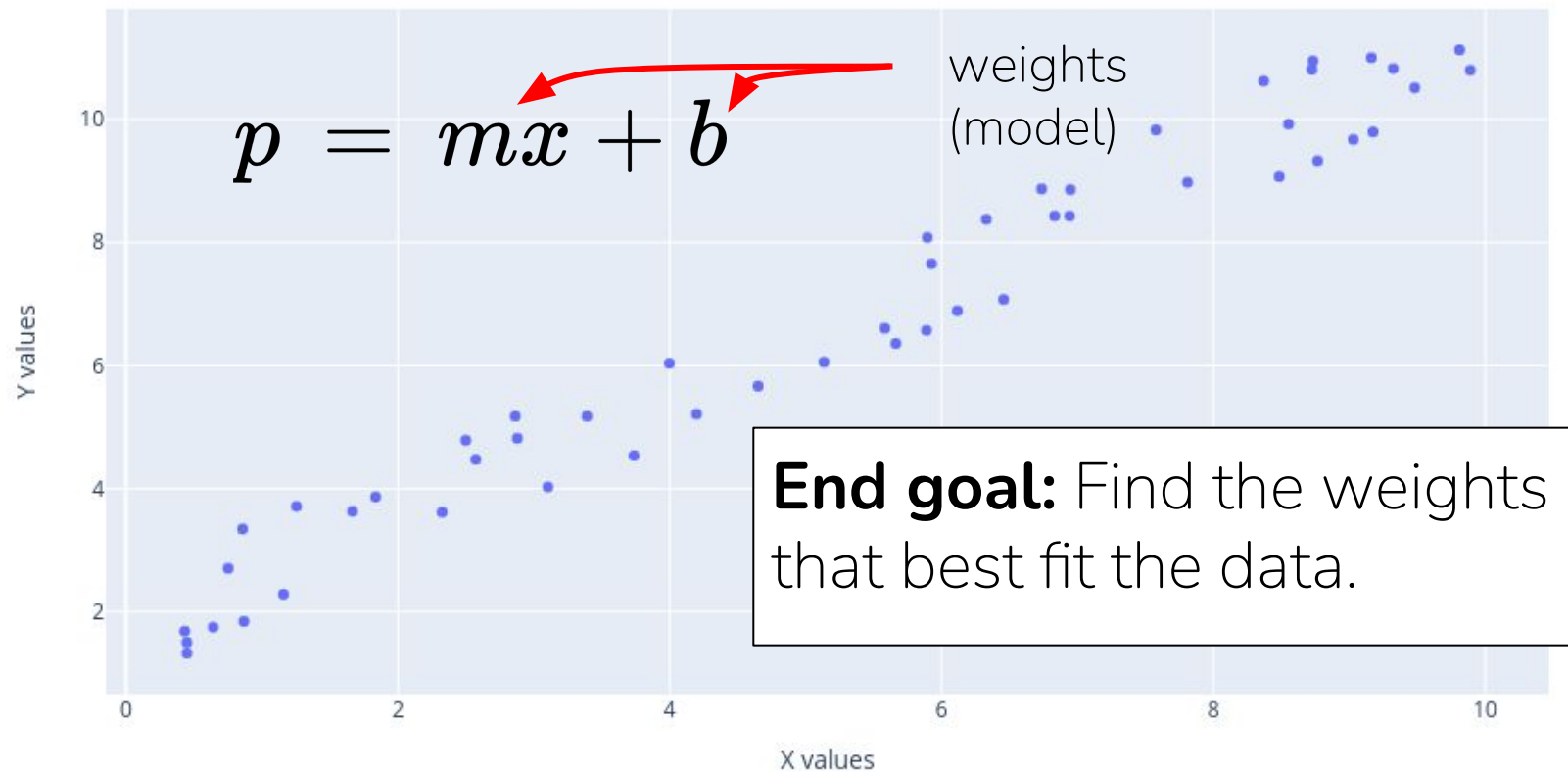
Let's **learn** a simple function



Let's **learn** a simple function



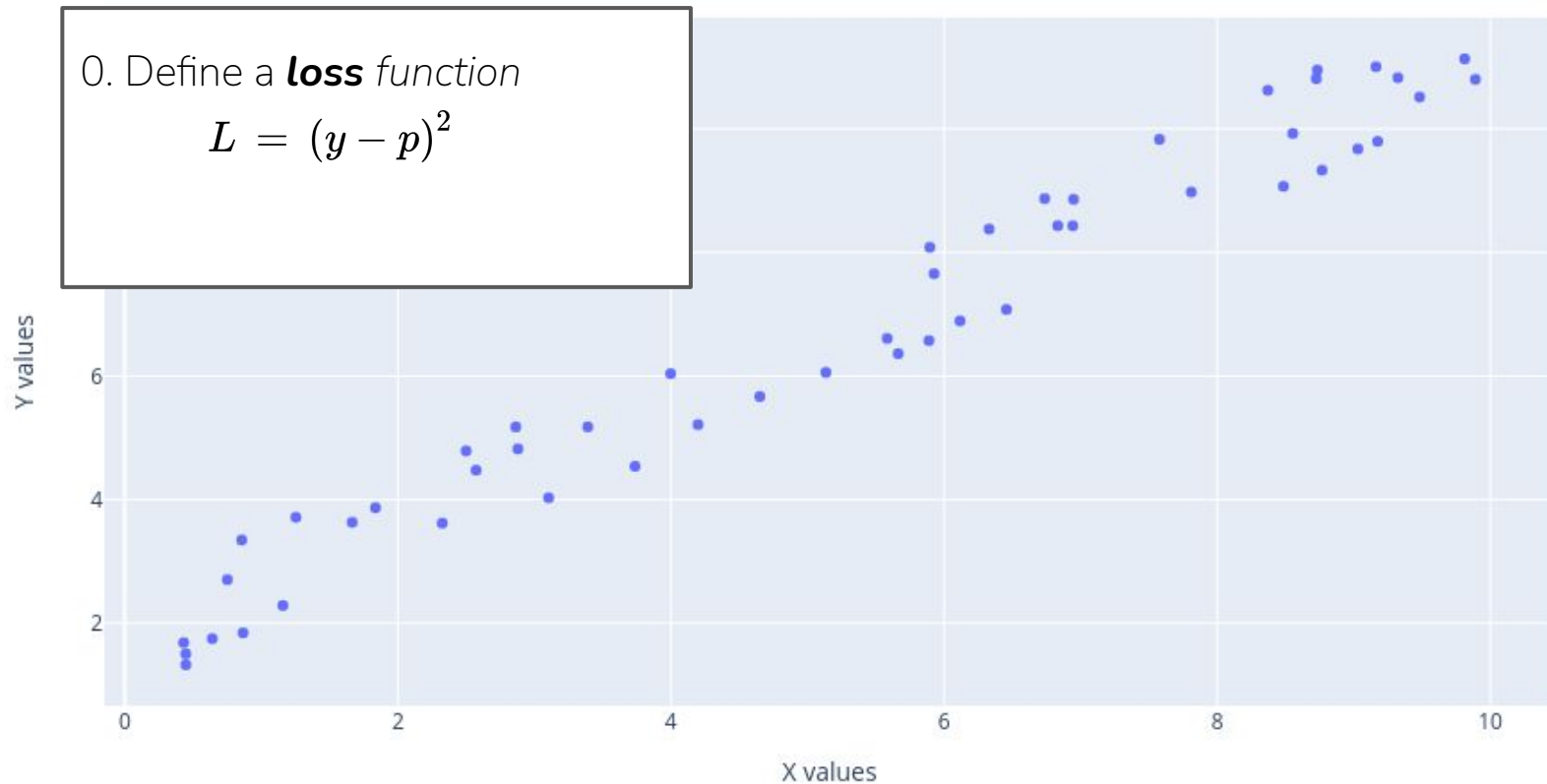
Let's **learn** a simple function



How to train a model:

0. Define a **loss** function

$$L = (y - p)^2$$

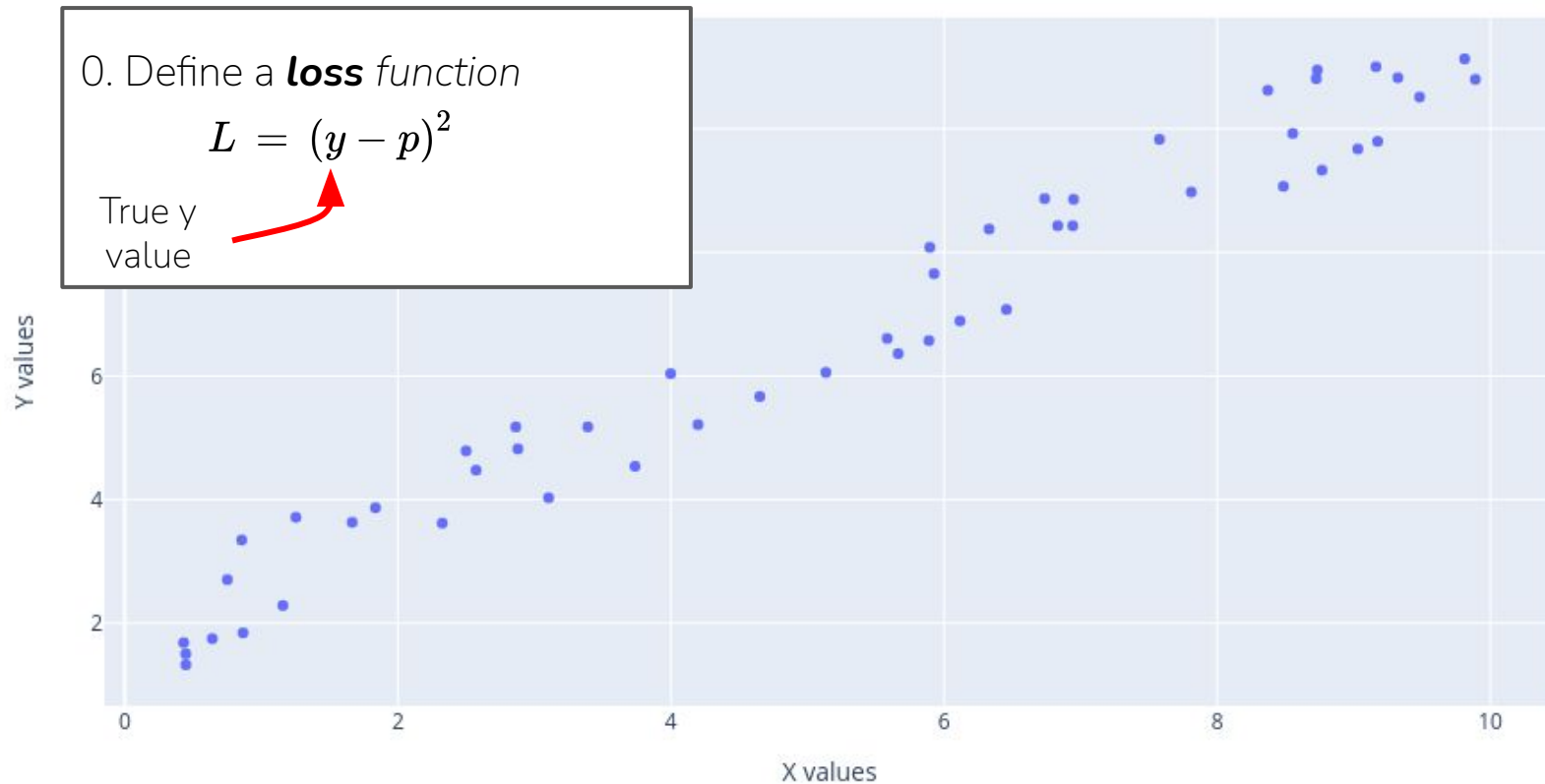


How to train a model:

0. Define a **loss** function

$$L = (y - p)^2$$

True y
value



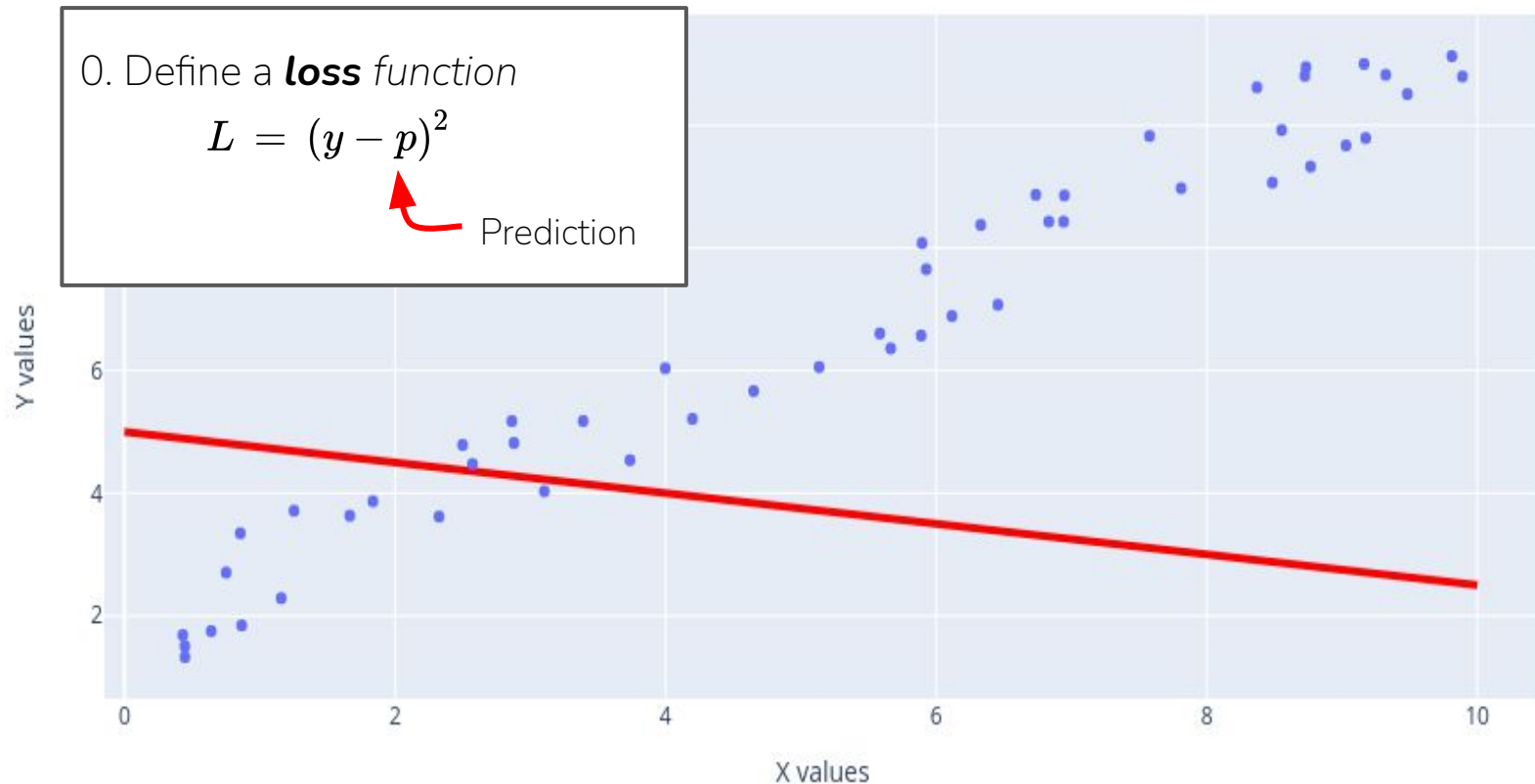
How to train a model:

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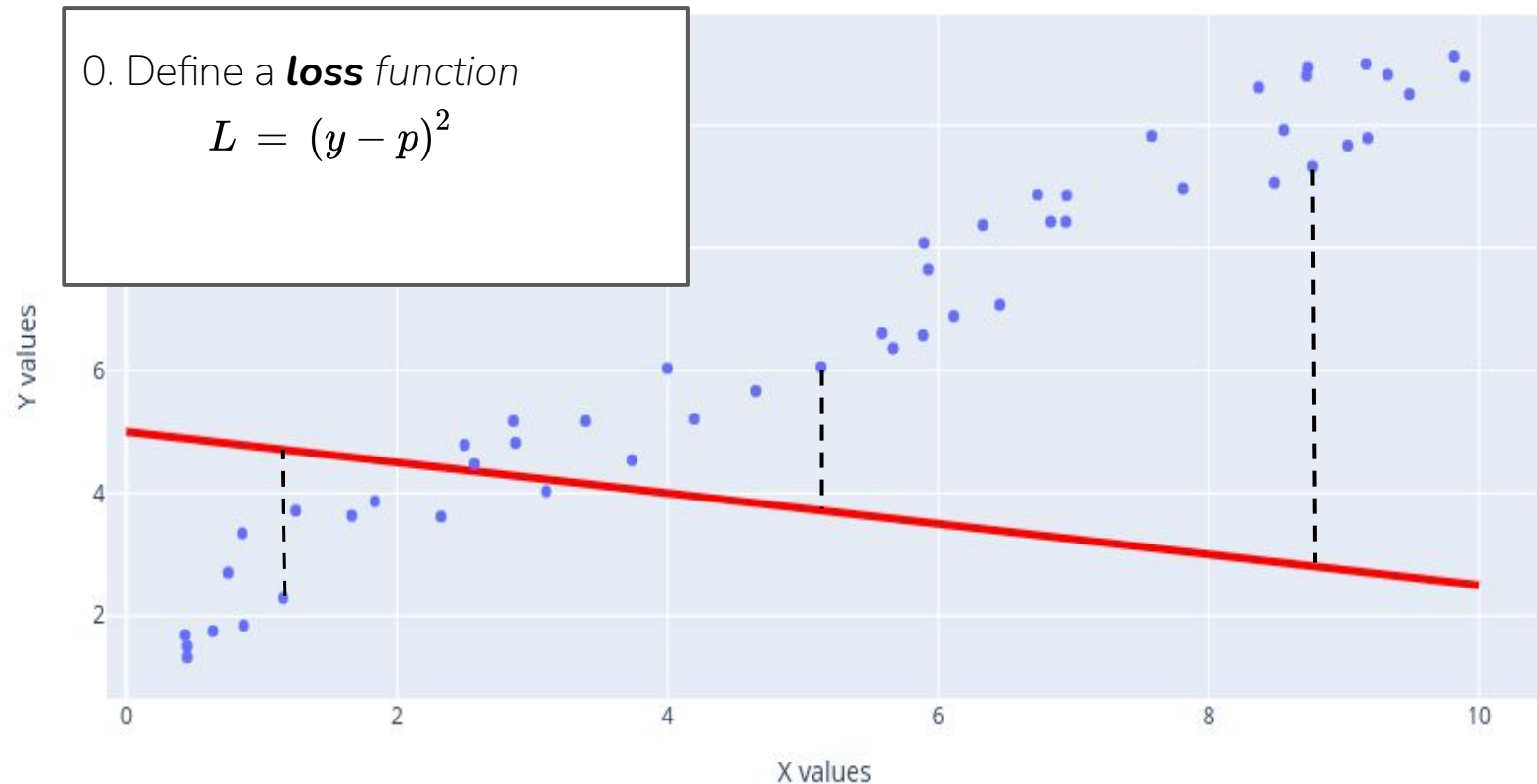
Prediction



How to train a model:

0. Define a **loss** function

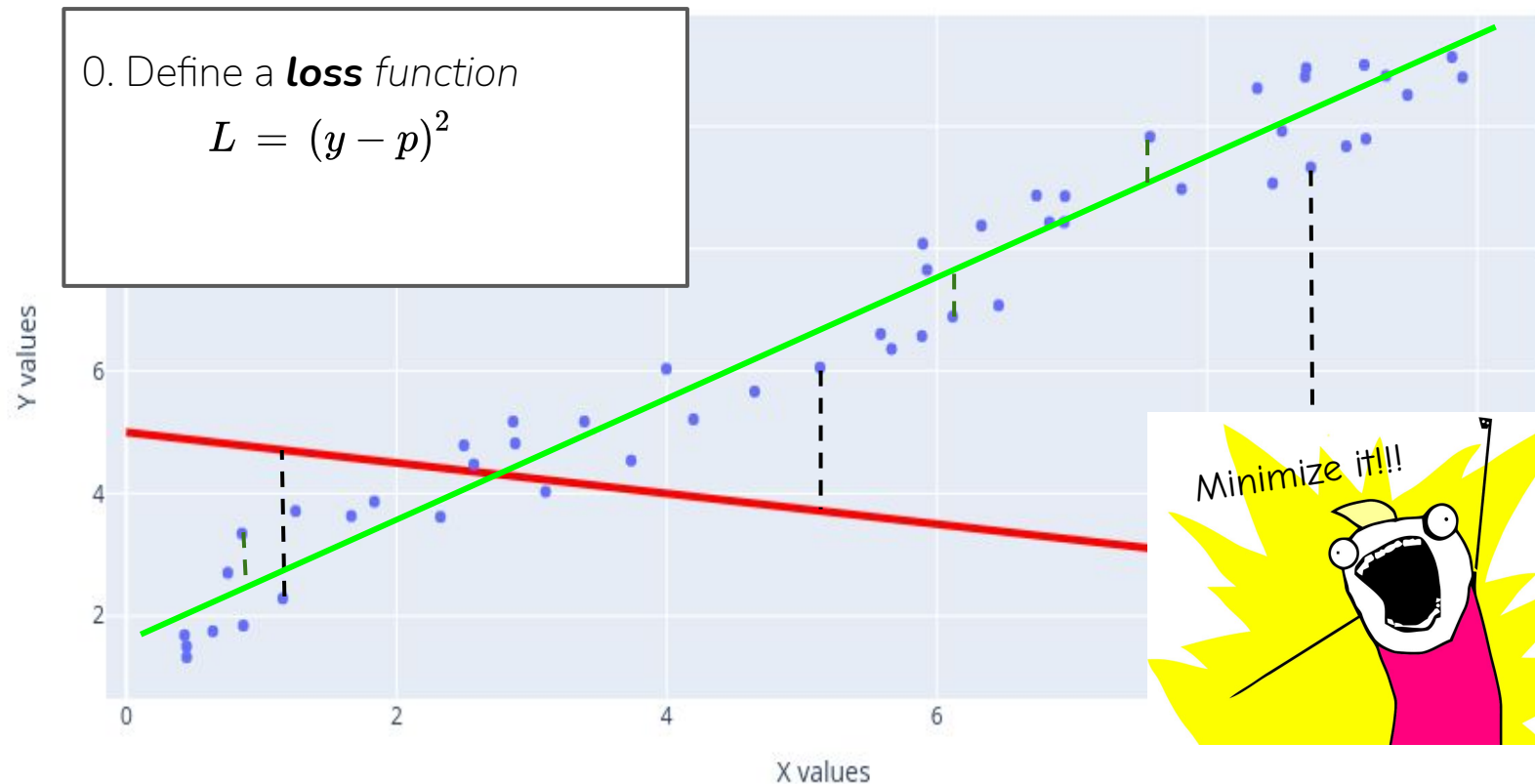
$$L = (y - p)^2$$



How to train a model:

0. Define a **loss** function

$$L = (y - p)^2$$



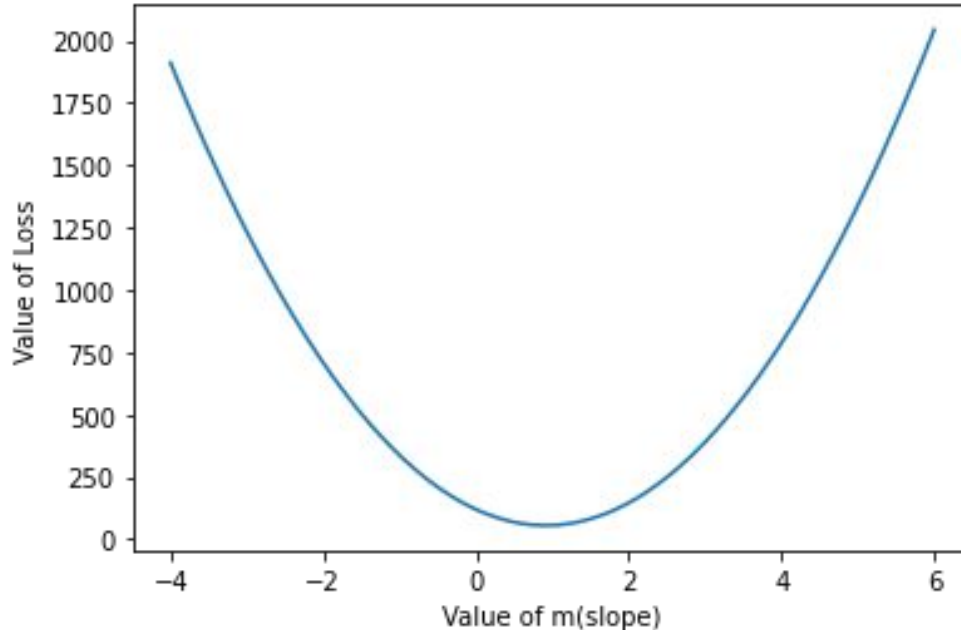
End goal: What values of **m** and **b** minimize **L**?

$$L = (y - p)^2 \quad p = mx + b$$

Dependent
variable

Independent
variable

$$L = (y - (mx + b))^2$$

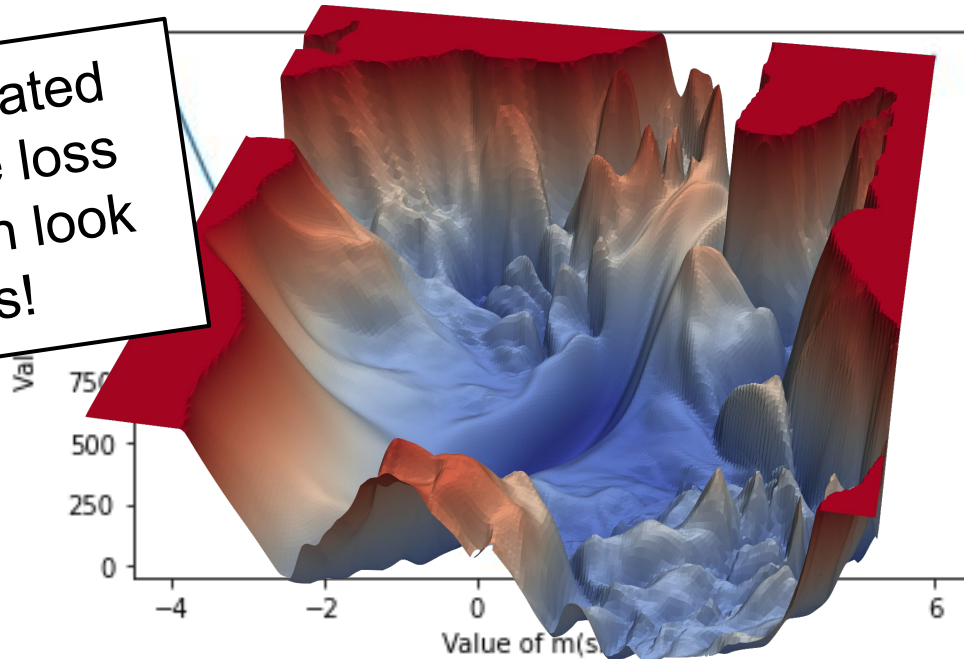
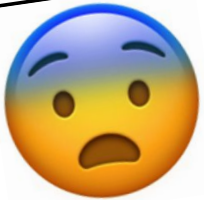


End goal: What values of **m** and **b** minimize **L**?

$$L = (y - p)^2 \quad p = mx + b$$

$$L = (y - (mx + b))^2$$

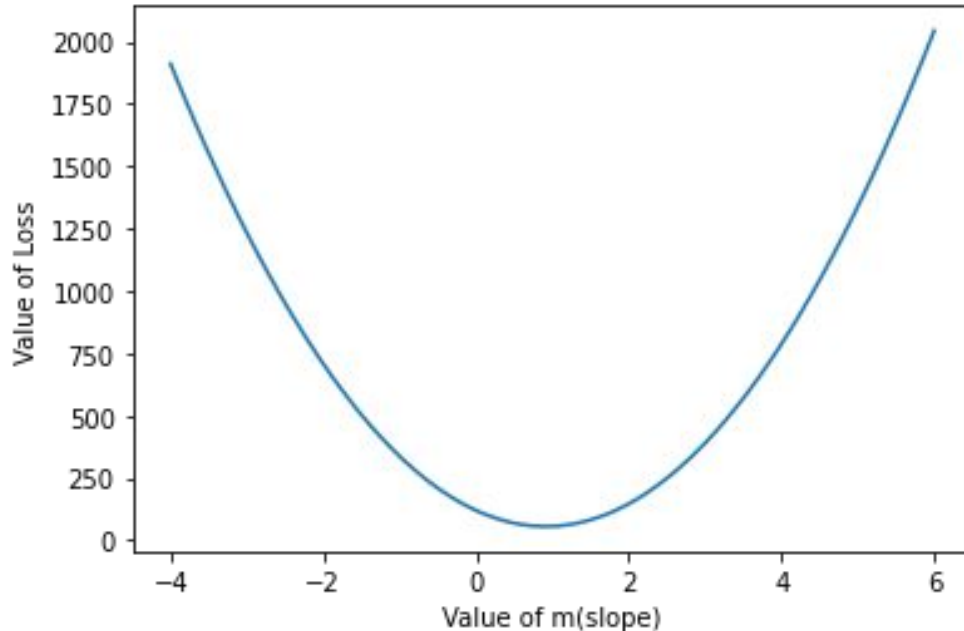
For complicated
models, the loss
function can look
like this!



End goal: What values of **m** and **b** minimize **L**?

$$L = (y - p)^2 \quad p = mx + b$$

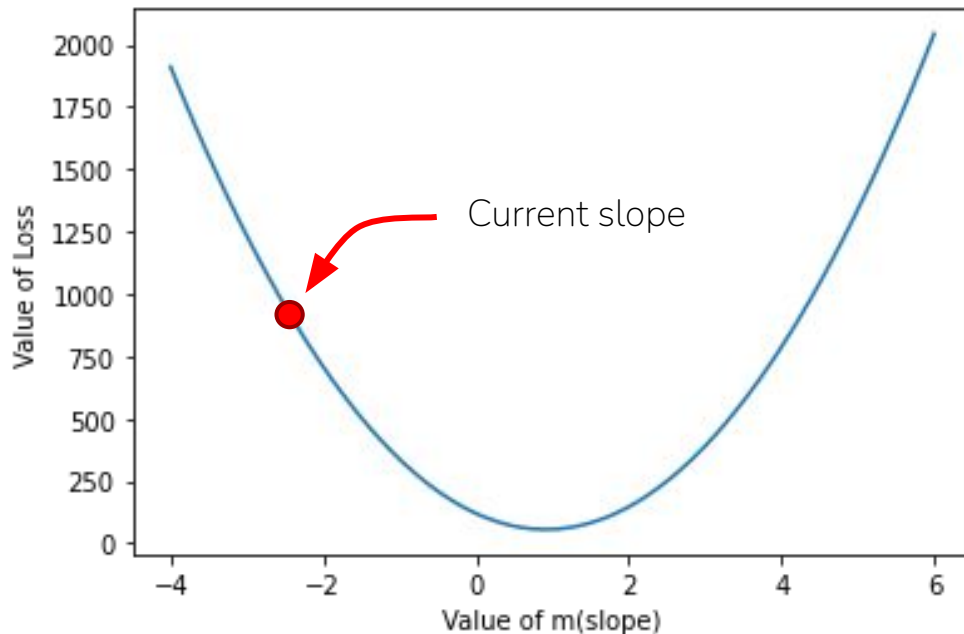
$$L = (y - (mx + b))^2$$



End goal: What values of **m** and **b** minimize **L**?

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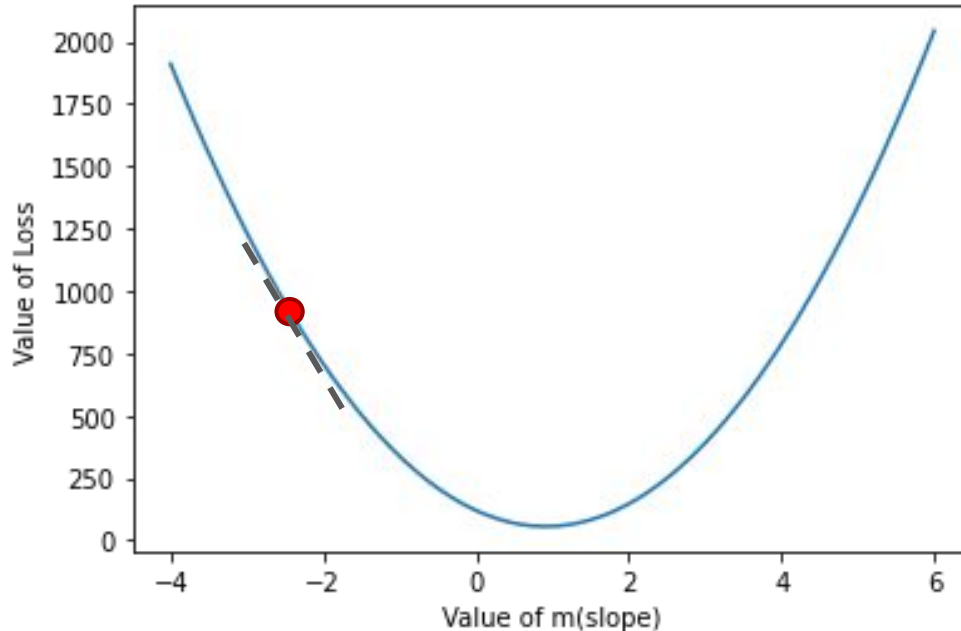
$$L = (y - (mx + b))^2$$



End goal: What values of **m** and **b** minimize **L**?

$$L = (y - p)^2 \quad p = mx + b$$

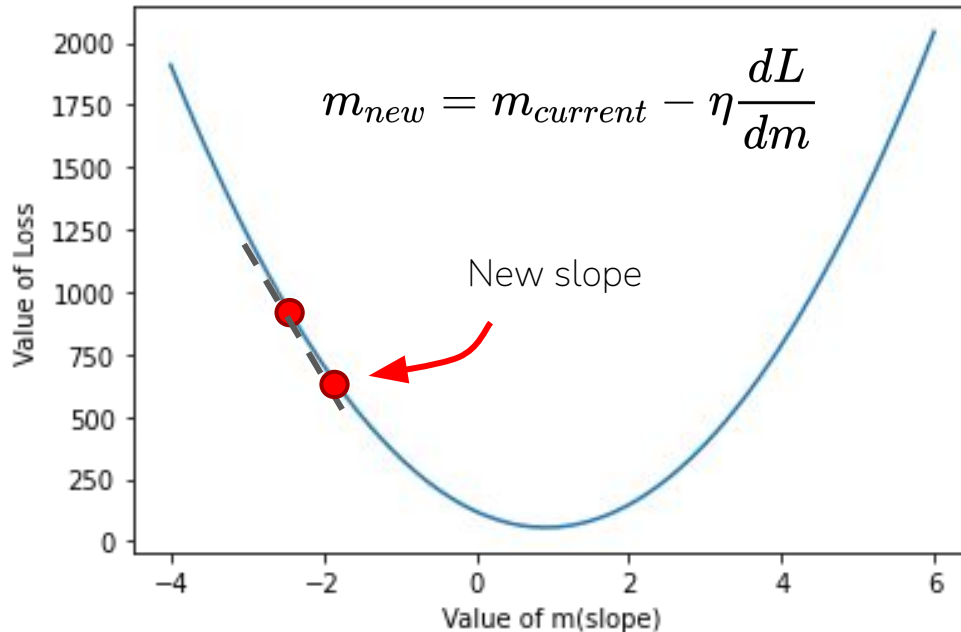
$$L = (y - (mx + b))^2$$



End goal: What values of **m** and **b** minimize **L**?

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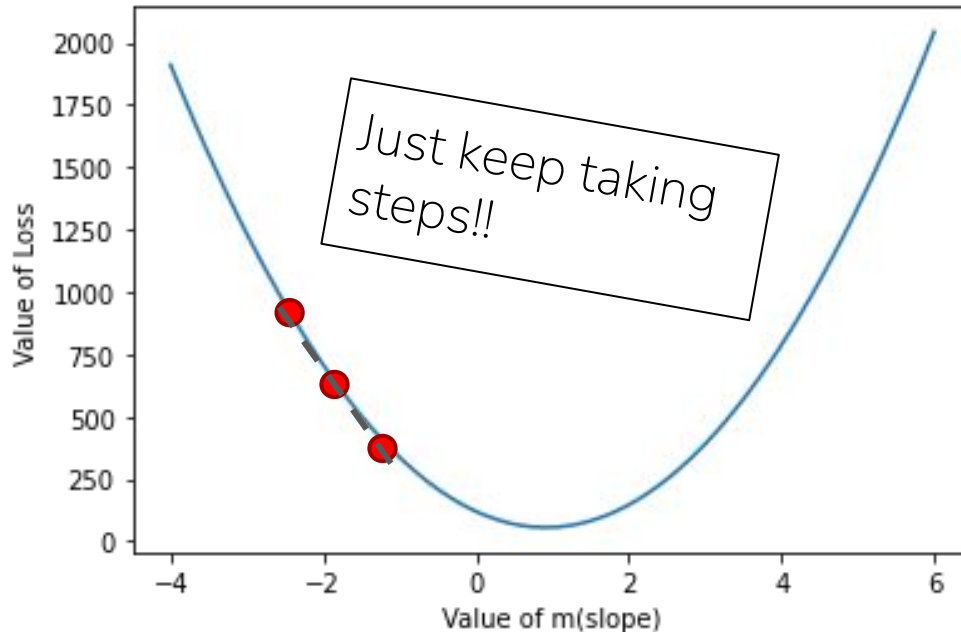
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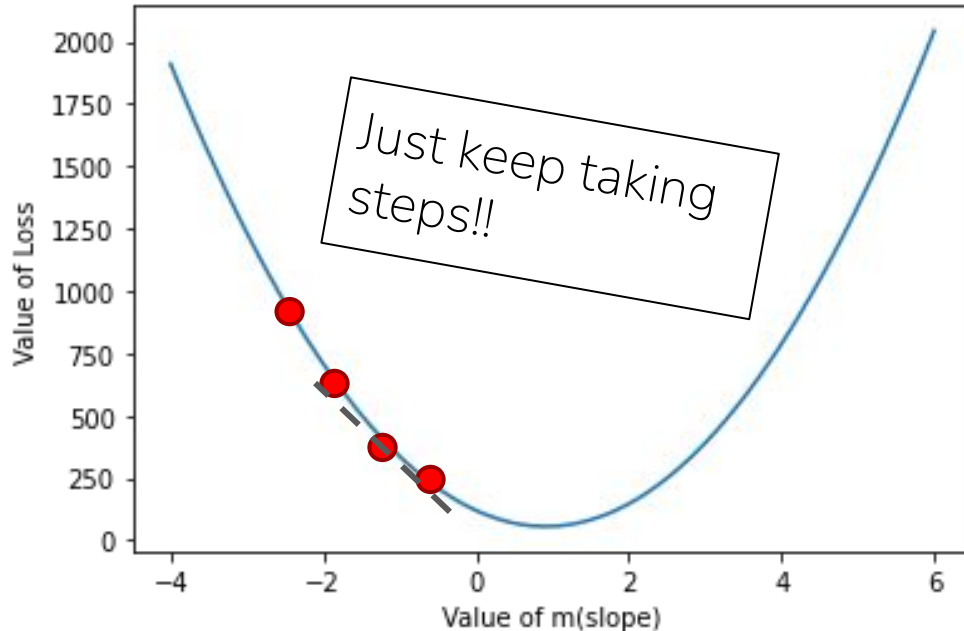
$$L = (y - (mx + b))^2$$



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$$L = (y - p)^2 \quad p = mx + b$$

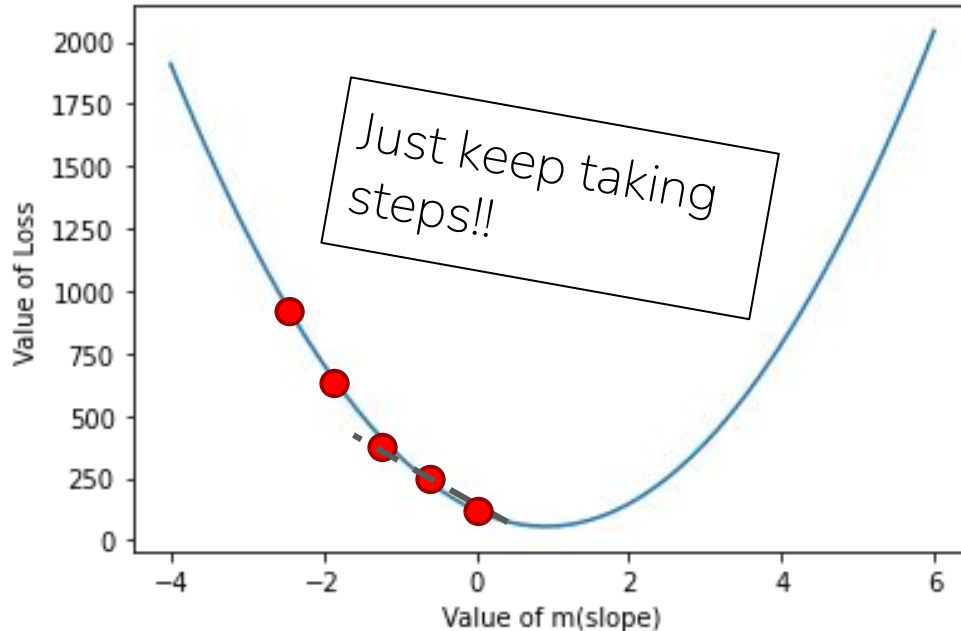
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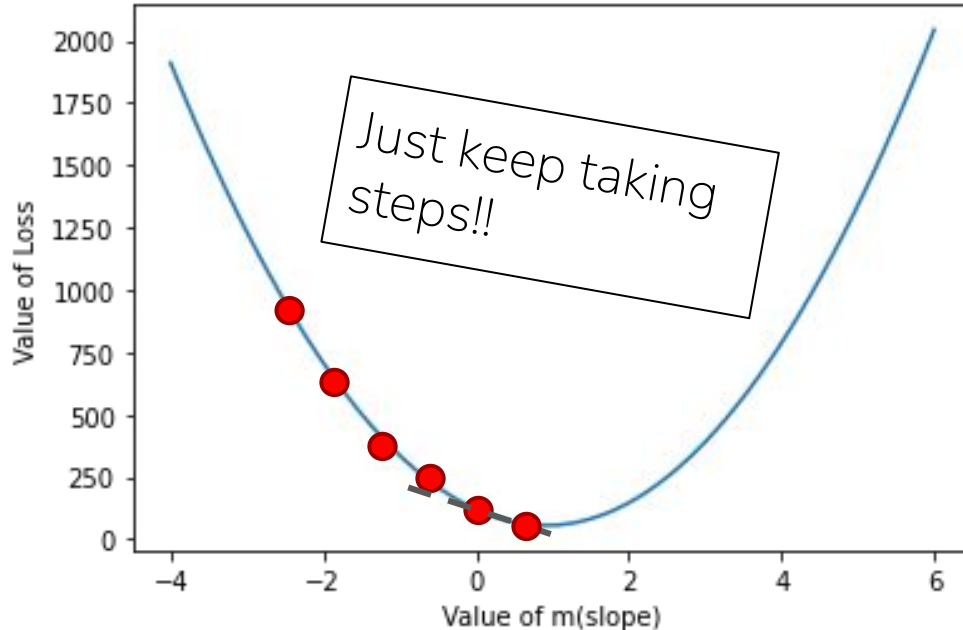
$$L = (y - (mx + b))^2$$



End goal: What values of **m** and **b** minimize **L**?

$$L = (y - p)^2 \quad p = mx + b$$

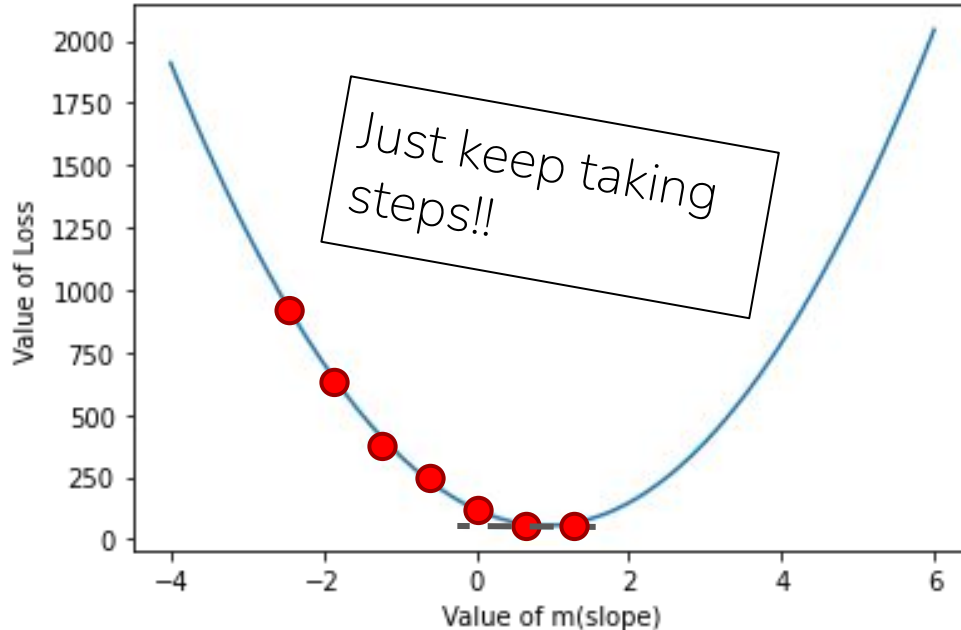
$$L = (y - (mx + b))^2$$



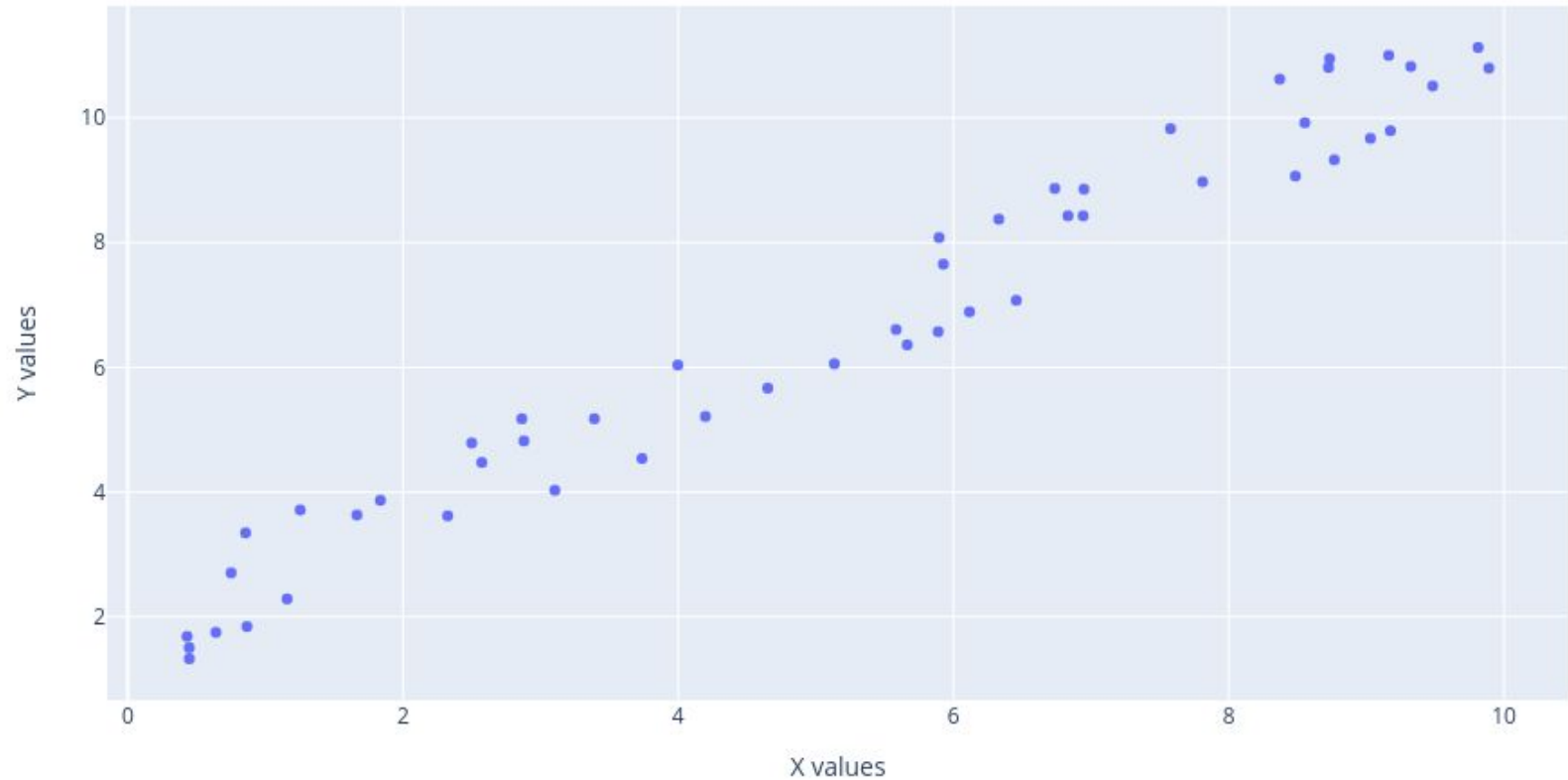
End goal: What values of **m** and **b** minimize **L**?

$$L = (y - p)^2 \quad p = mx + b$$

$$L = (y - (mx + b))^2$$



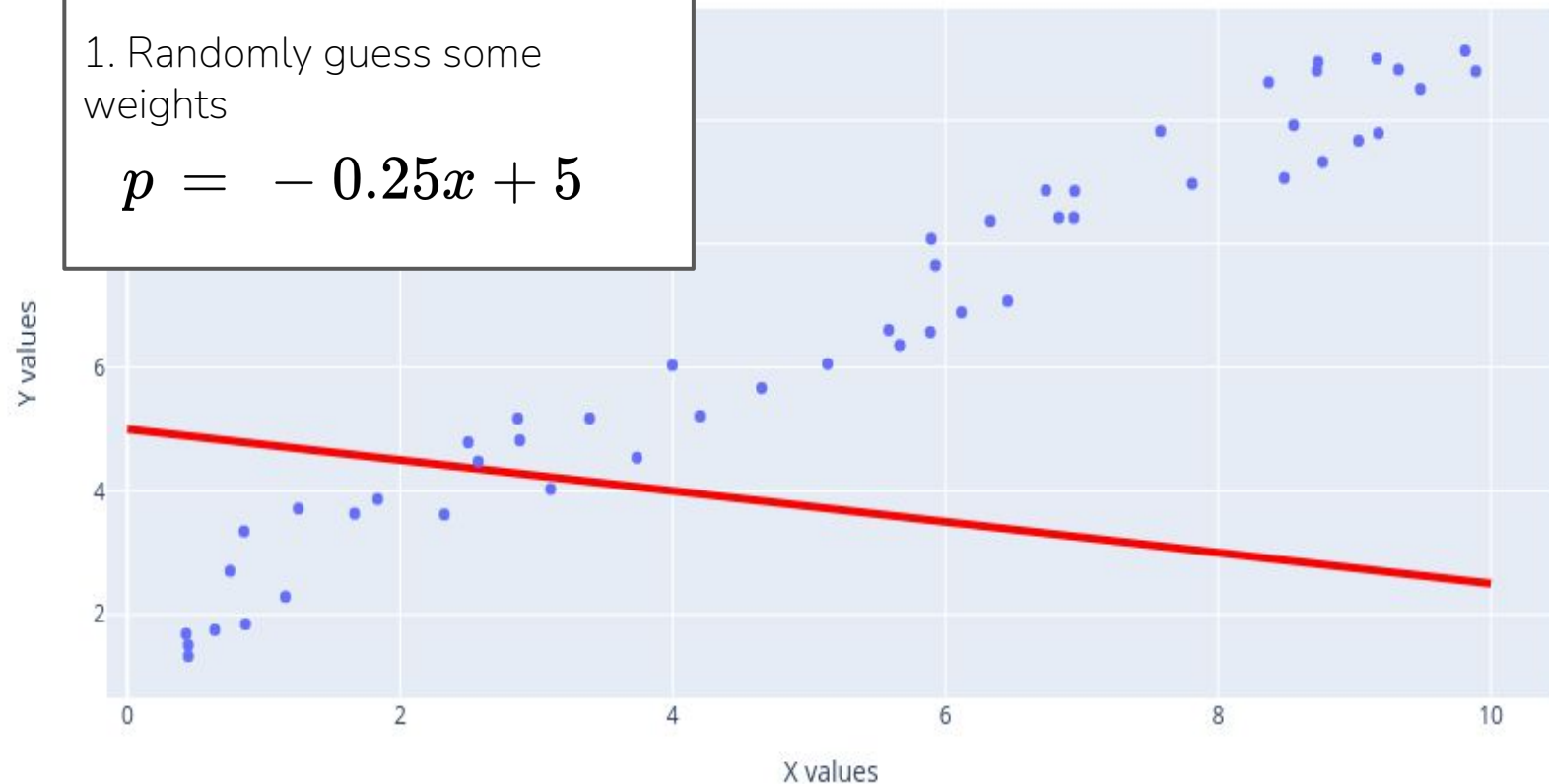
Let's do this for our problem now!



How to train a model:

1. Randomly guess some weights

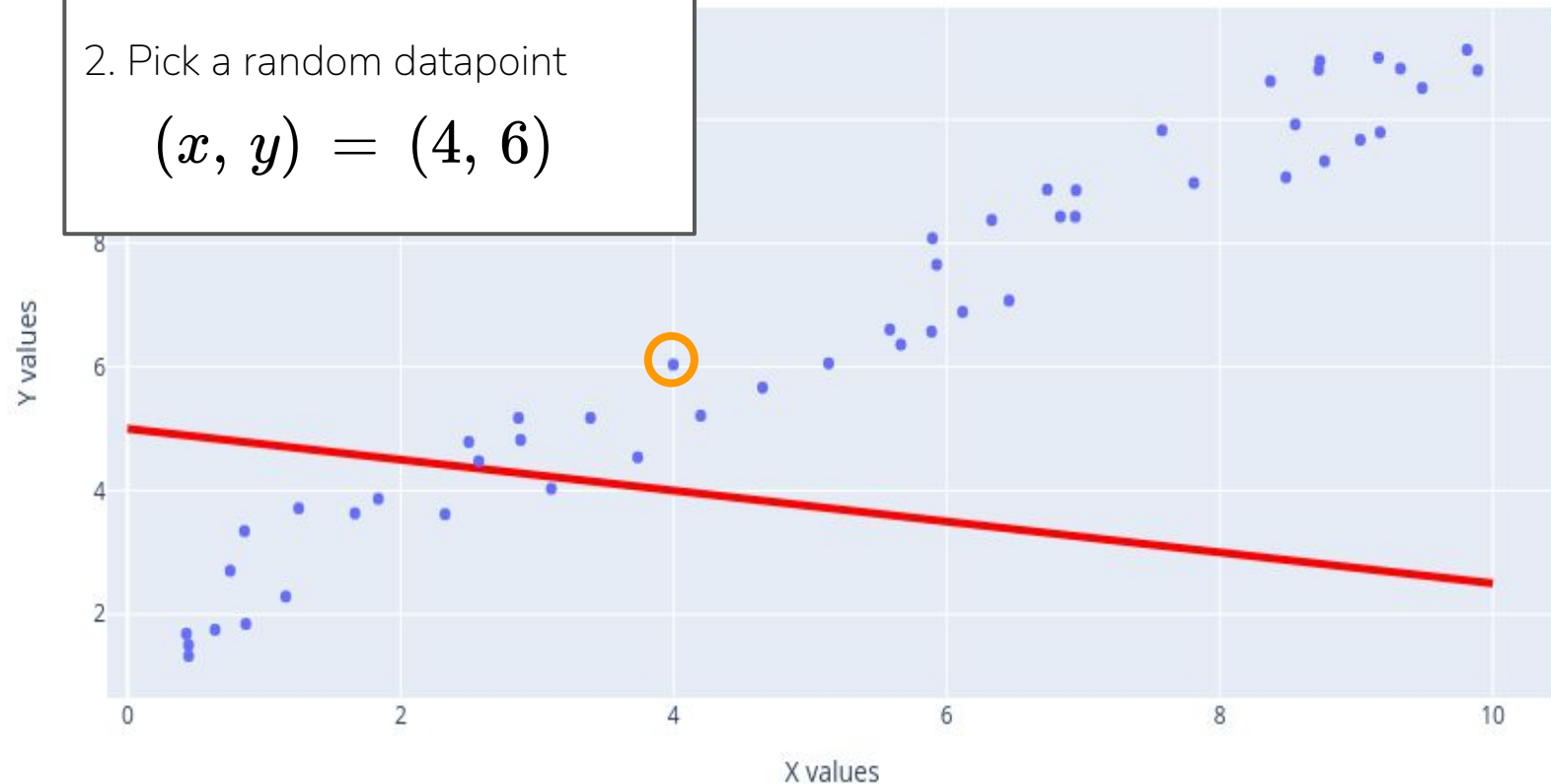
$$p = -0.25x + 5$$



How to train a model:

2. Pick a random datapoint

$$(x, y) = (4, 6)$$



How to train a model:

3. Calculate $\frac{dL}{dm}$

$$L = (y - (mx + b))^2$$

$$\frac{dL}{dm} = 2(y - (mx + b)) \left(\frac{d}{dm} (-mx - b) \right)$$

$$\frac{dL}{dm} = 2(y - (mx + b)) (-x)$$

$$\frac{dL}{dm} = -2yx + 2mx^2 + 2xb$$

$$\frac{dL}{dm} = -2(6)(4) + 2(-0.25)4^2 + 2(4)(5)$$

$$\frac{dL}{dm} = -8$$

Plug in our current point:

$$(x, y) = (4, 6)$$

$$(m, b) = (-0.25, 5)$$

How to train a model:

4. Do it again for $\frac{dL}{db}$!!!

$$L = (y - (mx + b))^2$$

$$\frac{dL}{db} = 2(y - (mx + b)) \left(\frac{d}{db} (-mx - b) \right)$$

$$\frac{dL}{db} = 2(y - (mx + b)) (-1)$$

$$\frac{dL}{db} = -2y + 2mx + 2b$$

$$\frac{dL}{db} = -2(6) + 2(-0.25)4 + 2(5)$$

$$\frac{dL}{db} = -4$$

Plug in our current point:

$$(x, y) = (4, 6)$$

$$(m, b) = (-0.25, 5)$$

How to train a model:

5. Update **m** and **b**

$$m_{new} = m_{current} - \eta \frac{dL}{dm}$$

$$m_{new} = 0.25 - 0.001 \cdot (-8)$$

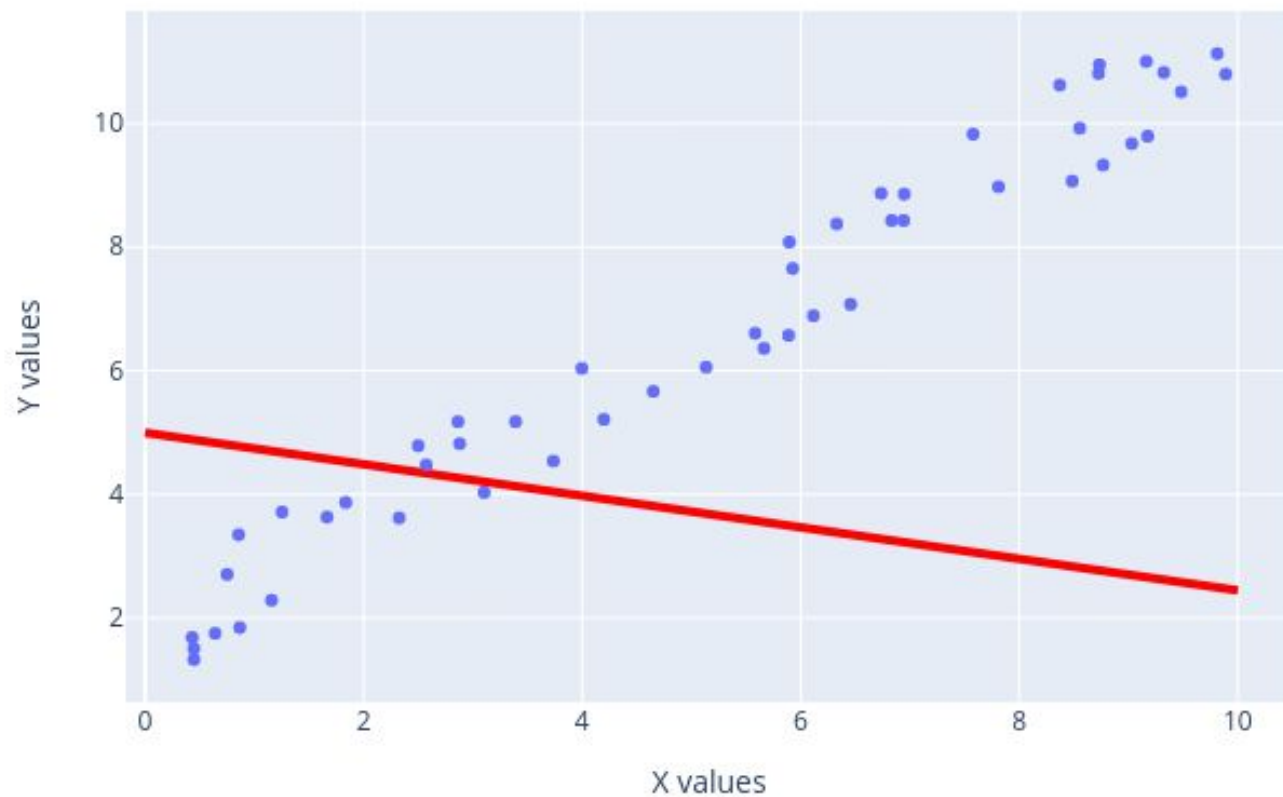
$$m_{new} = 0.242$$

$$b_{new} = b_{current} - \eta \frac{dL}{db}$$

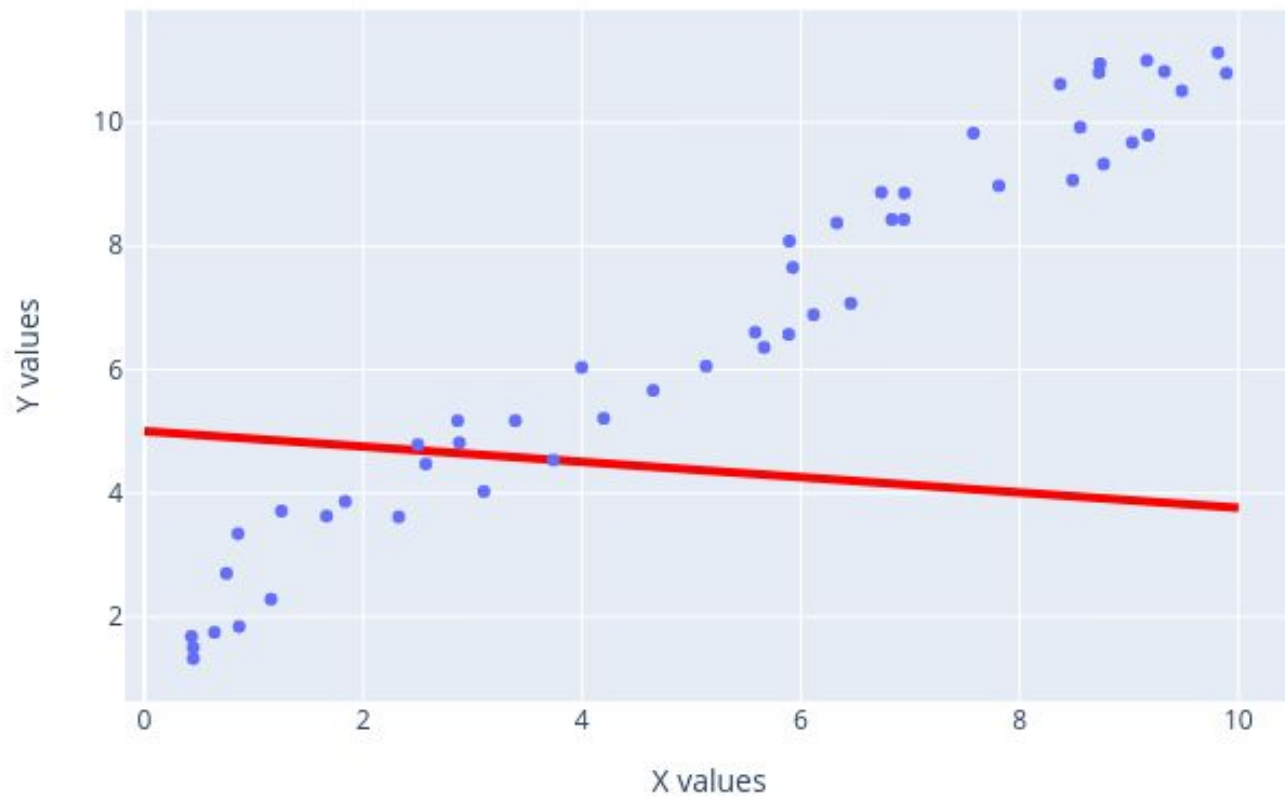
$$b_{new} = 5 - 0.001 \cdot (-4)$$

$$b_{new} = 5.004$$

Old ***m*** and ***b***:

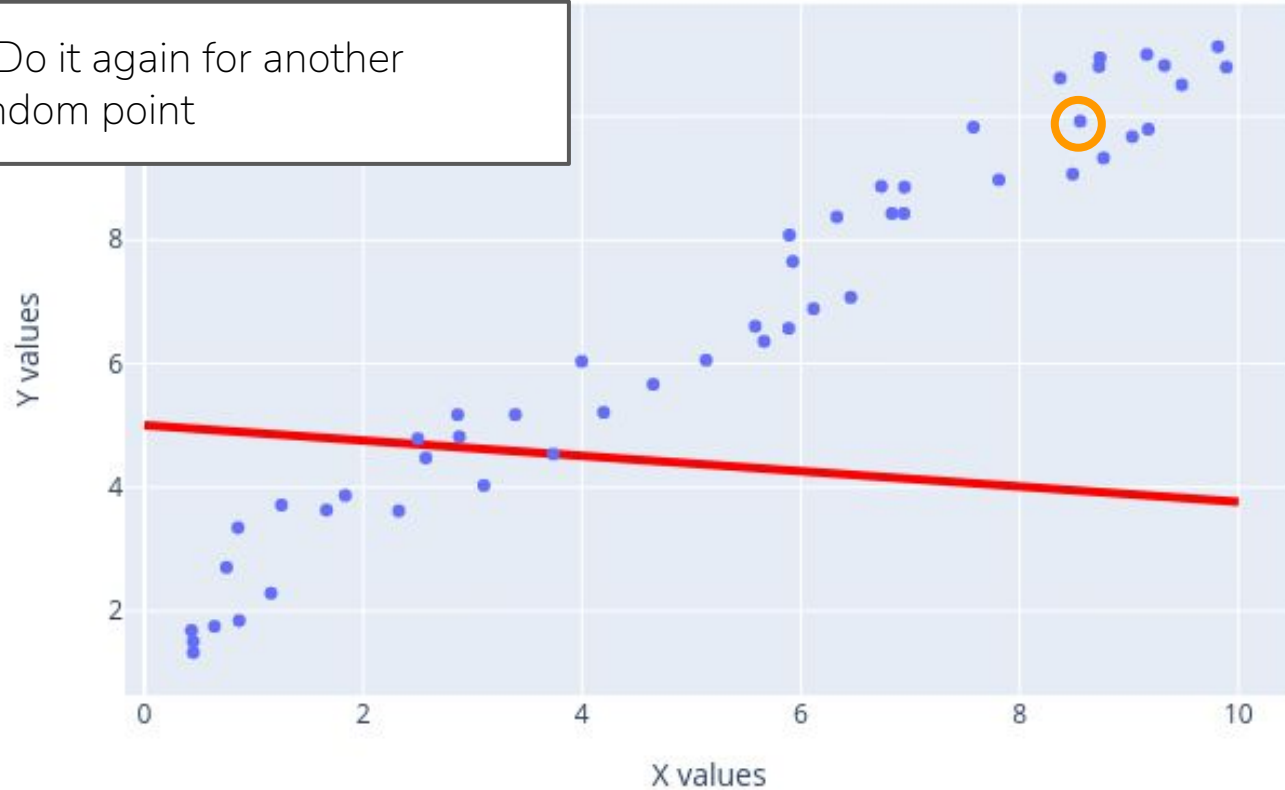


New ***m*** and ***b***:



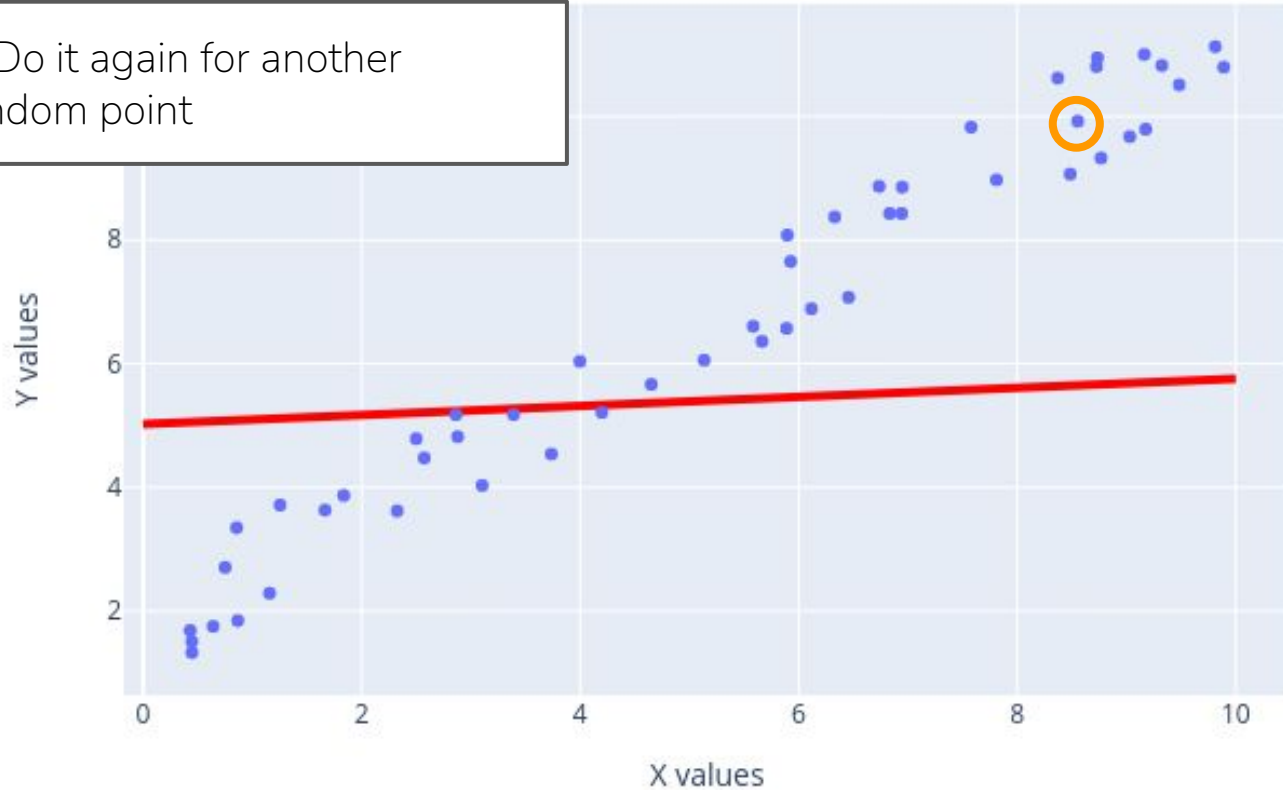
How to train a model:

6. Do it again for another random point



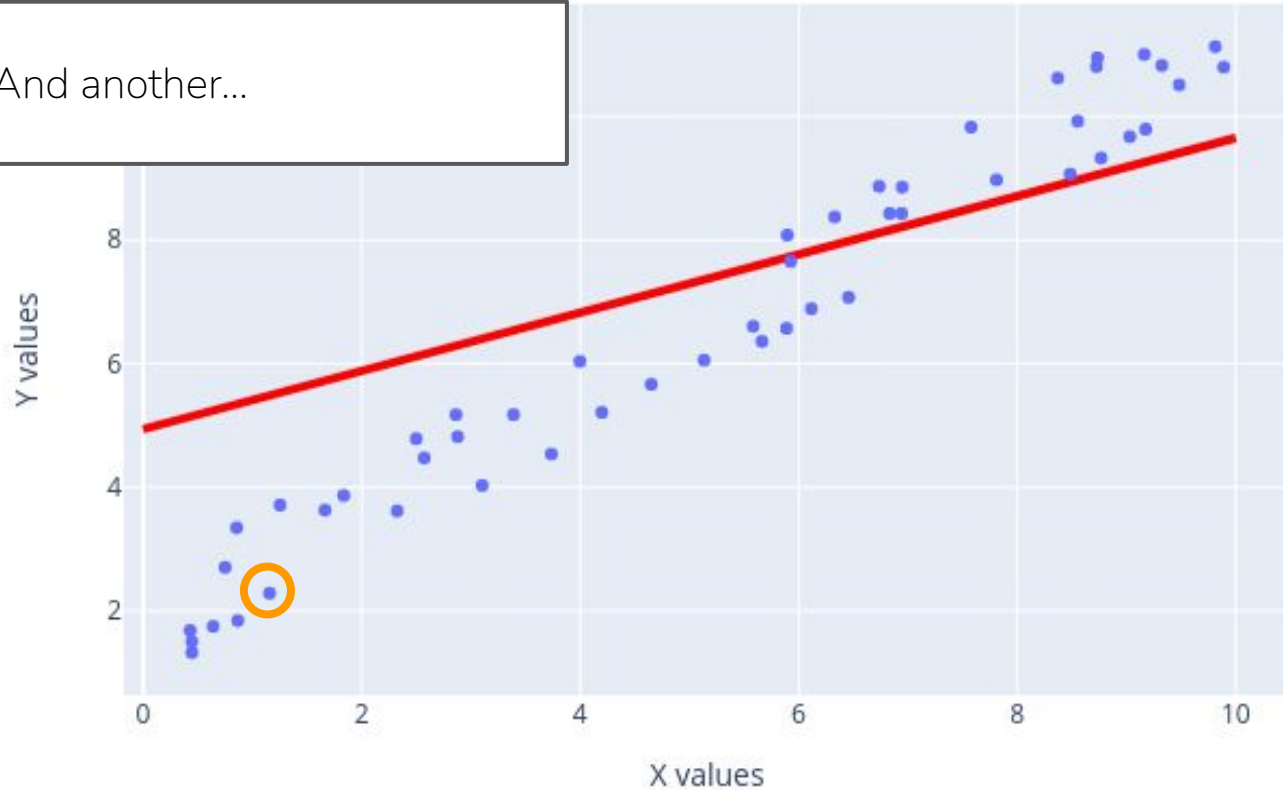
How to train a model:

6. Do it again for another random point



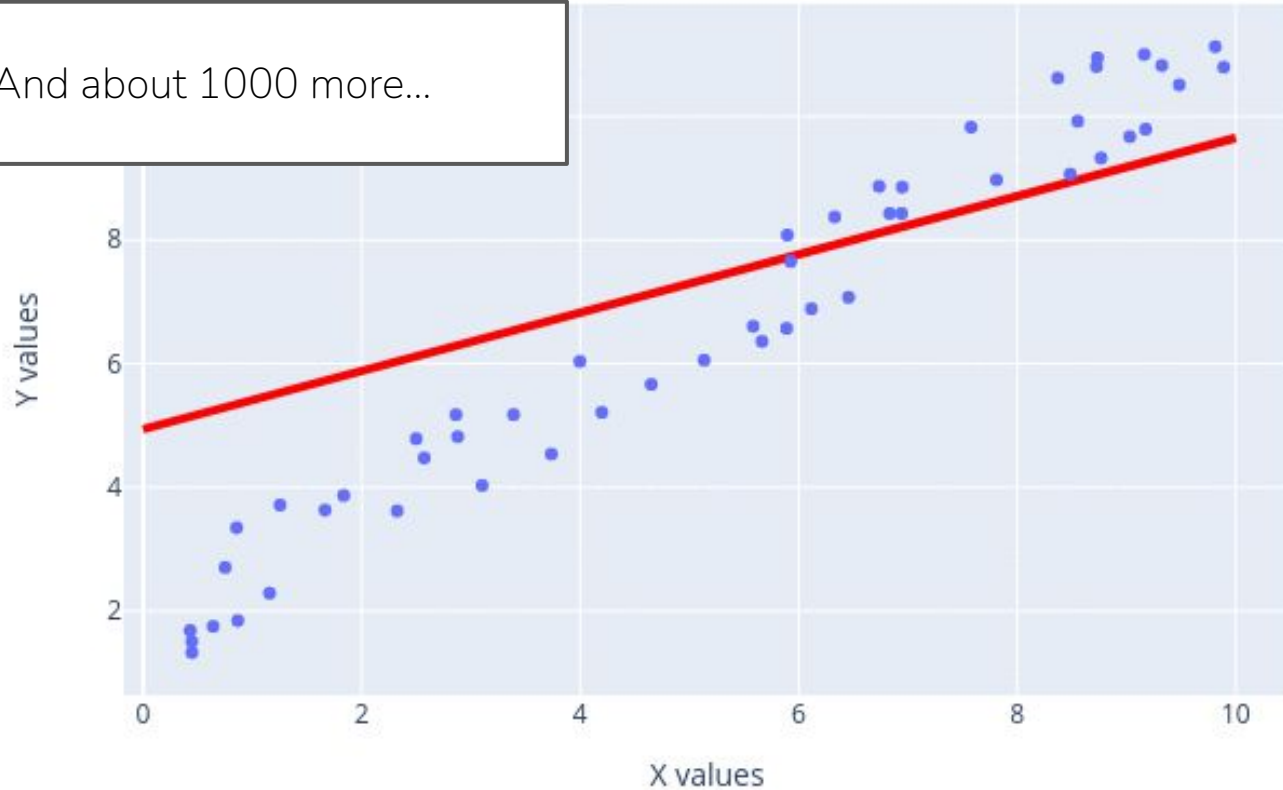
How to train a model:

7. And another...



How to train a model:

8. And about 1000 more...

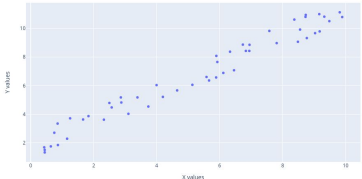

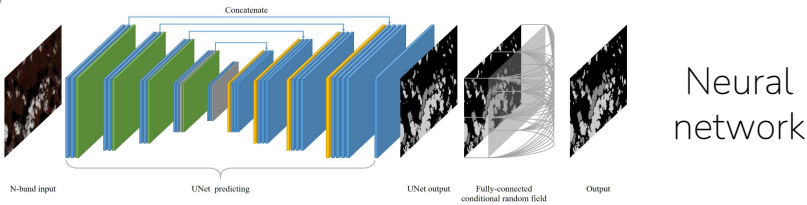



How to train a model:

9. Congrats! We just trained a model!



A more complicated problem.

	Our simple problem	Blue River's problem
Data		
Model	$p = mx + b$	
Loss	$L = (y - p)^2$	

You want more?!??!

[Better look at a neural network \(3blue1brown\)](#)

[Want a deeper non-technical dive? \(Andrew Ng\)](#)

[Another pretty cool form of machine learning \(CGP Grey\)](#)

[The high school guide to machine learning](#)

Code that generated these plots

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