INFORMATION THEORY AND MELODY

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INFORMATION THEORY AND MELODY

What is it about simple melodies that makes them so widely appealing? By considering music as a form of communication such questions can now be discussed in mathematical terms

by Richard C. Pinkerton

It is perhaps foolhardy to suggest that a machine might write a poem or compose a song. An artist is likely to exclaim indignantly at the very idea, and a scientist can only approach it with caution. The elusive creative processes of the human mind seem to defy analysis, to say nothing of duplication. And yet it can certainly do no harm to try to find what light we can on the subject. It may, indeed, be quite entertaining, as this article will attempt to show by considering an exercise in the analysis of melody.

Music has always attracted investigation by scientists. The experiments of Helmholtz on the pitch and quality of musical tones are classics in the field of physics. They showed how the choice of scales rests on simple physical relations, why certain combinations of notes sound harmonious and others dissonant, and so on.

Helmholtz described a melody as "a variation of pitch in time." Obviously this definition is only a small part of the story. Not every succession of different notes is a melody. What is it, then, that makes certain sequences of notes sound tuneful—that makes eyes light up and sets feet tapping?

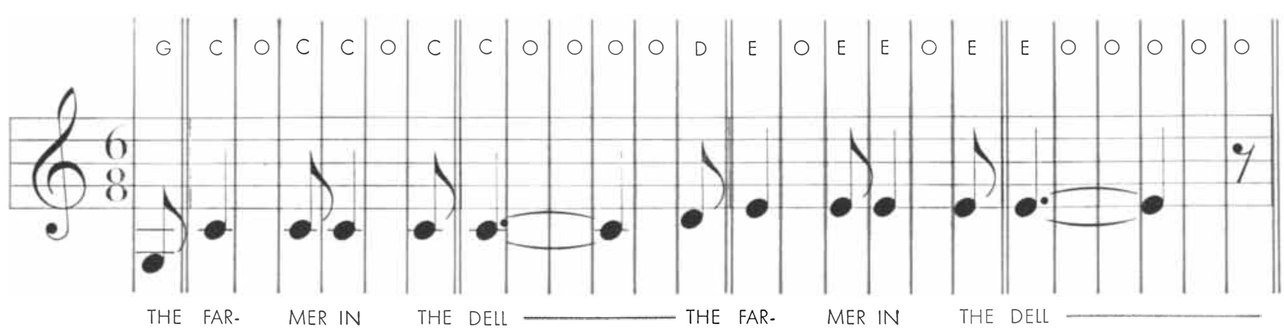
Some years ago the late Joseph Schillinger, who was a teacher of mathematics at Columbia University and also a musician, examined these questions in scientific terms. He was an experimenter who believed that art forms could be analyzed by mathematics and synthesized by "the technique of engineering." Schillinger evolved a mathematical system for composing music, and George Gershwin is said to have used this system while composing *Porgy and Bess*. Our aim in this article is something much less ambitious: we shall consider some simple nursery tunes and try to find the properties that make them tuneful.

Suppose we regard music simply as a form of communication. We can then get a great deal of help from modern communication theory [see "The Mathematics of Communication," by Warren Weaver; SCIENTIFIC AMERICAN, July, 1949]. In this theory a basically important element is the idea of entropy. Many years ago Sir Arthur Eddington observed that entropy should be placed in the same category with beauty and melody. Entropy is a description of the association between the elements in a system; it applies to the notes in a composition as well as to the molecules in a vessel.

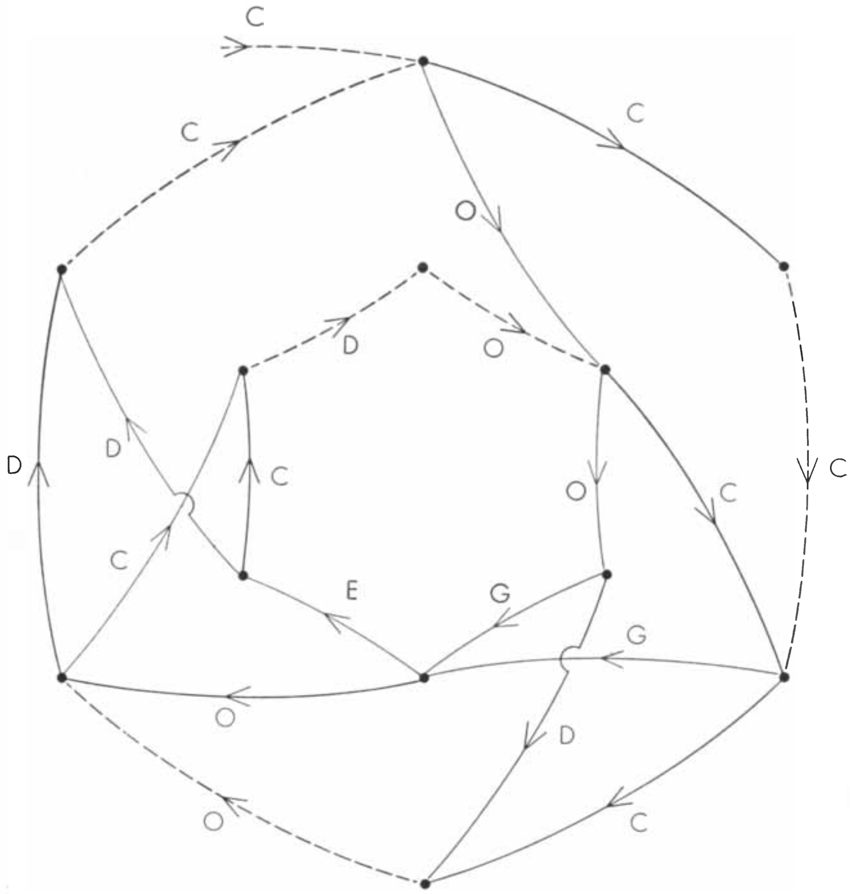
Now entropy is a numerical index of disorder. If we come upon a situation in which there is a high degree of uncertainty, in which everything is mixed up, we may say that the entropy is high. On the other hand, where there is a great deal of symmetry or patterned arrangement, the entropy is low. For example, a well-managed supermarket has low entropy: we are sure to find the canned vegetables grouped in a certain section, the breakfast food in another section, and so on. But in a store where we were equally likely to find the peas stacked with the waxed paper or heaped on the floor with the laundry soap, we would have to say that the entropy was high.

The higher the entropy, the more information can be conveyed. In a disordered store it might take a clerk days to tell us the location of all the items—*i.e.*, to communicate all the information contained in the situation. Similarly a map showing roads running helter-skelter contains more information than a piece of wallpaper with a monotonously repeated pattern.

We may ask: What is the entropy of nursery tunes? There are certain features common to all "whistling tunes." First of all, they have to sound somewhat familiar: some of the sequences



ANALYSIS OF TUNES begins by translating their notes into the usual alphabetical symbols, using "O" to represent a rest or the holdover of a note beyond its initial beat. The colored vertical lines mark off measures (*double lines*) and beats (*single lines*).

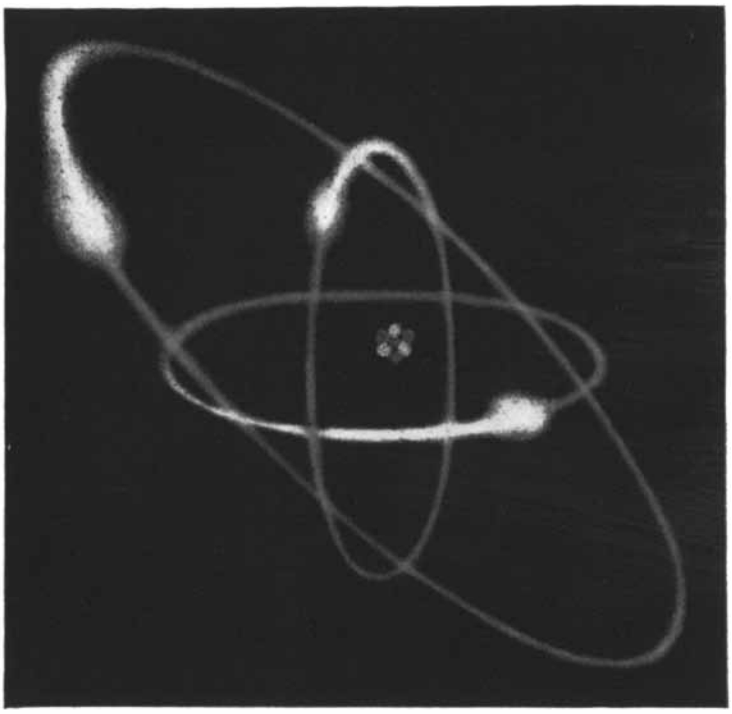


BANAL TUNE-MAKER produces simple, redundant melodies that sound like nursery tunes. A sequence of notes is obtained by following a path through the network, starting at the top, and writing down the note (or rest) attached to each segment traversed. Where there is a choice of paths, a coin is flipped. If it comes up heads, the black path is taken; if tails, the colored path. Broken lines show the path from a junction where there is no choice.

should have been heard before. Secondly, a melody always has certain basic qualities of regularity and symmetry: in terms of communication theory, it is somewhat redundant. Thus the composer of a melody must make the entropy of his music low enough to give it an apparent pattern and at the same time high enough so that it has sufficient complexity to be interesting. The question is, how high should the entropy be, and how can it be measured?

Let us make a statistical analysis of familiar nursery tunes. I selected a representative group of 39 from *The Golden Song Book* by Catherine Tyler Wessells (Simon and Schuster). The first step was to count the frequency with which the various notes of the scale appeared. To simplify matters I put all the songs in the key of C and treated all the notes as if they were in a single octave: thus middle C and C above middle C were both counted simply as C. To the seven different notes of the octave I added an eighth symbol, O, to signify a rest or the holding of a note for more than one beat. The set of eight symbols then is C, D, E, F, G, A, B and O. Translating "The Farmer in the Dell," for example, into this code, we get G/COCCOC/COOOOD/EOEEOE/-EOOOOO/GOOGOA/GOECOD/EO-EDOD/COOOO. (The song is written in 6/8 time and therefore has six beats per measure.) After writing all the 39 songs in this code, I counted the number of times each note appeared, and from this calculated the probability of occurrence for each. The probabilities are: C – .163; D – .112; E – .132; F – .066; G – .149; A – .045; B – .036; O – .297.

These probabilities are, of course, just what a musician would expect from his knowledge of tonal relations. The most pleasing sequences of notes are those in which the pitch, or number of vibrations per second, of the successive notes is related in the ratio of small whole numbers. Since C is our key note, we should expect G to appear fairly frequently, because it is related to C by the ratio 3/2 (*i.e.*, 396 to 264 v.p.s.). On the other



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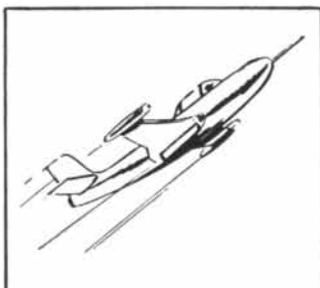

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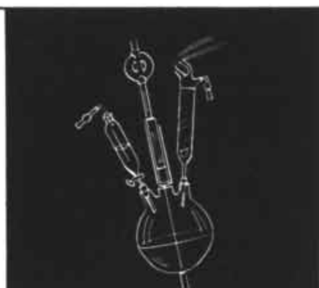

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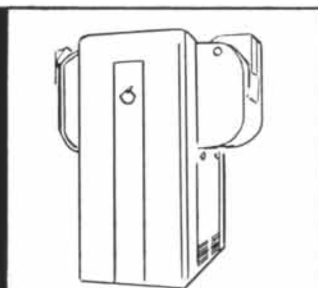

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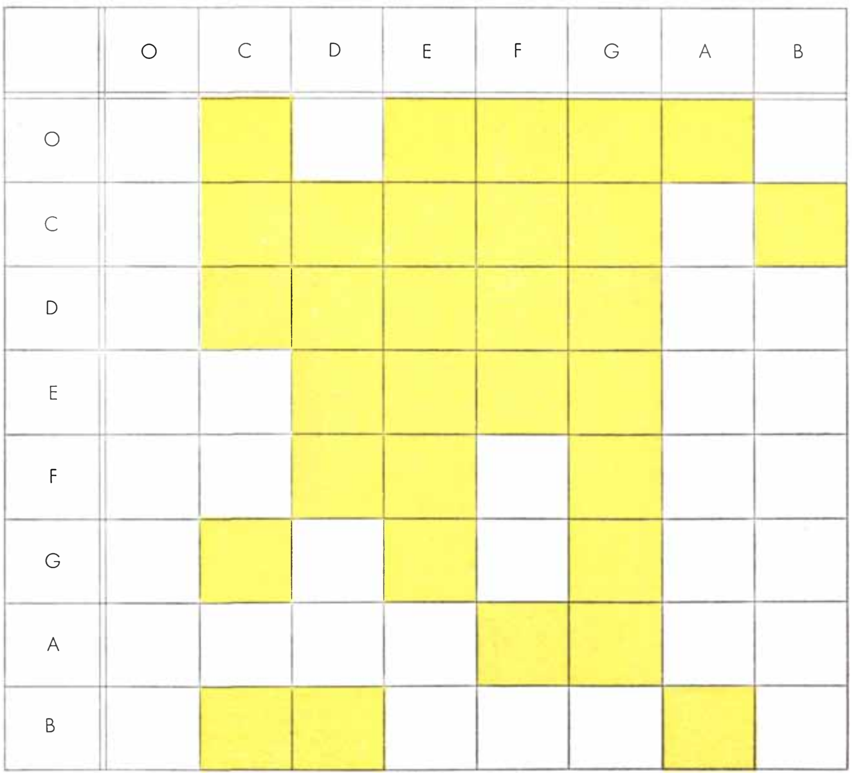
	O	C	D	E	F	G	A	B
O	0.38	0.17	0.10	0.10	0.06	0.13	0.03	0.02
C	0.36	0.23	0.13	0.07	0.02	0.10	0.03	0.07
D	0.26	0.20	0.21	0.19	0.03	0.06	0.01	0.05
E	0.22	0.15	0.18	0.16	0.16	0.12	0.01	0.00
F	0.15	0.00	0.14	0.35	0.14	0.20	0.01	0.01
G	0.29	0.14	0.00	0.16	0.06	0.26	0.08	0.00
A	0.17	0.05	0.07	0.00	0.02	0.36	0.15	0.17
B	0.18	0.30	0.12	0.01	0.01	0.08	0.21	0.08

TRANSITION PROBABILITIES show how frequently any note follows any other in the 39 nursery tunes. The first notes of all possible pairs are listed in the column at the left; the second notes, in the row at the top. Thus each number in the table gives the probability that the note at the top of its column will come after the note at the left of its row. The color pattern divides the table between likely transitions (*colored*) and unlikely (*white*).



TRANSITION PATTERN CHANGES when account is taken of the position of a note in the measure. This diagram shows the distribution of high (*colored*) and low (*white*) probabilities for the transition from the last note of a measure to the first note of the next.

hand, the note B, related to C by the more complex ratio 15/8, should have a lower probability.

Now we go back to our original question: What is the entropy of the nursery tunes? Obviously the situation of maximum entropy would be that in which all notes had an equal probability of occurring. To the degree that the selection of notes departs from complete randomness, the entropy is lowered. As a first approximation, then, we might estimate the entropy of the tunes from the relative frequency of occurrence of the several notes. There is a formula by which we can calculate the entropy per note when the probabilities are known: it is derived from the famous Boltzmann "H" theorem of thermodynamics. By this calculation, the maximum possible entropy for eight notes would be $\log_{10} 8$, or .903. (For convenience I used logarithms to the base 10 rather than the base 2.) According to their actual probability of occurrence in nursery tunes, the entropy of the notes is .821. This would make the entropy 91 per cent of the possible maximum; or, in other terms, the redundancy would be 9 per cent.

Obviously nursery tunes have a great deal higher redundancy than that. The redundancy of letters in English text is about 50 per cent, and nursery tunes are probably more redundant than our written language. To get an accurate estimate of their redundancy we must do more than merely count the number of times each note appears; we must also consider, among other things, the grouping of notes. Any given note must strongly influence our choice of the next note. For example, because it is easy to strike notes which are next to each other on the piano, adjacent notes have a high probability of following each other; thus once we have struck an F, the probabilities of hitting an E or a G are much higher than before. Again, an F seldom follows a B, because the interval between these two notes (called a diminished fifth) is a difficult one for the human voice to hit accurately.

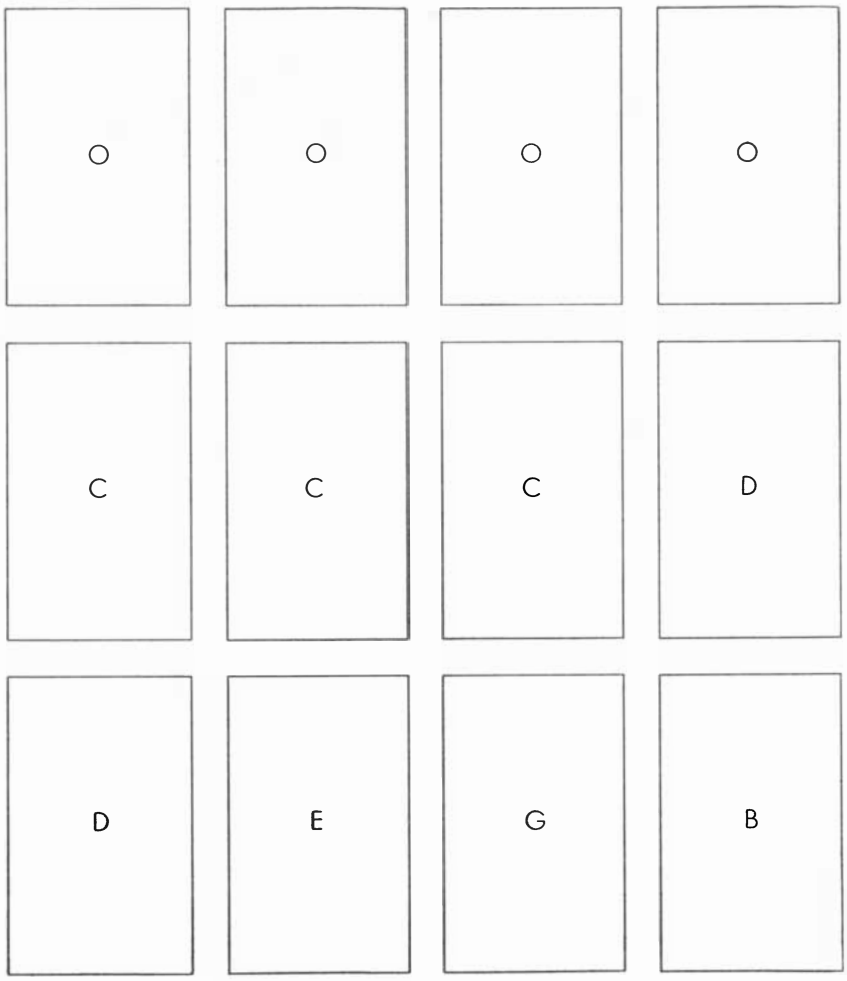
Taking the same nursery tunes, let us count the number of times specific notes are paired and make a table of these probabilities, or what the mathematicians call a "matrix of transition probabilities" [*see table on page* 82]. Now from this table we can begin to construct melodies, selecting each successive note by a method which weights the choices according to the probabilities. We may use 12 cards to make the choices. Suppose we start with the note C. The probability is about 4/12 that C

TRANSITION	C-O	C-C	C-D	C-E	C-F	C-G	C-A	C-B
PROBABILITY "p"	0.36	0.23	0.13	0.07	0.02	0.10	0.03	0.07
APPROXIMATE "p"	4/12	3/12	2/12	1/12	0	1/12	0	1/12

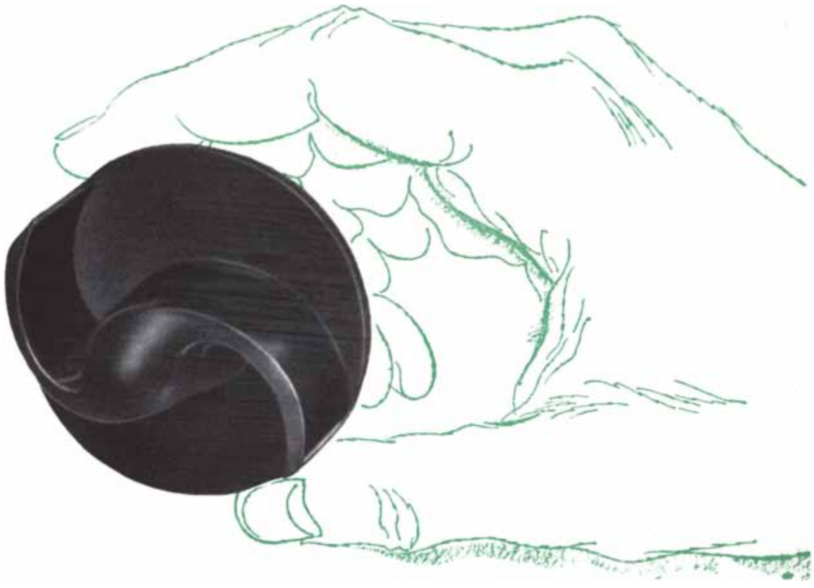
TRANSITIONS FROM C are listed above. The upper row of probability values is taken from material in the text. In the lower row, these values have been rounded off to 12ths.

will be followed by a pause, so we mark four cards O. There is a 3/12 probability that the note after C will be another C, so three cards are marked C. Similarly two cards are marked D and one card is marked E, one G and one B. Now we shuffle the 12 cards and draw one to choose the note after C. Say we draw G. We go back to the table, make up another set of cards based on the probabilities of the notes following G and choose our next note by drawing one of these cards. We continue this process until we have composed a sequence.

Now we find we are beginning to get somewhere. Snatches and phrases of the sequence sound quite tuneful. But something important is missing. What we have is a kind of be-bop nursery tune, with peculiar misplacements of emphasis due to erratic distribution of the pauses. They may pop up anywhere, even on the first beat of a measure. To construct a true nursery tune we need more redundancy, in the form of rhythm. In a simple tune pauses (O) are used in a rhythmic sense, and they never occur at the beginning of a measure. So we



NOTE AFTER C can be picked out by means of this set of 12 cards. The number of cards bearing each symbol reflects the probability (in 12ths) that the symbol will follow C.



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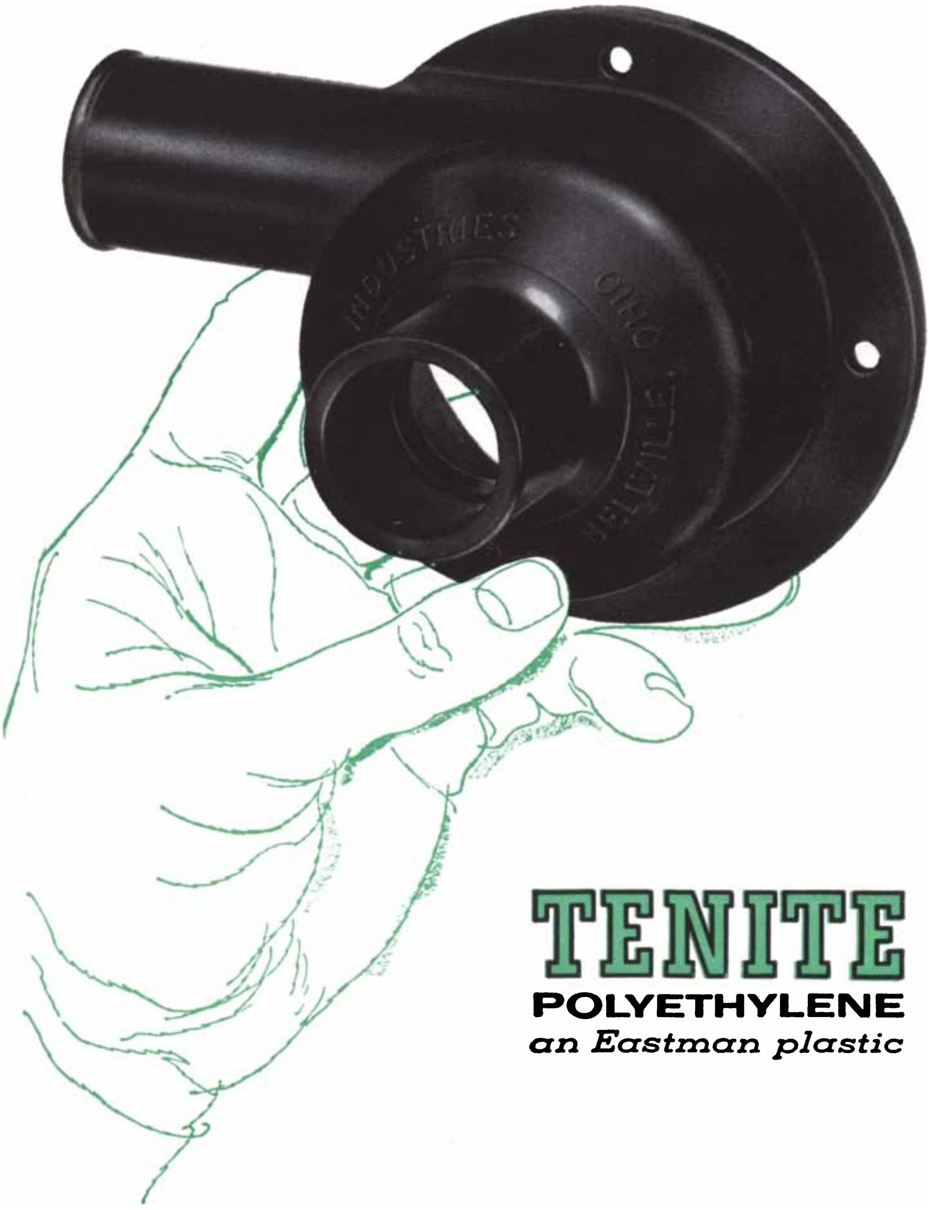

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BANAL TUNE which appears above is one of the melodies contained in the tune-making network on page

must make a whole set of supplementary tables, taking account of the probability of occurrence of notes and pauses in each position of a measure. To make nursery tunes in 6/8 time we need at least six tables.

Just for fun, I have made a set of six such tables and composed a number of tunes from them. (The probabilities were rounded off to convenient fractions so that I could make the choices with a deck of playing cards. Of course other devices could be used—say a roulette wheel with a certain number of positions or even an electronic device hooked up to an electric organ which would play the notes as they turned up.) The melodies produced are sometimes a little startling, but on the whole quite tuneful. They are all "easy to catch on to."

From the six tables I have picked out some of the most common note transitions as a basis for composing tunes. I call it the "banal tune-maker," since it is hardly more inventive than a music box. The transitions have been reduced to a series of binary choices, so that at each step one of the two choices offered can be selected simply by flipping a coin. The diagram on page 78 shows the scheme in a network form. We shall follow the network in the direction of the arrows, and at each junction where we have a choice, we shall flip a coin and choose the black path if it falls heads, the colored path if it falls tails. We start with the note C (at the top of the diagram). For the next beat we have a choice of C or O (a rest). Suppose the coin turns up heads. C, then, is our second note, and the third note also is C, because it is the only choice offered along this pathway. For the fourth note we have a choice of C or G; say the coin falls heads, selecting C. The fifth beat automatically becomes a rest. For the sixth note, ending the measure, the choice is between D and C; suppose the coin falls heads and selects D. Let us say that this procedure, followed through four measures, yields the following result (H standing for heads, T for tails and a dash for points where there is only a single choice): – H – H – H/ – T H T H T/ – – H T T H/ – – T T H H/. Written out on a musical staff, the four measures are as shown at the top of this page. Our banal tune-maker has produced a tune which is rather reminiscent of "Ride a Cock Horse to Banbury Cross."

If we wanted to, we could construct a network no more complicated than this which would actually duplicate nursery tunes. However, the point of the exercise is not that the tables of probabilities can reproduce nursery tunes but that they indicate the essential rhythmic and harmonic elements that underlie all simple melodies.

Returning to the question of redundancy, how "banal" is our banal tune-maker? We can calculate, from the relative probabilities of the various possible sequences in a measure (30 in this case), that its redundancy is higher than 63 per cent. Its tunes are highly monotonous. Yet many of them are less monotonous than some actual nursery tunes—such as, for one horrible example, "A Tisket, A Tasket."

We can apply a statistical treatment not only to melody (a sequence of notes) but also to harmony (a group of notes sounded simultaneously). Indeed, an 18th-century composer, Jean-Philippe Rameau, made a statistical analysis of harmony similar in spirit to the approach we have used here, and Allen Irvine McHose has analyzed the compositions of Bach by modern techniques. There are high and low probabilities in the

selection of notes for chords as well as for tunes.

Thus melody, rhythm and harmony can all be fitted into a statistical scheme. The clear implication is that we can build machines which will create music. A set of tables could be constructed which would compose Mozartian melodies or themes which would out-Shostakovich Shostakovich. We could get as close as desired to the style of any type of music without actually copying the melodies, and by altering the probabilities, we might evolve whole new styles.

Thinking of our network scheme, it is fun to speculate that a composer's individual style may reflect networks of nerve pathways in his brain. These patterns need not necessarily be derived from the composer's study of music; perhaps they may be formed through some other sense channel. There is actually a piece of music which was composed after the pattern of the New York sky line. I like to think that our own banal tune-maker came by the hexagonal form of its network through its author's staring too long at some hexagonal tiles or inhaling too much benzene.

What has been accomplished by this excursion into music? First of all, we have been able to calculate an entropy, or average information per note, for certain kinds of elementary melody. This gives us an indication of the amount of meaning or information that can be expressed by such melodies. We have demonstrated that a certain amount of redundancy or repetition is necessary in order to have tuneful melodies. It is possible to give a quantitative measure of this redundancy. Viewed as a statistical process, the choice of a melody has certain periodic properties imposed on it by the rhythm. These are reflected in any mechanical or mathematical representation we might make for the process. It is possible to build machines which could make music having any degree of inherent entropy, redundancy and periodicity we might desire. Arrangements are underway to set up a random choice system on an electronic computer which will be capable of turning out thousands of melodies, less banal than those made by the coin-flipping network.

Information theory may well prove generally useful for studying the creative processes of the human mind. I don't think we have to worry that such analysis will make our art more stilted and mechanical. Rather, as we begin to understand more about the property of creativeness, our enjoyment of the arts should increase a thousandfold.