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Information Theory Analyses of Four Sonata Expositions
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INFORMATION THEORY ANALYSES

I. Introduction†

Various authors have proposed for some years now that certain concepts of information theory should be useful both for studying structural details of musical compositions and, more broadly, for clarifying our understanding of the nature of musical communication. Even though the "theory of information", or "mathematical theory of communication", was developed from engineering studies of technical communication systems such as telephone circuitry, it nevertheless seems to embody

† This paper is partly abstracted from a thesis presented by Calvert Bean in partial fulfillment of requirements of the degree of Doctor of Musical Arts at the University of Illinois, Urbana, Illinois, April, 1961.

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OF FOUR SONATA EXPOSITIONS

LEJAREN HILLER

CALVERT BEAN

a number of unifying ideas that apply to all forms of human communication.

To date, very little has actually been done to test specifically whether concepts of information theory really do provide some sort of quantitative measure of significant details of musical structure. We propose here to demonstrate an elementary application of this theory to the study of a traditional musical form. Specifically, we shall be concerned with a comparative analysis completed in 1961 of four sonata expositions taken from the standard piano literature employing some simple processes of computation provided by the theory. In presenting the results of these computations we do not imply that the available techniques of information theory were exhausted or that a definitive and complete investigation of the particular musical

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examples was achieved. In fact, this is but the first of several investigations of the use of information theory for musical analysis to be carried out as part of a continuing research program at the Experimental Music Studio in the School of Music at the University of Illinois. Subsequent work will be the subject of future publications.

The literature concerned with the use of information theory for music analysis can be grouped into two broad categories. The first category consists of rather specific analyses of particular musical scores using statistical methods to obtain limited quantitative results. In these studies, a musical score is treated as a purely objective specimen that represents some set of clearly defined musical processes. The second category of publications, on the other hand, tends to be more speculative in nature. Here, the authors seem to be primarily concerned with how information theory might be used as a measure of the psychological response to music. References to much of the literature on musical applications of information theory published prior to 1959 have already been given elsewhere. *1 Since then, additional publications have appeared that should be noted as follows: First, Neumann and Schappert *2 have published a new account of experiments in music analysis and synthesis which were originally reported by Brooks, Hopkins, Neumann and Wright. *3 This involved the statistical analysis of 37 hymn tunes, followed by computer synthesis of tunes derived from this analysis. Second, authors such as Eco *4, Heike *5, and Cohen *6 have pointed out the general applicability of information theory to musical problems. Third, Dr. Wilhelm Fucks of the Institut für Physik in Aachen, Germany has published an important series of statistical studies of historical style from c. 600 to the present. *7 The last cited article by this author gives a particularly useful presentation of this work. Fucks was able to demonstrate that at least one statistical parameter, namely, excess, which depends on the ratio of the fourth moment to the fourth power of the standard deviation shows a linear progression from baroque to contemporary music except for recent serial compositions. Cohen *6 also reports that Brawley *8 has applied information theory to analysis of rhythms used in music dating from 1200 to the present. Among the second type of publications, we should note a book on information theory and aesthetic perception by Moles *9 and an article by Meyer *10 that suggests a correspondence between his theory of musical meaning and concepts of information theory. Finally, we should mention that brief preliminary reports of some of the work to be discussed here have appeared elsewhere as the consequence of lecture-demonstra-

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tions on information theory and music given by one of the present authors. *11

II. Relevant Concepts of Information Theory

Although many of the ideas of information theory have been presented elsewhere in a useful and comprehensive form (*12, *13, *14) it may still be helpful to give here a brief synopsis of those concepts which we shall directly apply to the analysis of the musical examples we have selected for this investigation.

In 1928 R. V. L. Hartley published a paper in the Bell System Technical Journal that laid the foundation of modern information theory. *15 Hartley's ideas can be summarized as follows:

1. Any communication system can be considered objectively and independently of the human sender and human receiver.

2. The commodity sent over the communication system is a sequence of signals or some physical representation of signals.

3. It is not important what the symbols mean, rather it is only important to know how many different kinds of symbols there might be and how fast they might be transferred through the system. In other words, we must estimate the maximum demands that might be put on the system.

This emphasis of system capacity led Hartley to define the word "information" as a measure of the number of symbols sent. We emphasize this quantitative definition as contrasted to the everyday use of the word "information" that also involves meaning.

Hartley then examined the random message because this is the easiest to treat mathematically. A random message, by definition, consists of a sequence of symbols chosen, one after another, independently but with equiprobability, from the available alphabet, whether that be letters, musical pitches and rhythms, spots on a television screen, or whatever. The choice sequence is statistical and is the simplest example of what is called a "stochastic process". This random message has maximum information content because the condition of equiprobability permits no economy to be effected by assuming that certain symbols will very seldom or never occur. Moreover, any

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conceivable message might possibly be generated since there are no restrictions on the system whatever. For this system, assuming an alphabet of N symbols, Hartley developed the following equation:

$$(H_N)_{max.} = \log_2 N \text{ bits/symbol} \tag{1}$$

where $(H_N)_{max.}$ is the information content in binary digits per symbol. (Binary digits are commonly called bits.) Then, if n symbols are communicated per second, the information rate is:

$$(H_T)_{max.} = n \log_2 N \text{ bits/second} \tag{2}$$

and finally, if the message lasts T seconds, the total information conveyed is:

$$H_{max.} = nT \log_2 N \text{ bits.} \tag{3}$$

In all these equations, $\log_2$ is equal to 3.32 $\log_{10}$, the common logarithm. The base 2 is employed because the fundamental unit of information is the binary digit, or bit. The binary number system, consisting only of the digits, 0 and 1, is employed because it is postulated that all questions of information content can be reduced ultimately to the binary choice of "yes-no" type decisions.

As an example of how to apply Hartley's equation, let us consider the chromatic octave made up of the twelve notes, C, C♯, etc., up to B. If these pitches are chosen sequentially with equiprobability to make up a sample of elementary random music, then N = 12 and Hartley's equation tells us that (H_n)max. = $\log_2 12 = 3.32 \log_{10} 12 = 3.58$ bits/symbol. This is the maximum information these twelve notes can convey per symbol. The information rate then depends on how fast we generate these notes, i.e., on the rhythm, the tempo, and the density of texture.

Because random messages are of relatively little interest, Hartley's paper did not attract much attention until Claude Shannon re-examined Hartley's ideas some twenty years later. Shannon was concerned with all kinds of messages, not just random messages. Shannon developed the basic equation for information content computation derived from the probabilities, p(i), where i = 1, 2, . . . N, associated with each symbol in a

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given alphabet of N symbols; and where the p(i) are not equal in value.

$$H_N = \sum_{i}^{N} p(i) \log_2 p(i) \text{ bits/symbol} \tag{4}$$

The summation denoted by $\sum_{i}^{N}$ simply means

$$p_1 \log_2 p_1 + p_2 \log_2 p_2 + \cdot \cdot \cdot + p_n \log_2 p_N$$

Since the magnitude of H_N in a message depends not only on the size of the alphabet, but also on the probability distribution of symbols drawn from that alphabet, it is immediately observed that information content in messages is analogous to entropy, the measure of distribution of energy in physical systems. Thus 'entropy' is often used more or less interchangeably with 'information' in the statistical theory of communication. Moreover, it is easily shown that Hartley's equation is a special case of Shannon's equation as follows: With an alphabet with N symbols, all equally probable, $p(i) = p_1 = p_2 = \ldots p_N = 1/N$. Then $(H_N)_{max.} = (1/N) \log_2 1/N + (1/N) \log_2 1/N + \ldots (1/N) \log_2 1/N$ for a total of N terms. Therefore, $(H_N)_{max.} = \log_2 N$, which is Hartley's equation.

Finally, just as with Hartley's equation, Equation (4) can be multiplied by n, to yield information rate:

$$H_T = -n \sum_{i}^{N} p(i) \log_2 p(i) \text{ bits/second} \tag{5}$$

and if the total information in the message is desired, by nT to yield:

$$H = -nT \sum_{i}^{N} p(i) \log_2 p(i) \text{ bits} \tag{6}$$

Random music as defined by Hartley's equation is "chaos" or "totally disorganized" music. Conversely, "totally organized" music, in our simple system developed from pitch choices only, is the single and unique choice of some one term from the repertory of symbols. Thus, if the probability of C is unity, i.e., a certainty, such that p(C) = 1, then all other p(i) must be zero and

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$$H = 1 \log_2 1 + 0 \log_2 0 + \ldots = \log_2 1 = 3.32 \log_{10} 1 = 0$$
bits/symbol

Thus, our scale of information content extends from maximum information content for total disorder down to zero information for total order. Conversely, if we define the redundancy of any message as:

$$R = \frac{H_{max.} - H}{H_{max.}} \times 100\% \qquad (7)$$

by this definition we have a measure of predictability that varies from 100% for total order to 0 % for total disorder.

Most music is obviously neither totally disorganized nor totally organized but falls somewhere between these extremes. Moreover, most music not only has an average information level somewhere between chaos and total order, but also usually has increases and decreases of information during its time duration. This factor is undoubtedly an important element in structural expectations, resolutions, intensities, and so on, as has already been observed by Meyer. *10 It is the exceptional composition that maintains constant information content from start to finish. Among the most obvious examples are, once again, random music and also certain types of highly predetermined music like "totally organized" serial music. As long as the repertory of symbols from which such music is constructed is maintained constant, this music remains invariant on the average from one section to the next. Every portion of it is statistically identical with every other portion, so it may be presented in any sequence and reshuffled, played backwards, and so on, without changing any part of its most essential characteristic.

In general, the concepts of information and redundancy seem most interesting as potential tools for the study of musical structures because of their resemblance to dipoles such as disorder-order, variety-unity and balance-imbalance. In this connection, a statement of Joseph Schillinger, formulated before the publication of The Mathematical Theory of Communication, is provocative:

> The behavior of sounding texture in any musical composition is such that it fluctuates between stability and instability, and so remains perpetually in a state of un-

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stable equilibrium. . . . Unstable equilibrium is a manifestation of life itself, and, being applied to the field of musical composition as a formal principle, contributes the quality of life to music. *16

III. The Method of Analysis

To test these ideas, we chose four examples of music taken from the piano sonata literature:

1. Mozart, Sonata in C Major, K. 545 (1788);
2. Beethoven, Sonata in E Minor, Op. 90 (1814);
3. Berg, Sonate (in B Minor), Op. 1 (1908; revised 1920);
4. Hindemith, II. Sonate (in G Major) (1936);

and subjected them to elementary information theory analysis. Since stylistic disparity is characteristic of this group of examples, we could thus evaluate how this quantitative measure might apply to the study of a tonal structure as used over a wide historical span. The investigation was further limited to the analysis of the pitch distributions in only one comparatively large formal unit, the exposition of the first movement of each of the sonatas in question.

The set of elements from which all four "messages" were drawn was the same, namely, the twelve notes of the well-tempered chromatic scale, assuming octave equivalence. However, we also analyzed the selected examples as deriving from a 21-note set in which the sharp, flat, and natural forms of each letter name were considered to be different elements. Although it is often difficult in highly chromatic music to distinguish which spelling of a note is the correct one, we nevertheless thought that by using this 21-note set we could take into account at least some differences in function between notes that are the "same" but are spelled differently, such as C♭ and B, E♭ and D♯, and so on.

Next, we subdivided each of the expositions at phrase endings or other natural dividing points. This sectionalizing of the four expositions by phrase demarcation was carried out so that changes in information content per symbol could be followed as the pieces progress in time. We proceeded to count the notes in these phrases in two ways, labelled Count A and Count B. Both counts were made on the basis of a 12-note and a 21-note set, so that a total of four different counts was made in all. In Count A, notes were tabulated without regard to duration; that is, only the number of attacks was counted. In Count

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TABLE 1

AN EXAMPLE OF INFORMATION CONTENT CALCULATIONS EMPLOYING COUNTS A AND B, 12-NOTE BASIS OF MOZART'S SONATA IN C, K.545

A. Actual note count:

Measures	1-4		5-12		13-17		18-21		22-26		26-28	
Notes	A	B	A	B	A	B	A	B	A	B	A	B
C	10	29	21	44	10	10	9	18	24	48	4	10
C♯	0	0	2	2	4	4	0	0	0	0	0	0
D	3	5	19	33	42	44	11	17	9	24	7	16
D♯	0	0	0	0	0	0	0	0	1	1	0	0
E	8	19	20	32	0	0	8	11	17	35	0	0
F	4	7	18	45	0	0	0	0	0	0	0	0
F♯	0	0	1	2	4	10	7	10	4	4	2	2
G	17	40	29	48	6	22	10	16	8	15	15	42
G♯	0	0	0	0	0	0	0	0	2	1.5	0	0
A	3	12	16	37	8	10	8	17	8	33	4	10
A♯	0	0	0	0	0	0	0	0	0	0	0	0
B	2	8	17	24	26	28	11	23	13	16.5	10	18
	47	120	143	267	100	128	64	112	86	178	42	98

B. Probability tables (Measures 1-4 only)

COUNT A

Measures

Notes	p(i)	1/p(i)	log 1/p(i)	3.32log 1/p(i)	$p_i$log 1/p(i)
C	10/47	4.70	0.67	2.23	0.475
C♯	0	—	—	—	—
D	3/47	15.66	1.20	3.97	0.254
D♯	0	—	—	—	—
E	8/47	5.88	0.77	2.55	0.435

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B. Probability Tables (Measures 1-4 only) – Continued

F	4/47	11.75	1.07	3.55	0.302
F♯	0	—	—	—	—
G	17/47	2.76	0.44	1.47	0.532
G♯	0	—	—	—	—
A	3/47	15.66	1.20	3.97	0.254
A♯	0	—	—	—	—
B	2/47	23.50	1.37	4.56	0.194

H_N = 2.446 bits/symbol

COUNT B

C	29/120	4.14	0.62	2.05	0.495
C♯	0	—	—	—	—
D	5/120	24.00	1.38	4.58	0.191
D♯	0	—	—	—	—
E	19/120	6.32	0.80	2.66	0.421
F	7/120	17.15	1.23	4.10	0.238
F♯	0	—	—	—	—
G	40/120	3.00	0.48	1.58	0.527
G♯	0	—	—	—	—
A	12/120	10.00	1.00	3.32	0.332
A♯	0	—	—	—	—
B	8/120	15.00	1.18	3.90	0.260

H_N = 2.464 bits/symbol

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B, durations of the notes were determined according to a scale of one sixteenth-note equal to one unit. Thus, a quarter note was given a weight of 4, a dotted quarter note a weight of 6, and so on. In three of the sonatas, there are rhythmic subdivisions smaller than a sixteenth note, but they occur only occasionally and were disregarded.

After all note occurrences in each exposition subsection were tabulated, we could proceed directly to the calculation of frequencies of note occurrences and, in turn, of information contents measured in bits per symbol (i. e., bits per note). For this second calculation, we employed Equation (4), using the computed frequencies of note occurrences for the probabilities, p(i) = p(C), p(C♯), . . . p(B).

Next, the total information content of the symbols in each subsection was found by Equation (6). In this equation, the product nT is, once again, the number of notes transmitted in each section. Then, to find the total information content in each complete exposition, we simply added the information values of the subsections. An average information value, H_N, of the symbols in the whole of each of the expositions was next calculated. This quantity was found by dividing the total number of bits by the total number of notes. This result is a standard against which fluctuations of information associated with notes in a given formal unit can be measured.

We computed redundancy figures by means of Equation (7), starting with H_N for the complete expositions. The maximum information for the 12-note set of symbols is 3.58 bits per symbol, and for the 21-note set, it is 4.39 bits per symbol, as computed by means of Equation (1).

Finally, we calculated rates of information transmission, H_T, by finding the number of notes transmitted per second in each subsection and then multiplying this figure by the information content per note already found in each subsection by means of Equation (4). The average rate of note transmission for each subsection was found by dividing the total number of notes in each subsection by the elapsed time of the subsection. An analogous computation was also carried out for the four complete expositions, to yield four average information rates for these complete expositions against which fluctuations in information rate could be measured.

An example of how these calculations were carried out is shown in detail in Table 1. A complete record of these calculations

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has been provided elsewhere. *17

IV. Analytical Results

A. Statistical data. The results of these calculations are given in Tables 2 through 5, one table for each musical example. In the first column of both Parts 1 and 2 of the tables are listed the following formal units as indicated by Roman numerals: I includes the first subject; II, the transition from first to second subject; III, the second subject; IV, the extension of second subject material, if any; and V, the closing theme, or codetta, if any. It is possible that groups III and IV need not have been separated, but their separation seemed more useful than their combination for the purpose of this analysis. Part 1 of each table includes the information content, H_N, in bits per symbol, the corresponding redundancy, R, given as percentage, and the rate of information transmission, H_T, in bits per second, as determined on the 12-note basis for both Counts A and B. Part 2 includes the same results calculated on the 21-note basis. These values are given in Columns 3, 4 and 5, respectively, of each table for subsections indicated by measure numbers in column 2. At the ends of each of these columns are listed the average information content, H_N, its corresponding redundancy, and the average rate of information transmission, H_T, respectively.

In the case of the Mozart exposition only one set of calculations of information contents was actually necessary because the only accidentals which occur in this example are written and function as sharps. Thus, the number of symbols for which information values were calculated is the same on both the 12- and the 21-note bases. Redundancy percentages differ between the two bases, however, because the two maximum information contents differ.

In the Beethoven exposition, the second subject is not extended (no Section IV), while in the Hindemith exposition, there is no closing theme (no Section V), but rather a direct continuation into the brief development section. In the Mozart exposition, mm. 18-26 might have been treated as one unit, as were mm. 5-12. A striking textural change, between mm. 21 and 22, is the reason for not having done so. In the Hindemith example, mm. 1-26 might have been treated as one unit, but division into two sections was made because of the reharmonized statement of the principal thematic material of the first subject, beginning and ending in the tonic, starting with m. 17. In the Berg Sonata exposition, there is more overlapping of phrase beginnings and

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TABLE 2

MOZART SONATA IN C MAJOR, K.545

Formal units	Phrases by measures	H_N (Bits per symbol)	R (Percentages)	H_T (Bits per second)
PART 1: 12-note basis				
COUNT A: Strike tones only				
I	1-4	2.45	31.7	13.4
II	5-12	2.89	19.4	23.7
III	13-17	2.27	36.7	26.5
IV	18-21	2.79	22.2	20.7
	22-26	2.78	22.4	25.8
V	26-28	2.31	35.5	15.9
Average values:		2.63	26.6	21.2
COUNT B: Tones weighted for duration				
I	1-4	2.46	31.2	34.5
II	5-12	2.86	20.2	44.5
III	13-17	2.46	31.4	36.7
IV	18-21	2.76	23.0	36.1
	22-26	2.65	26.0	50.8
V	26-28	2.19	38.9	32.8
		2.62	26.9	39.4

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PART 2: 21-note basis

COUNT A: Strike tones only

I	1-4	2.45	44.2	13.4
II	5-12	2.89	34.2	23.7
III	13-17	2.27	48.3	26.5
IV	18-21	2.79	36.5	20.7
	22-26	2.78	36.7	25.8
V	26-28	2.31	47.3	15.9
Average values:		2.63	40.0	21.2

COUNT B: Tones weighted for duration

I	1-4	2.46	43.8	34.5
II	5-12	2.86	34.8	44.5
III	13-17	2.46	44.0	36.7
IV	18-21	2.76	37.1	36.1
	22-26	2.65	39.6	50.8
V	26-28	2.19	50.1	32.8
Average values:		2.62	40.3	39.4

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TABLE 3 BEETHOVEN SONATA IN E MINOR, OP.90

Formal units	Phrases by measures	H_N (Bits per symbol)	R (Percentages)	H_T (Bits per second)
		PART 1: 12-note basis		
COUNT A: Strike tones only				
I	1-8	2.98	16.9	23.6
	8-16	2.83	21.1	10.2
	16-24	2.93	18.2	17.4
II	24-44	3.29	8.2	24.3
	45-54	3.31	7.5	71.8
III	55-66	2.41	32.8	27.3
IV	—	—	—	—
V	67-81	2.70	24.5	11.0
Average values:		3.00	16.2	25.3
COUNT B: Tones weighted for duration				
I	1-8	2.98	16.8	90.5
	8-16	2.79	22.0	73.7
	16-24	2.81	21.6	110.2
II	24-44	3.01	10.3	88.9
	45-54	3.27	8.6	131.4
III	55-66	2.39	33.1	51.9
IV	—	—	—	—
V	67-81	2.70	24.7	83.8
Average values:		2.90	18.7	89.7

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PART 2: 21-note basis

COUNT A: Strike tones only

I	1-8	2.98	32.0	23.6
	8-16	2.88	34.3	10.4
	16-24	2.93	33.2	17.4
II	24-44	3.42	22.1	25.2
	45-54	3.31	24.5	71.8
III	55-66	2.41	45.2	27.0
IV	—	—	—	—
V	67-81	2.70	38.4	11.0
Average values:		3.03	30.8	25.6

COUNT B: Tones weighted for duration

I	1-8	2.98	32.1	90.5
	8-16	2.79	36.2	73.7
	16-24	2.81	36.0	110.2
II	24-44	3.21	26.8	94.9
	45-54	3.27	25.4	131.4
III	55-66	2.39	45.4	51.9
IV	—	—	—	—
V	67-81	2.70	38.5	83.8
Average values:		2.94	32.9	91.1

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TABLE 4

BERG SONATA IN B MINOR, OP.1

Formal units	Phrases by measures	H_N (Bits per symbol)	R (Percentages)	H_T (Bits per second)
PART 1: 12-note basis				
COUNT A: Strike tones only				
I	1-10	3.53	1.5	14.7
II	11-16	3.54	1.2	16.4
	16-28	3.54	1.2	20.3
III	29-36	3.43	4.2	15.4
IV	37-48	3.55	0.9	26.4
V	49-55	3.53	1.4	10.8
Average values:		3.52	1.6	17.8
COUNT B: Tones weighted for duration				
I	1-10	3.51	2.1	48.7
II	11-16	3.50	2.2	55.1
	16-28	3.53	1.4	57.7
III	29-36	3.42	4.4	50.5
IV	37-48	3.54	1.2	68.5
V	49-55	3.52	1.7	32.7
Average values:		3.51	2.1	52.8

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PART 2: 21-note basis

COUNT A: Strike tones only

I	1-10	3.89	11.1	16.3
II	11-16	3.88	11.6	18.0
	16-28	3.81	13.2	21.8
III	29-36	3.57	18.7	16.1
IV	37-48	3.98	9.2	26.6
V	49-55	3.75	14.6	11.5
Average values:		3.83	12.7	19.3

COUNT B: Tones weighted for duration

I	1-10	3.85	12.3	53.1
II	11-16	3.76	14.3	59.2
	16-28	3.73	15.0	60.8
III	29-36	3.62	17.5	53.4
IV	37-48	3.97	9.5	76.7
V	49-55	3.87	11.9	35.9
Average values:		3.81	13.3	57.3

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TABLE 5

HINDEMITH SONATA NO.2 IN G MAJOR

Formal units	Phrases by measures	H_N (Bits per symbol)	R (Percentages)	H_T (Bits per second)
		PART 1: 12-note basis		
COUNT A: Strike tones only				
I	1-16	3.25	9.4	19.4
	16-26	3.36	6.1	18.2
II	26-40	3.38	5.7	23.6
III	41-55	3.45	1.0	23.6
IV	36-63	3.28	8.3	27.4
V	—	—	—	—
Average values:		3.35	6.3	21.1
COUNT B: Tones weighted for duration				
I	1-16	3.19	11.0	68.6
	16-26	3.02	15.7	57.1
II	26-40	3.32	7.2	64.8
III	41-55	3.44	3.8	69.8
IV	56-63	3.17	11.5	80.0
V	—	—	—	—
Average values:		3.25	9.2	67.4

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PART 2: 21-note basis

COUNT A: Strike tones only

I	1-16	3.34	23.8	20.0
	16-26	3.45	21.4	18.3
II	26-40	3.44	21.5	24.0
III	41-55	3.62	17.5	24.8
IV	56-63	3.43	21.9	28.6
V	—	—	—	—
Average values:		3.41	22.4	22.5

COUNT B: Tones weighted for duration

I	1-16	3.26	25.7	70.2
	16-26	3.07	30.1	57.9
II	26-40	3.39	22.6	66.2
III	41-55	3.62	17.6	73.3
IV	56-63	3.31	24.5	83.6
V	—	—	—	—
Average values:		3.35	23.6	69.4

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endings than in any of the other examples. The measure groupings, then, are not always precise indications of each phrase length, but they are within one or two beats of being so.

B. H_N and R Results

1. Fluctuation of information contents. If we compare the figures in the H_N and R columns in Tables 2 through 5, we see that there is little common ground among the ranges of information fluctuation from one sonata exposition to another. This is generally true of Counts A and B, on the 12- and the 21-note basis, although there is a slight sharing of values between the Beethoven and the Hindemith examples.

There is a decided increase in average information contents from the Mozart to the Beethoven example, from the Beethoven to the Hindemith example, and from the Hindemith to the Berg example. In the order just listed, there is consequently a corresponding decrease in redundancy from the Mozart through the Berg examples.

A comparison of the extreme low and extreme high figures in Count B, on the 21-note basis, also illustrates this. In the Mozart exposition, Section V has an information content of only 2.19 bits per symbol, with 50.1% redundancy; in the Berg Sonata exposition, Section IV has information content of 3.97 bits per symbol, with 9.5% redundancy. In the same section of the Berg Sonata, an even more striking instance of high information content is shown in Count A on the 12-note basis: the information content is 3.55 bits per symbol (compared to a possible maximum of 3.58 bits per symbol for random music), and the corresponding redundancy is only 0.94%.

As an index of the differences in the amounts of chromaticism that are found from example to example, the 21-note counts are particularly revealing. None of the phrases of the Mozart example and only two phrases of the Beethoven example draw upon the larger symbol set. However, all phrases in the two twentieth century sonatas draw upon the 21-note set, so there is a noticeable difference between the information contents calculated from it and the smaller set.

Of the Berg and Hindemith sonatas, the former has the more nearly equal distribution of symbol choice, each phrase but one, the second subject in Count A, having an information value on the 21-note basis higher than the possible maximum for random music based on a 12-note set (3.58 bits per symbol).

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Only once in the Hindemith Sonata, on the other hand, does a phrase have a higher value on the 21-note basis than the possible maximum for the 12-note set. This phrase is the second subject, which has a comparatively large information value in both Count A and Count B.

This difference between the two modern sonatas is evident too in the 12-note counts, in that the information content of many phrases in the Berg example approaches the level of an almost equally likely symbol choice, while there is a consistently lower level in the Hindemith: no phrase has an H_N value over 3.45 bits per symbol, but in the Berg Sonata, only one phrase has an H_N value lower than 3.5 bits per symbol.

These patterns of information fluctuation, as given in the H_N columns of Tables 2 to 5 are also shown in Figures 1-4. There is one graph each for Count A and Count B on both the 12- and 21-note bases. The general contour of fluctuation from one graph to another remains similar for each musical example.

2. Comparison of the four expositions. In the Mozart example, similarities of contour from one count to another are stronger than dissimilarities. This similarity of contour from one count to another is true also of the Beethoven example. The amount of variation in Section I is less in Counts B than in Counts A. Of the three phrases in that section, the second has the lowest information content of all, and the third has somewhat lower information content than the opening phrase. There is one difference in the pattern of information-content change from the fourth to the fifth phrase, noted only in Count A on 21-note basis: a slight decrease occurs, instead of a slight increase.

The existence of greater amounts of information in transitional material than in thematic statements is reasonable on the basis of our musical experience. There is a sense of completeness associated with thematic statement. A definite change of texture and a lack of clear thematic profile tends to arouse a sense of expectancy in the listener. The emergence of a new thematic statement or the return of a familiar one satisfies this sense of expectancy.

The change in information content between Sections II and III further substantiates this generalization. Not only is there a relatively great decrease at this point, but we find a drop to the lowest information quantity found in all calculations to be observed in the Beethoven example and also in calculations

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FIGURE

1

COUNT A (12-NOTE BASIS)

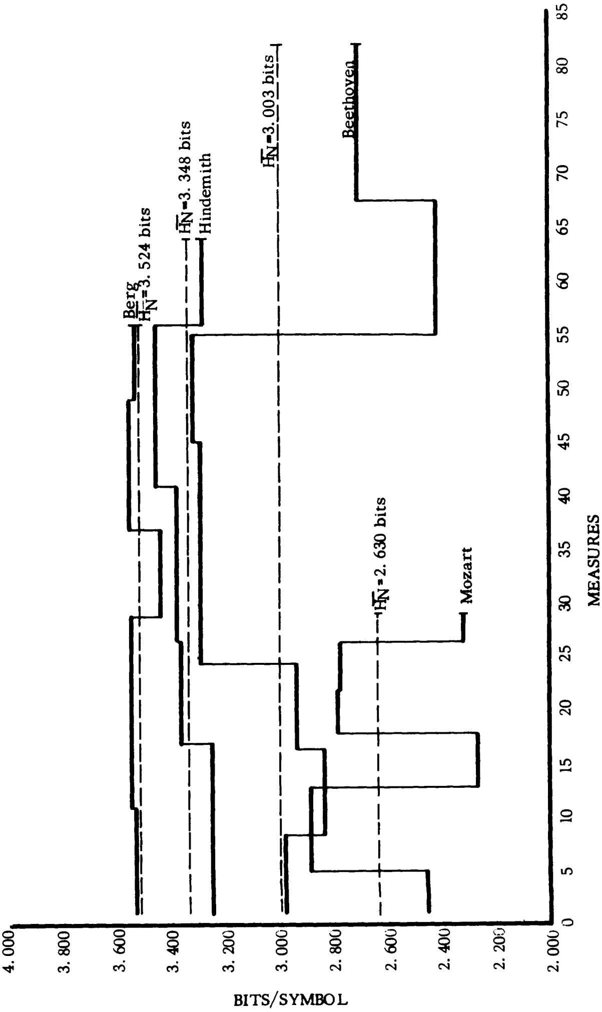

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FIGURE

2 COUNT B (12-NOTE BASIS)

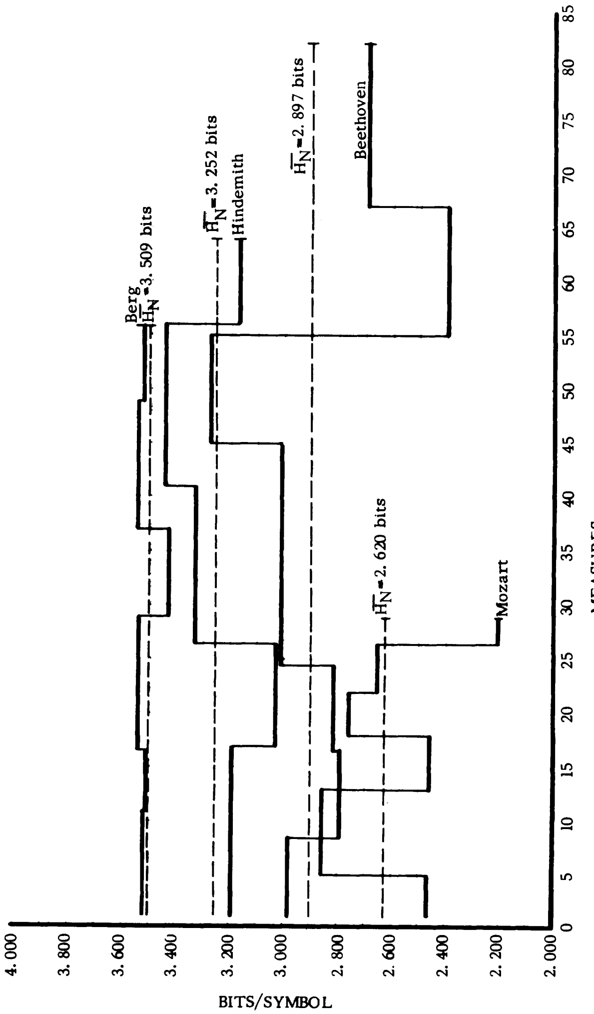

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FIGURE 3 COUNT A (21-NOTE BASIS)

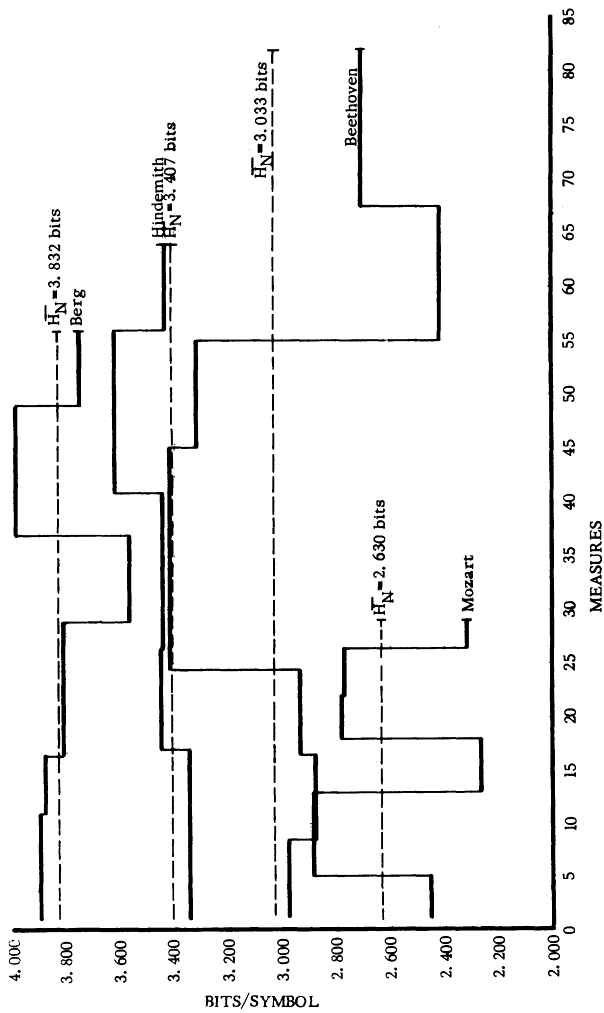

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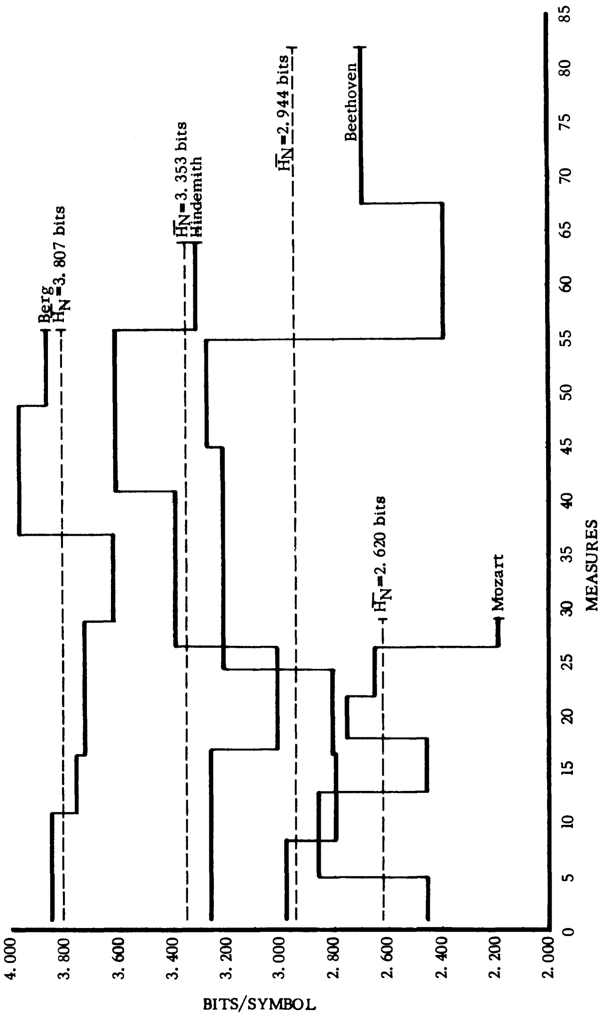
FIGURE

4 COUNT B (21-NOTE BASIS)



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based on Count A in the Mozart example.

Recalling the descriptive terms we have been using for information and redundancy, we could say that the second theme is more stable and less uncertain than the first theme, and also, of course, than the transition to it from the first theme. This statistical result, applied to the relationship of first and second themes as they are tonally based, indicates a greater weighting of the key center of the second theme. In each of these expositions, the key center of the second subject is the dominant, and in each of them the dominant note occurs more frequently, on the average, than does the tonic note itself, from the beginning of the piece. It seems, at least in these examples, that the first subject does more to prepare the dominant than to stabilize the tonic as a focal point of the exposition section.

Turning to the other pair of sonata expositions, we see that from Section I to Section II in the Berg Sonata exposition, the contour of change determined on the 12-note basis in Counts A and B is rather similar to that noted between the corresponding sections in the Mozart and Beethoven examples. However, there is a small decrease instead of an increase of information content on the 21-note basis between Sections I and II. This is true also of the contour change between phrases within Section II. Another noteworthy point is that whether the changes are to greater or lesser quantities, the amounts of change in either case are considerably smaller in this example than were the amounts of increase from Section I to II in the Mozart and Beethoven expositions. Finally, the second subject of the Berg Sonata has a lower information content in all of the calculations than that of the preceding phrases, and this is consistent with the pattern of the earlier examples.

It is only in the Hindemith exposition that a different contour emerges. The second subject has the highest information content of all the sections in the exposition, while the first subject, in Counts A, has the lowest information content. In Counts B, however, there is a decrease in information amounts from the first to the second phrase within Section I (the first subject), and then a subsequent increase in the third phrase, Section II. The average information content for the two phrases making up Section I is nevertheless lower than that for Section II, so that we can say that the direction of the change from Section I to II is ultimately the same in Counts B as that noted in Counts A.

The greater stability of Section I, as compared to those follow-

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ing, can be partially explained by a pedal point on the tonic note for the first six measures, next on B, mm. 13-15 1/2, and then on the tonic G, mm. 17-22. The greater redundancy of the second phrase of Section I noted in Counts B can be attributed to the stabilizing influence of metrical patterns.

In neither of the two modern examples does the dominant function as the tonic of the second subject as it does in both the Mozart and Beethoven sonata expositions. In the Berg sonata, the subdominant, E, functions as temporary tonic for the second subject. The seventh scale step, F, serves in that capacity in the Hindemith exposition. There is the further difference, in the latter example, of a modal change from major for the first subject to minor for the second subject. There is much less abrupt shifting in frequencies of note occurrence from section to section in the Berg example, and, as might be surmised from its generally high information level, there are more equal note occurrences than in the other examples.

Motion from the second subject onward is opposite to that which led into the second subject in all cases. Thus, there is an increase of information after the second subject statement in the Mozart, Beethoven, and Berg examples, and a decrease of information in the Hindemith example, where there is repetition of thematic material — repetition varied by modal and textural changes.

In the Mozart and Berg examples, new thematic material and a more elaborate texture in Section IV follow the second subject statement. In the Beethoven example, the closing subject, Section V, immediately follows Section III, without intervening extension of the second subject. The amounts of information in bits per symbol in the final section of the Mozart Sonata exposition are the lowest found for all phrases, as calculated from Count B. As calculated from Count A, however, the amounts of information in the final section are not as low as those of the second subject. An explanation of this difference in amounts of information in the final section between Counts A and B is the higher degree of uniformity in the rhythmic patterns of this codetta — an elaborated authentic cadence with a broken-chord figure as melodic motive — as contrasted to the more varied rhythmic form of the second subject. In the Berg Sonata, the final section of the exposition has a higher information content than that of the second subject in all calculations, while the decrease in information from Section IV to Section V is similar to that pattern noted in the Mozart Sonata at this formal point.

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3. Differences between Count A and Count B. The separation of counts on the basis of strike-tones and time durations points up the general tendency of rhythmic values to lower amounts of information. However, in Section I of both the Mozart and Beethoven examples, exceptions to this generalization do occur. The H_N value determined from Count B is larger than that determined from Count A, although the differences are slight, particularly that in the Beethoven example (Tables 2 and 3).

Such a difference suggests an "unusual" distribution of rhythmic values. One influence might be the use of rests as an important means of articulating a musical idea. In any event, the calculation of information contents of the rhythmic values themselves would presumably reveal how this slight difference arose.

4. Tonality. How the process of tonal motion becomes more blurred when we proceed from the Mozart through the Beethoven and Hindemith to the Berg example is strikingly demonstrated in the comparisons of information and redundancy values. The approach to randomness of symbol choice noted in the Berg example reveals the extent to which indefiniteness of tonal progression (instability of equilibrium) was an effect intended to disguise the direction of that motion. The considerable redundancy noted in the Mozart example illustrates the opposite tendency, that is, the use of definiteness of tonal progression (stability of equilibrium) to emphasize the direction of that motion.

C. H_T results. The measurement H_T of the rate of transmission of information can serve as a guage of the texture or density of a musical structure. It reflects varying degrees of harmonic thickness and of contrasting rhythmic values. In Figures 5 through 8, we have presented graphically the values of H_T tabulated in Tables 2 through 5. There is one graph for Counts A and B on the 12-note and the 21-note basis in Figures 1-4.

Of all the sonata expositions analyzed, the Mozart shows the smallest range of information-rate variation in Count B, and the next smallest in Count A. There is also the least difference in the rates of transmission between Count A and Count B calculated for each example. In Count A the second theme has the highest rate of information transmission of all the phrases in the exposition, while this same section has the lowest information content per symbol. The larger number of strike tones that occurs per second in this theme, compared to those occurring in the other sections, explains this increase in the

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FIGURE 5

COUNT A (12-NOTE BASIS)

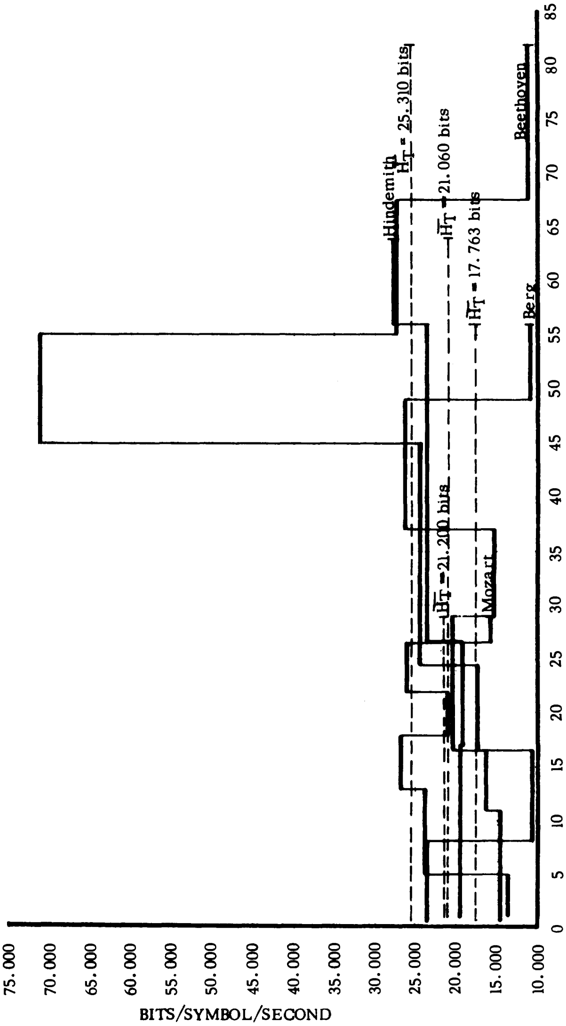

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FIGURE 6

COUNT B (12-NOTE BASIS)

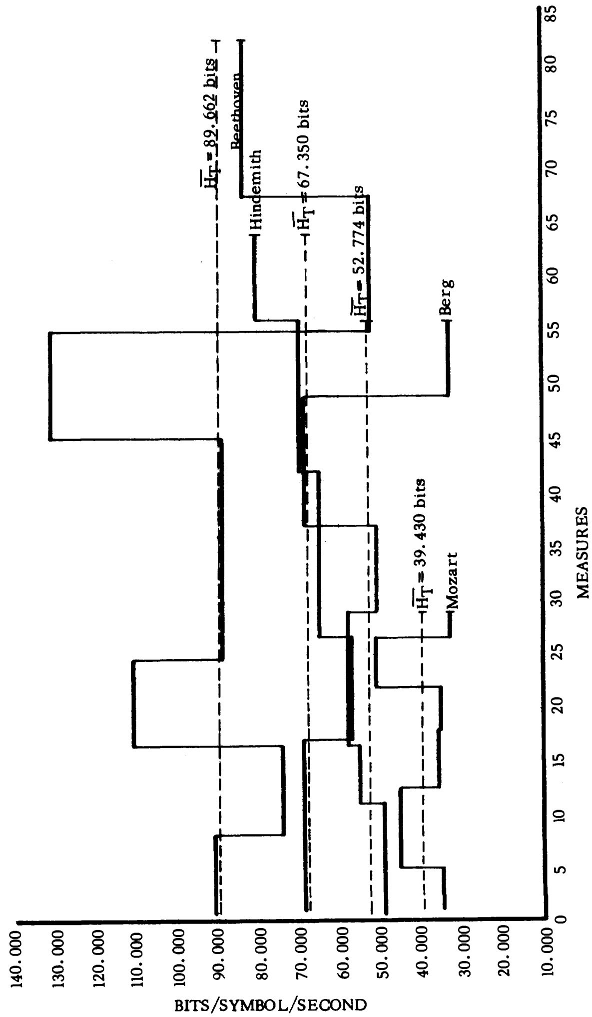

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FIGURE 7

COUNT A (21-NOTE BASIS)

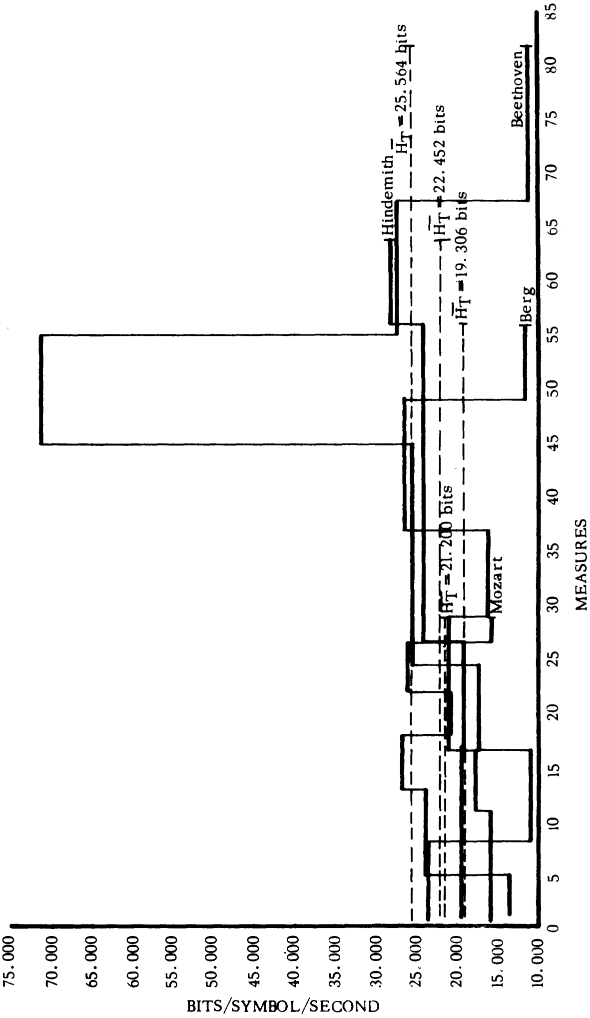

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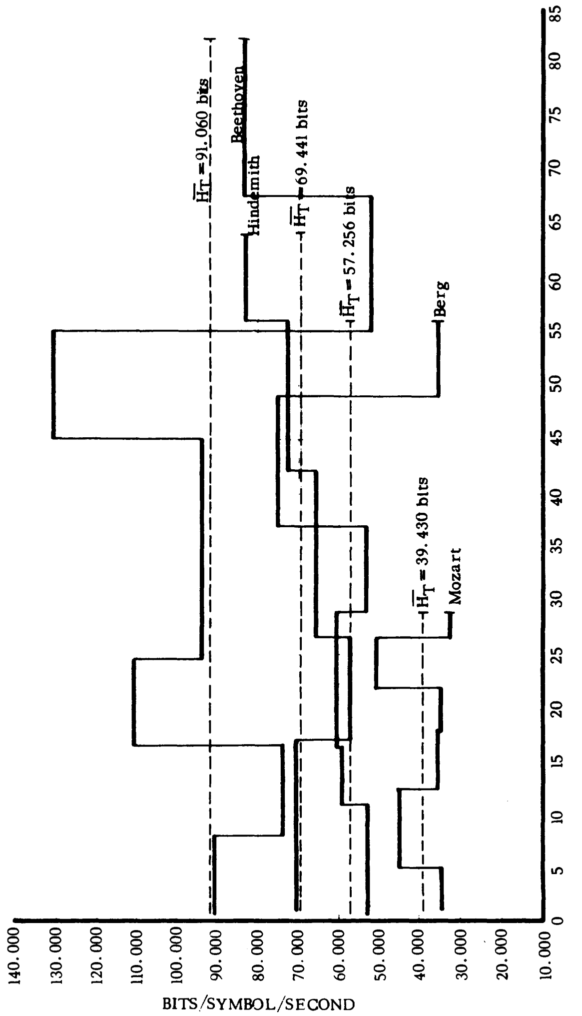
FIGURE

8 COUNT B (21-NOTE BASIS)



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rate of information transmission. In Count B, however, the H_T value determined for this section is a little less than the average, as is the H_N value, for this section. This difference from Count A is explained by the fact that the accompaniment figure has only one rhythmic value, the sixteenth note, and this lack of durational variety offsets the large number of strike tones. Any extended rhythmical sameness generally tends to increase redundancy and correspondingly to decrease information, since repetition of one rhythmic pattern over a period of time is a control on the freedom of choice of rhythmic values.

Consistent with results obtained from the H_N calculations for the Beethoven Sonata is the fact that the greatest variation from section to section in the rates of information transmission also occurs in this example. The greatest amount of information in bits per second is found in the phrase just preceding the second subject statement, which section also had the highest H_N value.

Low amounts of rate and content do not coincide. In Count A the rate of information transmission of the closing theme is lower than that of the second theme, on both the 12- and the 21-note bases, but in Count B the situation is reversed. There is constant use of a sixteenth-note broken-chord figure as accompaniment to the melody in octaves and single notes of the second subject, while a chordal texture predominates in the closing theme, in which there is a more varied use of rhythmic values, especially of many sustained tones.

Fluctuation of the rate of information transmission from section to section in the Berg sonata follows the contour observed for the fluctuation of information content. Increases and decreases of amounts occur at the same formal points. This is true of both the 12- and the 21-note bases. Only in this example does such a similarity of the contour of fluctuation between content and rate hold throughout. This interesting singularity of the Berg Sonata, as compared with the other examples, emphasizes a strong characteristic of its musical texture. The large variety of rhythmic values that occur in the piece is used in the same way as are pitches: they generally persist throughout, rather than emerge strongly in one section or another.

As in the Beethoven example, the highest H_N and H_T amounts coincide in the Berg example, but the lowest amounts do not. Variation between the two Counts B is greater than in the Hindemith example, but variation between the Counts A and B is less than that noted between those counts in either the Beethoven or

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Hindemith example, as was also true for the variation in H_N amounts.

Finally, in the Hindemith sonata, the fluctuation of information rate remains similar from one count to another, with the slight exception occurring in Count A on the 12-note basis between Sections II and III. The main difference between the patterns of fluctuation of information content and of rate is noted between Sections III and IV: in the H_N values, there is a decrease between these sections, while in the H_T values there is an increase. A four-voice harmonic texture is constant in Section IV, but in Section III there was variation from one- to five-voice texture, with two- and three-voice textures being most frequently used.

V. Conclusions

Statistical methods of musical analysis are often regarded as academically interesting at best or unmusical at worst because such results are frequently presented as complete and revealing in themselves and are only infrequently employed to support farther-reaching investigations into musical processes. Here we shall attempt to apply the results so far obtained, admittedly of narrow significance in themselves, to a few broader theoretical considerations.

What seems most promising in the analytical methods suggested by information theory is that the fluctuation of elements that creates a musical structure might be judged from a basis applicable to any musical style and, moreover, given a meaningful quantitative measure. The results of such analysis can thus be expressed in terms independent of historical context and unweighted by qualitative connotations. On this level of the "technical problem" of communication, we might well develop unequivocal bases for comparing significant aspects of various stylistic practices and thus be able to proceed from this level to semantic and esthetic levels of communication, those most pertinent to a full evaluation of any work of art. Even the fluctuations in information contents at the simple level of analysis presented here are interesting by comparison with the order-disorder scale represented by the range from a completely fixed choice of symbol (complete certainty) to an equiprobable choice of symbols (maximum uncertainty). Toward the upper limit of this scale, there is wider use of the potentialities of a given symbol set, so that the message becomes correspondingly more complex, less predictable, and less unmistakable. The converse is also true: the use of fewer sym-

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bols lessens the complexity of a message and increases its definiteness, its unmistakability.

In the Mozart, Beethoven and Berg expositions, similar contours of information fluctuation occur. Differences between amounts of variation in the Mozart and Beethoven examples, though not great, are yet noticeable. By way of contrast, though, there is a considerable difference between the amounts of variation noted in these expositions and in that of the Berg Sonata.

Three generalizations about musical styles seem to be supported by these results:

(1) The broadening of the pattern of information fluctuation noted in the Beethoven exposition compared to the Mozart graphically emphasizes the richer development by Beethoven of earlier established musical procedures.

(2) The extreme contour of information flow noted in the Beethoven sonata exposition substantiates the common view that his style is characterized by highly dramatic contrasts. The kind of surprise most often associated with his technique is of a dynamic or accentual kind. While this investigation does not account at all for dynamic changes or accents, except through durational values, it nevertheless underlines this aspect of his music.

(3) The more even pattern of information fluctuation in the Berg exposition seems indicative of a practice weakened almost to the vanishing point. Berg seems to be following classical procedure while attempting to disguise it as much as possible. This disguise depends on the comparatively high equality of symbol choice, which does not allow a strong enforcing of a tonal center through tonal repetition.

In this connection, it is interesting to recall comments made by Schoenberg on the tonal process:

> My school, including such men as Alban Berg, Anton Webern, and others, does not aim at the establishment of a tonality, yet does not exclude it entirely. The procedure is based upon my theory of the "emancipation of the dissonance". Dissonances, according to this theory, are merely more remote consonances in the series of overtones. Though the resemblance of the more remote overtones to the fundamental tone gradually diminishes,

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> their comprehensibility is equal to the comprehensibility of the consonances. *18

Berg was not excluding tonality, but its affirmation was comparatively faint. Only mm. 1-3, 29-33, and 53-55, in which the exposition is repeated, are exceptions in that they emphasize a tonic feeling by authentic cadences. These measures coincide with theme beginnings.

The contour of information fluctuation in the Hindemith sonata exposition is sharper in contrasts than the Berg, but it follows a different course than the other examples considered. It is tonally clearer than its chronological predecessor, but it gives evidence of a structural procedure that differs from the traditional, at least as revealed within the quantitative limits of study.

In his ordering of the harmonic materials of music, Hindemith proposed a hierarchy of chord values determined by the amount of consonance in sonorities. The most "valuable" chords are the major and minor triads, "noblest of all chords". Other groups and subgroups of chords are described qualitatively as they are ranked in a table of chord-groups. The term "harmonic fluctuation" denotes motion from one sub-group to another sub-group. In this process, there is a "shift of harmonic gravity". Hindemith states that:

> The step from a more valuable to a less valuable chord is in the harmonic sense, then, a descent, a fall, and conversely a step from a less valuable to a more valuable one is an ascent. But since in our chord-table the harmonic tension of chords increases from section to section and from sub-group to sub-group in the same proportion as the value decreases, the progression from a higher to a lower chord represents an increase in tension, and a step in the opposite direction a decrease. It is this up-and-down change of values and tensions which we shall term harmonic fluctuation. This fluctuation may be gradual or sudden according to the relative values of the chords which make up the progression. *19

Similarities between information-theory and Hindemith-theory terminologies are apparent. There are, of course, many differences between this notion of harmonic fluctuation and shifting information-redundancy relationships, particularly since the Hindemith theory is obviously more special than the musical application of information theory. Hindemith posits a defi-

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nite set of elements and rules for their use, while information theory concepts can be used to deal with any sets of elements and without assuming restrictions in their use. The similarity consists in the "forces" with which these two theories are concerned. Tension is roughly analogous to information, both factors relating to uncertainty or disorder, while value is roughly analogous to redundancy, both factors relating to certainty or order. (Hindemith's use of the word "value" in this context seems unfortunate, to say the least.)

Hindemith, concerned principally with defining a set of symbols, prescribes few means for broader structural processes. For example, Landau notes that:

> Hindemith did not clearly recommend specific practices in the fluctuation of chord values; neither did he disapprove of others. . . it hinges completely on expressive purpose, which could conceivably justify any plan of fluctuation no matter how haphazard or arbitrary it may appear to be. *20

The "plan of fluctuation" that begins to reveal itself in our analysis of Hindemith's Second Piano Sonata exposition seems to be not at all haphazard. There is a steady increase in information per section from the beginning to the second theme, after which there is a decrease, as that theme is restated in modified form. The element of "surprise" is more strongly associated with the second theme in this work than in the other examples. Its tonal area is less prepared than those of the other examples. The distinctiveness of this "plan of fluctuation" compared to the other examples seems to be that it depends basically on shifting tonal emphases as the section progresses, not on marked increase of harmonic tension.

The idea of fluctuation of musical materials as form-generating thus is one that persists in recent music theory. What is "good" or "bad" fluctuation is a problem that is recognized as highly pertinent to musical structures in which any chord-forms, or tonal aggregates, might be found. Besides the Schoenberg, Hindemith and Schillinger considerations of this problem, Felix Salzer has extended the theory of Schenker to the analysis of musical literature of periods before and after those with which Schenker himself was concerned and in which he believed the best musical structures were created. Schenker's main contribution to music theory deals precisely with the problem evaded by Hindemith, namely, how it is that large coherent musical forms are assembled from small components parts

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(Schenker's notion of an Ursatz). This raises the question of whether information theory calculations can be extended beyond the independent choice processes taken as a model for the present analysis. The answer is that indeed they can be so extended.

Our present analysis is limited to the structural models based on what are called zeroth-order and first-order stochastic processes. By definition, a zeroth-order approximation to some message is simply that generated by the random choice process governed by equiprobable choices that we have already discussed. A first-order approximation to the same message consequently is based on a probability distribution that is derived from frequencies of occurrence of the symbols but does not account for ways in which one particular choice might affect subsequent choices. The four musical examples examined in the present analysis were thus treated as if they were generated by first-order stochastic processes and the results must be strictly interpreted within this limitation.

The next step in applying information theory then is the consideration of the time domain, i. e., the long-range structure of music. We can treat this problem with "conditional" or "transitional" probabilities of the general form, $p_i(j)$, $p_{ij}(k)$, etc., where $p_i(j)$ means the probability of event j given previous event i, $p_{ij}(k)$ means the probability of event k given previous events i and j, and so on. These probabilities can be entered into Shannon's equation just as were simple probabilities, and information contents can be calculated as before.

A typical simple conditional probability calculation might involve computing second-order probabilities for pitches in position j in a musical composition, the pitch just preceding at point i, having already been chosen. For this, we obtain 12^2 or 144 conditional probabilities for a system derived from the twelve tones of the ordinary chromatic scale. Similarly, to compute third-order $p_{ij}(k)$, we require 12^3 or 1728 such values. Furthermore, we can calculate non-transitional probabilities, p(ij) for note-pairs or digrams selected independently as such, also for note trigrams, and so on. We can also consider other musical elements such as rhythm, note duration and instrumentation, and we can consider any combination of these elements. Clearly, the possibilities become enormous in number, and the amount of calculation so overwhelming that the only practical way of handling the problem is with an electronic digital computer. As described in a recent article *21, we have written a computer program to handle these types of calculations. We have recently also employed this program for

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musical analysis and plan to report some of the results so obtained in a future publication.

The measurement of the rates of information transmission seems not to be so immediately applicable to established analytical methods, but then established analytical methods are weakest precisely in dealing with the handling of musical time. It is only stating the obvious to point out that we possess no theory of duration, tempo and rhythm comparable to harmonic theory. Thus, the measurement of information rate may eventually provide more useful data than measuring information content because it deals with how much activity, i.e., how many notes and metrical values occur within a given time unit, and more importantly how various metrical values are distributed within this time unit. Obviously, one extremely important set of calculations that is required is the evaluation, first, of rhythmic structure by itself and, second, of the interaction of pitch and time elements as they mutually influence one another.

One final word of caution is in order here in presenting results such as those given in Table 2. This has to do with the reliability of the results so presented. It should not be forgotten that these are statistical calculations, which means that the results are quantitatively valid only in relation to sample sizes of the musical passages under examination. In the present study, only relatively small samples were analyzed throughout, which means that the results are probably only good to two significant figures at best and that small differences in information contents or rates are undoubtedly not statistically significant and should not be taken seriously. In this earliest study of the application of information theory to music analysis, we did not attempt to evaluate the precise statistical significance of the results because we were primarily concerned in setting up an exploratory study. We wished first to see what sort of interesting results, if any, might be obtained and, second, to demonstrate clearly how one proceeds in this type of investigation. The results seem to us to demonstrate quite convincingly the utility of information theory as a new and useful tool for music analysis. Consequently, in future publications, we shall show how this type of analysis can be extended to both more detailed and more precise measures of musical structure that will incorporate measures of the statistical significance of the analytical results.

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