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Structure and Information in Webern's Symphonie, Op. 21

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Structure and Information

in

INTRODUCTION

In this article, we shall compare two analyses of the first movement of Anton Webern's Symphonie, Opus 21: (1) A structural analysis and (2) an information theory analysis.† We hope to demonstrate how these two analytical procedures complement each other. Since it is the more novel process, we also shall demonstrate how an intensive information theory analysis of a musical composition can be carried out, showing

† This paper is partly abstracted from a thesis presented by Ramon Fuller in partial fulfullment of requirements of the degree of Doctor of Musical Arts at the University of Illinois, Urbana, Illinois, February, 1965.

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Webern's Symphonie, Op. 21

LEJAREN HILLER

RAMON FULLER

the procedures, the types of results that can be obtained, and how restrictions such as sample size affect the final results. We chose this composition for comparative study because it is a classic example of highly-ordered serial music. This means, in turn, that its structural organization should be directly reflected in information content measurements. Since the first movement exemplifies certain recent tendencies (e.g., its rigidly controlled pitch structure), and since its three principal sections are each sufficiently long for a statistical analysis, it alone is considered. The second movement, while very interesting, consists of such short variations that a valid statistical analysis cannot be performed without combining the variations. Moreover, its structural plan is much more self-evident, and its tone rows are used in a more conventional melodic manner.

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STRUCTURAL ANALYSIS

The primary purpose of a structural analysis of Webern's Symphonie, Opus 21, is to reconstruct precompositional schemes presumably used by Webern himself. It is, therefore, conditioned by logic applied by the composer and cannot offer any evaluation independent of his plan. Several similar analyses of the first movement of Webern's Symphonie can be cited, including some that have been published (*1, *2, *3, *4, *5) and one that is not published (*6). With the exception of Austin's treatment, the published analyses typically show a reduction of the first fourteen bars or so with a few paragraphs of explanation, and then conclude with a statement to the effect that the rest of the movement continues along the same lines. On the other hand, the unpublished analysis by Goldthwaite covers both movements completely. The following analysis includes data found in the above cited analyses, together with new material. Since this is a familiar composition, of which certain basic features are well known, we shall present this structural analysis as concisely as practical.

The Tone Row

As shown in Figure 1, the second half of the tone row used in this composition is the retrograde form of the first half, transposed an augmented fourth. Each half of the row includes all the tones enclosed within a perfect fourth – A down through E in the first hexachord, and E♭ down through B♭ in the second. The distribution of intervals within the row is weighted in favor of minor seconds: Each hexachord contains one major third, one minor third, and three minor seconds; in addition, there is the augmented fourth between the two center tones, where the hexachords touch, to provide the complete row under the given conditions. This means that six out of the eleven possible intervals are minor seconds and that, of the six basic intervals in the minor second-through-tritone range, two (the major second and the perfect fourth) are absent. There are two forms of redundancy within the row itself: (1) The predominance of the minor second, which limits the harmonic and motivic possibilities of the row, and (2) the retrograde relationship between the two hexachords, which reduces to 24 the total number of transpositions of the row by the elimination of the retrograde and retrograde inversion. For convenience, we shall number the 24 non-equivalent transpositions of the row as follows: The row (original form) which begins on A, is labeled O-1; that row beginning on B♭ is labeled O-2, and so on through O-12. The twelve transpositions of the inversion

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of the row are similarly labeled: I-1 through I-12.

Sectional Plan

The repeat signs, the general character of the music, and certain features such as the retrograde structure found in bars 25b through 44 suggest the form, |: A :|: BA' :|. The A' section is, in effect, a recapitulation of the A section. We shall therefore adopt the familiar terms "exposition", "development", and "recapitulation" as names for A, B, and A' sections, respectively. The exposition consists of bars 1 through 26; the development comprises bars 25b through 44, and the recapitulation, bars 44 through 66b. There is some overlapping of the three sections, as the above bar numbers indicate. The tone rows in the exposition and recapitulation appear in the following order: O-1, I-1; I-9, O-5; O-6, I-8; I-4, O-10; I-9, O-5; O-6, I-8. Each row is paired with the row which is its literal mirror. Because of the canonic structure of this movement (see below), there is considerable overlapping of tone rows. The development section consists of rows I-8, O-8; I-4, O-12; O-6, I-10; O-2, I-2. It is differentiated from the other two sections by its retrograde structure. The second half of the development is an exact retrograde of the first half, the only exceptions being the missing B♭s in bars 31 and 33, and the grace-note rhythms.

Canonic Structure

Each section is a complete four-voice double canon in which the two consequent voices mirror the two antecedent voices. Each voice consists of a number of statements of the tone row; therefore, at most points within the movement, there are four different transpositions of the tone row being simultaneously presented. O-1, which appears at the beginning of the exposition and recapitulation, is linked by the common tones, C and E♭, to I-4 (bars 11-12), and these two rows are mirrored by I-1 and O-10 to form a two-part mirror canon. The rhythms are identical in the original voice and its mirror, and all intervals are mirrored exactly. There is another equally strict two-part mirror canon formed by the linkage I-9, O-6; I-9, O-6; and its mirror O-5, I-8; O-5, I-8; proceeding simultaneously with the previously described canon. In the first half of the development, there are again two canons: One formed by I-8 and its mirror, O-8, and a second formed by I-4 and its mirror, O-12. The last four rows, as previously mentioned, are the literal retrograde of the first four. There is an interesting but probably not too significant analogy to modulation

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FIGURE

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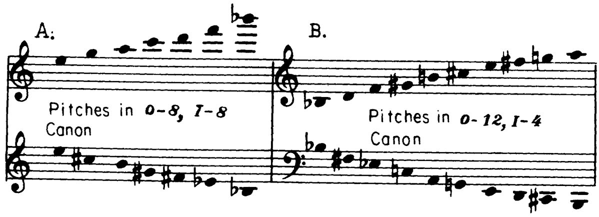

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FIGURE

5

6

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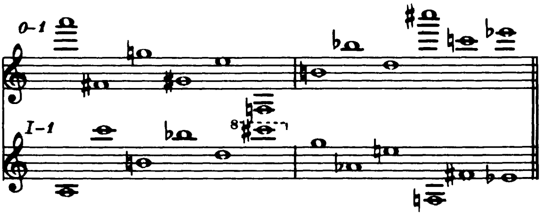

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to an apparent dominant here, since the first four rows of the development are a perfect fifth higher than the first four rows of the exposition and recapitulation

Vertical Structure

The number of pitches used, especially in the exposition and recapitulation, is small. If we compare the two pairs of exposition tone rows in Figure 2, we find two of the compositional restrictions which bring about this condition: (1) With the exception of E♭, which appears in two different octaves, each tone is found in only one octave; e. g., use of the tone A is restricted to A_3. (2) Each tone, in whatever row it appears, is always mirrored by one other particular tone, as follows: A is mirrored by A, F♯ by C, G by B, G♯ by B♭, E by D, F by C♯ and E♭ by E♭. Figure 3 shows the manner in which tones are mirrored by each other as well as the octave in which each appears. The "fourth-chord" structure that results is shown in the second part of the example. Tones connected by dotted lines mirror each other. Both of the exposition canons are entirely restricted to these thirteen pitches.

The pitch structure is more complex in the development than in the exposition; the two canons in the development do not use the same set of pitches, but each has its own symmetrically organized pitch structure. The O-8, I-8 canon is centered on E, while the O-12, I-4 canon is centered on B♭ (Figures 4 and 5).

In Figure 6, the first four tone rows in the development are reduced to the span of a major seventh to show that, as in the exposition, one given tone is always mirrored by another particular tone, regardless of the row involved. However, because there are two pitch structures, and because each of these pitch structures is composed of more than twelve pitches (the B canon uses 20 pitches), a given tone and its mirroring counterpart in the other voice are usually found in more than one octave. Thus the development section, even though it is appreciably shorter than the other sections, reflects a considerable expansion of resources and a relaxation of octave restrictions.

The recapitulation returns to the same sequence of row transpositions as found in the exposition, as well as to a highly restricted vertical structure. Figure 7 shows the mirror note pairings for the first two rows of one of the two-part canons. As in the exposition, these pairings hold for the entire section, regardless of the row transpositions involved, and except for

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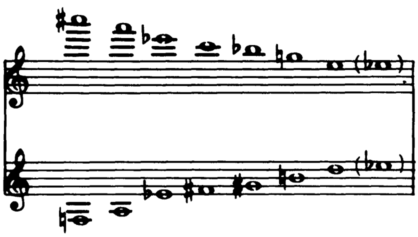
FIGURE

8



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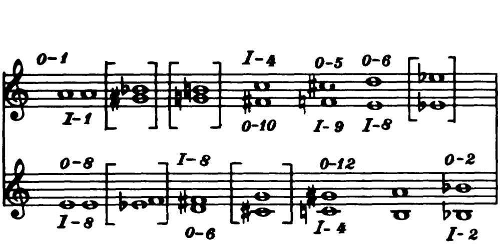

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TABLE

1 TONE ROWS USED IN THE FIRST MOVEMENT OF WEBERN'S SYMPHONIE, OP. 21

1st Tone	Row	Number of times used			
		Exposition	Development	Recapitulation	Total
A	O-1	2	0	2	4
B♭	O-2	0	2	0	2
B	O-3	0	0	0	0
C	O-4	0	0	0	0
C♯	O-5	4	0	4	8
D	O-6	4	2	4	10
E♭	O-7	0	0	0	0
E	O-8	0	2	0	2
F	O-9	0	0	0	0
F♯	O-10	2	0	2	4
G	O-11	0	0	0	0
G♯	O-12	0	2	0	2
A	I-1	2	0	2	4
B♭	I-2	0	2	0	2
B	I-3	0	0	0	0
C	I-4	2	2	2	6
C♯	I-5	0	0	0	0
D	I-6	0	0	0	0
E♭	I-7	0	0	0	0
E	I-8	4	2	4	10
F	I-9	4	0	4	8
F♯	I-10	0	2	0	2
G	I-11	0	0	0	0
G♯	I-12	0	0	0	0
					64

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TABLE

2 INSTRUMENTATION OF THE TONE ROWS

Exposition

Rows	Grouping	Instruments (in order of appearance within the row)
O-1, I-1	4-4-4	Horn, Clarinet, Strings
I-9, O-5	1-2-2-3-2-2	Harp, Strings (pizz), Strings (arco), Harp, Horn, Harp
O-6, I-8	2-2-1-3-2-2	Harp, Horn, Strings (pizz), Strings (arco), Harp, Strings (arco)
I-4, O-10	4-4-4	Strings, Clarinet, Horn
I-9, O-5	2-2-3-3-1-1	Strings (arco), Harp, Strings (pizz), Strings (arco), Harp, Strings (arco in I-9 and pizz in O-5)
O-6	1-7-3-1	Harp, Strings (arco), Harp,
	(first ending)	Strings (pizz)
I-8	1-3-4-3-1	Harp, Strings (pizz), Strings (arco), Harp, Strings (pizz)
O-6	1-7-3-1	Harp, Strings (arco), Bass Clarinet,
	(second ending)	Harp
I-8	1-3-7-1	Harp, Strings (pizz), Strings (arco), Harp

Development

Rows	Grouping	Instruments
I-8, O-8	3-1-2-5 (final B-flat missing from both of these rows)	Clarinet, Strings, Harp, Strings
I-4	3-1-2-6	Strings, Horn, Harp, Clarinet
O-12	3-1-2-2-2-2	Strings, Horn, Harp, Bass Clarinet Strings, Harp
O-6, I-10	Retrograde of O-12 and I-4, with missing B♭s in Strings	
O-2	Retrograde of O-8	
I-2	Retrograde of I-8	

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octave positions are identical with those in the exposition. The vertical structure of the recapitulation consists of fourteen pitches symmetrically centered on E♭ (which, however, does not appear in the center, but is split between two octaves, as shown in Figure 8). The tone A, (a tritone from E♭, and hence the other axis of symmetry) is also split between the two octaves. Thus there is a considerable difference between the vertical structures of the exposition and recapitulation in such details as octave range and type of symmetry (e.g., the absence of fourth-chord structure in the recapitulation.)

Transpositions of the Row

In Table 1, we have shown the number of times each row appears. There is a total of sixty-four presentations of the row (or thirty-two, discounting the repeats) but only thirteen of the twenty-four available transpositions are used. This, plus the fact that all the row transpositions found in the music are not used an equal number of times, indicates in itself a considerable compositional constraint. Figure 9 shows how the row transpositions are arranged symmetrically around the double axis, A and E♭ in the exposition and recapitulation, and around the E and B♭ double axis in the development. Unbracketed pairs of tones are the first tones of rows that mirror each other. Bracketed pairs of tones represent rows not used, even though they would have fit into the scheme. The axes, and indeed all the pairings of tones, are identical with those found in the vertical structures of the corresponding sections. This is why each tone, in whatever row it appears within a given section, is always mirrored by one particular tone rather than by different tones for different appearances. In other words, if the first tone of a row is the same distance (in the opposite direction) from a central point as the first tone of the mirroring row, it follows that all the other pairs of tones which mirror each other in the two rows will also be centered on this same point. Likewise, if all pairs of rows which mirror each other are centered on the same point, any given tone will always be mirrored by the same tone each time it appears, regardless of the row, because only one other tone can be the same distance from the center. In summary, the fixed mirroring scheme is explained by the symmetry of the transposition scheme, and by the symmetry inherent in a mirror canon.

Instrumentation

In the exposition and development, the canonic structure is reflected in the instrumentation; i.e., the instrumentation of the consequent or mirroring row is analogous to that of the

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antecedent row. As shown in Table 2, O-1 in the beginning of the exposition is divided into three four-note groups played by horn II, clarinet, and cello, respectively; its mirroring row, I-1, is divided into three four-note groups played by horn I, bass clarinet, and viola, respectively. While all rows do not follow this pattern of equal division, almost all of the mirroring rows are exact instrumental counterparts of the corresponding antecedent rows; i.e., motives which mirror each other are played by similar (or the same) instruments. The most important exceptions in the exposition and development are as follows:

> In the first ending of the exposition, seven tones (O-6 and I-8) played arco in the viola are mirrored by the cello playing three tones pizzicato and four tones arco.
>
> In the second ending of the exposition (again in O-6 and I-8), three tones in the viola are mirrored by three bass clarinet tones.
>
> In the development, I-4 and its mirroring row O-12 are instrumentally analogous in their first 8 tones; however, the last four tones of I-4, which are played by the clarinet, are mirrored by the cello and harp. Since this aberration occurs at the place where the retrograde begins, and since the instrumentation, pitch, and rhythm are all retrograde exactly, it seems possible that Webern purposely introduced different mirroring instruments at this point to avoid having too many tones of one timbre occurring in one place. Moreover, the bass clarinet cannot play the low B assigned to the harp in bars 34 and 35.

In the recapitulation, we could not find instrumentation canons such as those found in the exposition and development; as far as we can determine, there seems to be no general pattern for the use of instruments in this section. In two cases, rows linked by a common tone are similarly instrumentated, namely, the pairs O-1, I-4 and I-1, O-10. In the latter pair, the groupings are inverted symmetrically — one clarinet tone, five string tones, and six clarinet tones in O-10 are mirrored by six string tones, five clarinet tones, and one string tone in I-1. Otherwise, there seem to be no further similarities between the instrumentation of the antecedent and mirroring rows in this section.

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Rhythm

The durations of the tones were counted in an attempt to learn whether there is any overall rhythmic scheme. We were unable to find any such pattern. The results can be seen in Table 3. We can only point out that (1) the exposition and recapitulation are both double canons in rhythm as well as in pitch, and (2) as George Perle has also pointed out*7, the two antecedent voices are rhythmically similar in the development (identical except for some grace notes), resulting in a four part rhythmic canon. In summary, there exist the following primary ordering processes, each associated with certain specified secondary processes:

1 Serialization of the pitches, which produces: (a) first-order randomization of the pitch structure – i. e., the occurrence of all 12 tones about equally often (at higher-order levels, however, this is an ordering process: knowledge of the tone row makes it possible to predict the order of the last ten tones of the row from the first two); and (b) weighting of certain intervals within the row.

2 The canonical structure, which produces: (a) ordering of the actual pitch content of the sections into symmetrical vertical schemes; (b) the literal repetition of rhythms first played by the principal voice of each canon; (c) some literal repetition of the instrumentation of the principal voices; and (d) retrograding of all parameters of the development section.

The ordering of pitches and intervals accomplished by serialization is considerably disguised when four tone rows are played together as canonical voices, since intervals not contained in a given presentation of the tone row are sounded both as adjacencies and as simultaneities. This alters all the interval probabilities and all except first-order pitch probabilities. Thus, although the row is the entire melodic basis for the canons, it is not the only (perhaps not even the most important) stylistic determinant. Moreover, the canonic structure represents a significant constraint on how any single tone row structure can be employed.

APPLICABLE CONCEPTS OF INFORMATION THEORY

Information theory is a mathematical tool developed by electrical engineers to measure the capacity of communication systems to transmit information (*8, *9, *10, *11). This theory

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TABLE

3 DURATIONS OF TONES IN THE FIRST MOVEMENT OF WEBERN'S SYMPHONIE, OP. 21

Value in 8th Notes	Number of Occurrences			
	Exposition	Development	Recapitulation	Total
1/2†	16††	28	36	80
1	0	96	112	208
2	160	0	12	172
3	0	0	36	36
4	48	0	0	48
5	0	0	0	0
6	12	16	24	52
7	0	0	24	24
8	16	0	24	40
9	0	16	0	16
10	0	0	0	0
11	0	0	0	0
12	4	16	0	20
13	0	16	0	16
Total	256	188	268	712

Actual durations of the sections (in 8th notes)

Exposition	Development	Recapitulation
400	320	384

† These grace-note durations are arbitrary, since grace-notes are not, in practice, given a definite rhythmical value.
†† These figures are based on two statements of each section.

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is strictly concerned with quantitative measure and not with significance. When it is applied to the analysis of a particular "message" such as a musical composition, that portion of the theory applied to "discrete, noiseless systems" is used. The equations for the discrete, noiseless case suffice for the analysis of any message composed from an alphabet of discrete symbols, signs, or musical notes (*12, *13). The three most important factors that determine the information content of a message are:

(1) The size of the alphabet used in the message. Other things being equal, a message which uses, say, thirty-two symbols, will contain more information per symbol than one which uses only eight symbols; that is, the wider the range of possible events, the greater the uncertainty and the greater the information content.

(2) The probabilities or relative frequencies of the symbols used in the message. A message in which some of the symbols are used more times than others will contain less information than one in which all symbols are used the same number of times. Information theory calculations based on the relative frequencies of the symbols are called first-order calculations, and are calculated by the equation (*14)

$$H(i) = -\sum_{i} p(i) \log_2 p(i) \qquad (1)$$

where H(i) is the average information content per symbol. This equation, with proper substitutions, is also used for calculating the information content of specific ways of grouping symbols, e.g.:

$$H(i,j) = -\sum_{i,j} p(i,j) \log_2 p(i,j) \qquad (2)$$

where H(i, j) is the average information content per digram (a combination of two symbols which may exist in two permutational forms, each of which is counted as a separate digram). Equations analogous to Eqs. (1) and (2) enable the analyst to calculate the average information content for n-grams of any size, where n is the number of symbols per n-gram in a particular calculation.

(3) The manner in which symbols are placed in relation to each other. If a certain symbol is always followed by another particular symbol, this will reduce the information content of the

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message in comparison with what it would be if all the symbols were equally likely to follow each other. Calculations which measure the average information content per symbol when second-order probabilities – notated $p_i(j)$, which is read, "the probability of j given i" – are given, are called second-order calculations. Similarly, third-order calculations measure the average information content per symbol when the two previous symbols and the third-order probabilities – notated $p_{i,j}(k)$ – are given, and so on, through n^{th}-order calculations. The equation for second-order calculations is

$$H_i(j) = \sum_i \sum_j p(i, j) \log_2 p_i(j) \tag{3}$$

where $H_i(j)$ is the average information content per symbol when the previous symbol and all $p_i(j)$ are known. The third-order equation is similar:

$$H_{i,j}(k) = \sum_i \sum_j \sum_k p(i, j, k) \log_2 p_{i,j}(k) \tag{4}$$

It is, of course, necessary to count the number of appearances of each symbol in all the various possible sequences at each level of investigation to obtain all the probabilities needed in the calculations. In our study of the Webern Symphonie, Opus 21, the counting was done with the aid of the ILLIAC I electronic digital computer. The maximum information value H_{max} is defined as the logarithm to the base 2 of the total number of symbols in an alphabet of N symbols; it represents the amount of information that would be generated if all symbols of the alphabet occurred equally often. The above equations provide the means to answer questions such as:

(1) How nearly random is the arrangement of symbols in the message? There may be more than one answer to this question, depending upon how many calculations are performed – that is, there will be an answer for each n^{th}-order calculation performed. The actual information value can then be compared with the theoretical maximum value for the size of the alphabet. The closer H is to H_{max}, the nearer the message is to the random state at the particular n^{th}-order level.

(2) How redundant is the message? The following equation is used for calculating redundancy:

$$R = [(H_{max} - H)/H_{max}] \times 100\% \tag{5}$$

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Redundancy of the message is thus computed as a percentage of H_{max}. Under proper conditions, a redundancy figure can be found for any n^{th}-order information content; there are, however, certain practical limitations when analyzing short messages, as will be seen. Redundancy is an indication of the degree of order present in the message; the lower the redundancy, the nearer the message is to the random state.

(3) How does one section of the message compare with another in redundancy and information content? This question can be dealt with by performing a set of calculations for each major section of the message and plotting results on a graph. The analyst must be careful not to subdivide the total "message" into sections which are too short, since information theory analysis is most valid for long (theoretically, infinitely long) sequences.

ANALYTICAL METHOD FOR OBTAINING MEASUREMENTS

We chose the following parameters for analysis with information theory:

1 Pitch.
2 Four types of intervallic relationships between pitches.
3 One aspect of rhythm, hereafter called the "attack interval".
4 Pitch and attack intervals combined.

Each of these parameters consists of a set of physically measurable and perceptually distinct units which can be readily coded for computational purposes. Next we carried out the following sequence of operations:

(1) Initial counting and coding process. Each symbol used in a given parameter was assigned a number; for example, the thirteen pitches used in the exposition were numbered from one to thirteen. The musical symbols for the parameter were then translated from the score into a sequence of numbers representing, as far as possible, the temporal sequence of symbols as found in the music. Where two or more tones are attacked simultaneously, the symbols were arbitrarily counted from the lowest tone upward.

(2) Use of the ILLIAC I electronic digital computer for further counting and calculating. The sequences of numbers obtained above were punched out on coded perforated tape, and together

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with a suitable program (*15) were entered into the ILLIAC I electronic digital computer, which then counted monograms, digrams and trigrams, calculated both first-order and higher-order conditional frequency distributions, and calculated values of $p(i)\log_2 p(i)$ for monograms. Results were printed out in the format shown in the Appendix.

(3) Completing the calculations. A hand-operated adding machine was used to complete information theory calculations. From the results of these final calculations, two graphs were prepared for each parameter, and two for the combined pitch and attack interval calculations. These are the following:

(a) Information content plots for each section, each consisting in turn of the following plots for single parameter calculations:

1 $H(i)$ – first-order monogram calculation.
2 $H(i, j)$ – first order digram calculation.
3 $H(i, j, k)$ – first-order trigram calculation.
4 $H_i(j)$ – second-order monogram calculation.
5 $H_i(j, k)$ – second-order digram calculation. (digram following a monogram.)
6 $H_{i,j}(k)$ – third-order calculation (monogram following a digram.)
7 Maximum values (shown in dotted lines) for $H(i)_{max}$, $H(i, j)_{max}$, and $H(i, j, k)_{max}$. $H(i, j)_{max}$ and $H(i, j, k)_{max}$ are often so nearly equal that only one value is shown, representing both. The pitch-attack interval information plots are labeled $H(x)$, $H(y)$, $H(x, y)$, $H_x(y)$, and $H_y(x)$; these quantities, analogous to $H(i)$, $H(i,j)$, and $H_i(j)$, respectively, are defined later.

(b) Redundancy plots, consisting of one redundancy plot for each of the above named information plots except:

1 H_{max} plots, which, of course, have zero redundancy.
2 Most of the conditional information plots, for which accurate redundancy values could not be calculated, due to the shortness of the sections.

INFORMATION CONTENT OF THE PITCH STRUCTURE

The actual pitch content of each section was used as the symbol alphabet rather than the 12-tone scale. Thus, each section has its own unique pitch alphabet: There are thirteen pitches in the exposition, twenty-seven in the development, and fourteen

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FIGURE

10 PITCH INFORMATION

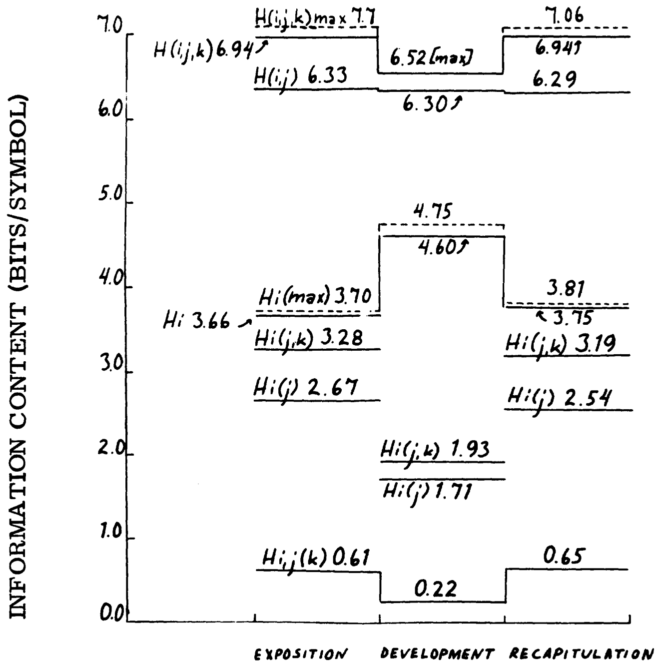

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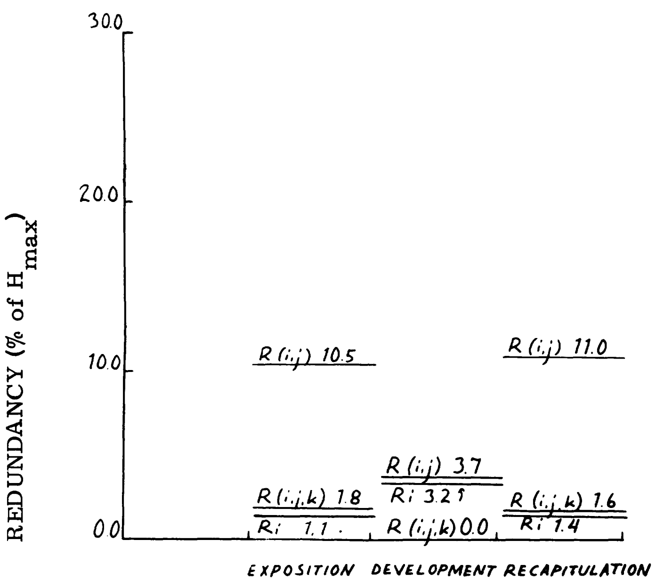
FIGURE

11 PITCH REDUNDANCY



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in the recapitulation.

Information Content Plot

In this plot, (Figure 10) the exposition and recapitulation are quite similar to each other; the development, however, appears to be structurally quite different from the other sections. Only the H(i, j) plot shows all three sections to be similar. The development has the highest information content in the H(i) plot, and the lowest in all other plots except that for H(i, j). In this latter plot, the development is closer to its H_{max} than in the other sections. In the H(i, j, k) plot, the information content equals its maximum value in the development because all trigrams are unique in this section, indicating exhaustion of the data.

Redundancy Plot

In the development, monogram redundancy (Figure 11) is somewhat higher than in the other sections, while digram redundancy is somewhat lower. Since all trigrams are unique in the development, trigram redundancy R(i, j, k) is zero there; in the other sections, trigram redundancy is very low, because nearly all trigrams are unique.

Chi-square tests, some examples of which are mentioned later, show that the 3.2% first-order redundancy in the development is statistically significant, while the somewhat lower redundancies in the other two sections are not. This redundancy was produced by the manner in which the composer assigned tones to the various octaves; that is, in the development, nearly every one of the twelve tones of the chromatic scale is found in more than one octave, while only one tone in the exposition and two in the recapitulation are thus distributed. Moreover, in the development, some tones are more evenly distributed among the octaves than other tones are, with the result that some of the 27 pitches found in this section occur more often than others, thereby creating some redundancy. In the development, half the attacks are accounted for by one-third of the pitches. Even so, this weighting, heavy enough to be statistically significant, accounts for only 3.2% redundancy, or 0.15 bits per symbol loss of information from the maximum possible.

INFORMATION CONTENT OF THE INTERVAL STRUCTURES

Intervals can be differentiated according to whether they are

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harmonic or melodic. Melodic intervals can be classified according to direction (up or down); all intervals can be harmonically inverted and/or expanded. Of the various possible methods of obtaining interval alphabets for analysis, we chose the following:

(1) Type I Intervals. Those seven intervals from the unison through the augmented fourth. The top tone of any interval larger than an augmented fourth was lowered successively by octaves until the interval was smaller than or equal to an augmented fourth. Upward melodic intervals, downward melodic intervals, and harmonic intervals were thus considered equivalent.

(2) Type II Intervals. The twelve intervals from the unison – through the major seventh. Larger intervals were reduced by the process described above. However, in order to preserve a correspondence between the pitch and interval structures, downward melodic intervals were inverted harmonically to become upward melodic intervals (e.g., a downward minor third became an upward major sixth).

(3) Type III Intervals. Intervals larger than a major seventh were reduced to their less-than-an-octave counterparts, as with Type II intervals; however, the distinction between upward and downward moving melodic intervals was kept. This procedure yielded an alphabet of 23 intervals.

(4) Type IV Intervals. Here, as with Type III intervals, a distinction was made between upward and downward melodic intervals, and harmonic intervals were considered equivalent to upward intervals. However, larger intervals were not reduced; all sizes of intervals were included in the alphabet. This yielded a different size of alphabet for each section: 36 intervals in the exposition, 37 in the development, and 46 in the recapitulation.

In all interval calculations, intervals were counted exactly as pitches were in their temporal sequence. Where more than two tones are attacked simultaneously, intervals were counted from the bottom upward, following which the interval between the top tone of the aggregate and the lowest pitch of the next attack was counted. The final tone of each section was counted as a unison, so that we maintained one interval for each tone in each section.

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Two problems arose in connection with the Type III and Type IV interval calculations: (1) the ratios between the sizes of the alphabet and the lengths of the sections are quite critical because of the increased size of the interval alphabets; and (2) since harmonic intervals (i.e., the intervals between tones attacked simultaneously) were considered equivalent to upward moving melodic intervals, we created a weighting in favor of upward intervals. Table 4 shows the percentage of harmonic intervals in each of the three sections. Alternatively, we could have counted half of the harmonic intervals as upward melodic intervals and half as downward melodic intervals. This would have eliminated the weighting and retained the same size of alphabet. However, it seems inconsistent to regard a set of intervals in two different ways for no other reason than mathematical neatness. On the other hand, we could have counted harmonic intervals as additional symbols in the alphabet. This would have further enlarged the alphabets so that sample sizes would almost certainly have been too small for these alphabets.

Some further limitations arose because at least three general classes of intervals are formed from temporally adjacent tones in this music, namely:

> (1) Intervals which belong to a motive. Both tones of such an interval are found in the same canonic voice and are played by the same instrument. Because other tones often occur between the two tones forming a motivic interval, many motivic intervals are not accounted for in these calculations.
>
> (2) Quasi-melodic intervals formed from tones successively attacked by different instruments, and which belong to different motives and/or voices. All these intervals are included automatically in these calculations due to the nature of the counting method.
>
> (3) Harmonic intervals, which can be classified into two types: (a) those formed by pitches attacked simultaneously, and (b) those formed by successively attacked pitches which sound together because the first pitch is held through at least part of the duration of the other pitches.

Type 1 and 3b intervals are not always included in the interval calculations while Types 2 and 3a always are. Other more obscure interval relationships could also be mentioned, e.g., those between tones which are not in the same voice and are separated by other attacks, but which are quite perceptible because of comparative nearness in time and pitch range.

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TABLE 4 PROPORTIONS OF HARMONIC INTERVALS PRESENT

Section	% Harmonic Intervals
Exposition	29.4%
Development	14.9%
Recapitulation	8.9%

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84

FIGURE

12 TYPE I INTERVALS INFORMATION

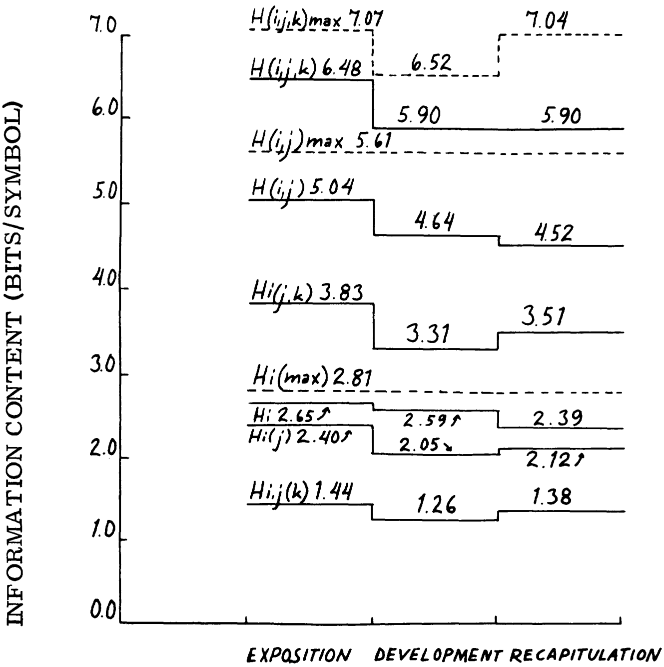

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FIGURE

13 TYPE II INTERVALS INFORMATION

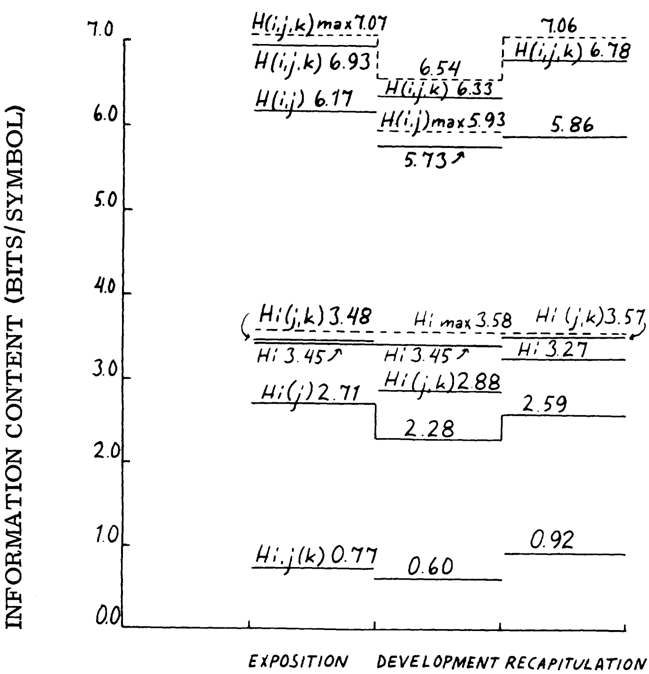

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FIGURE

14 TYPE III INTERVALS INFORMATION

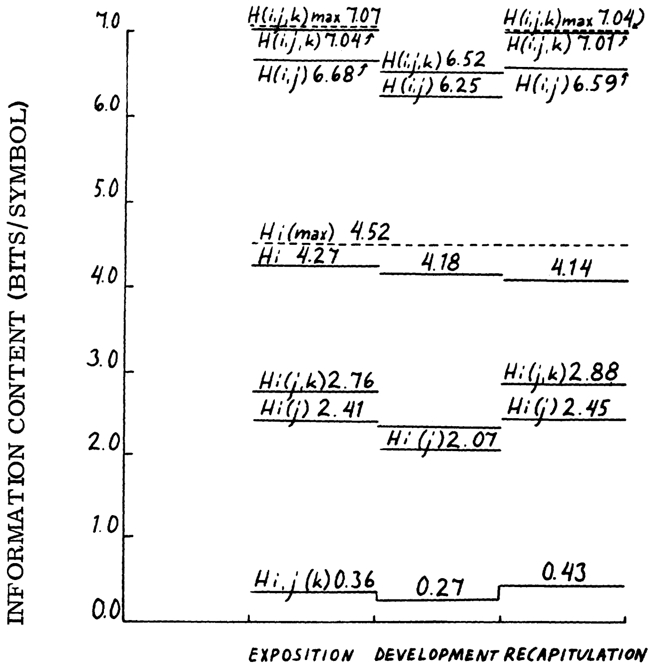

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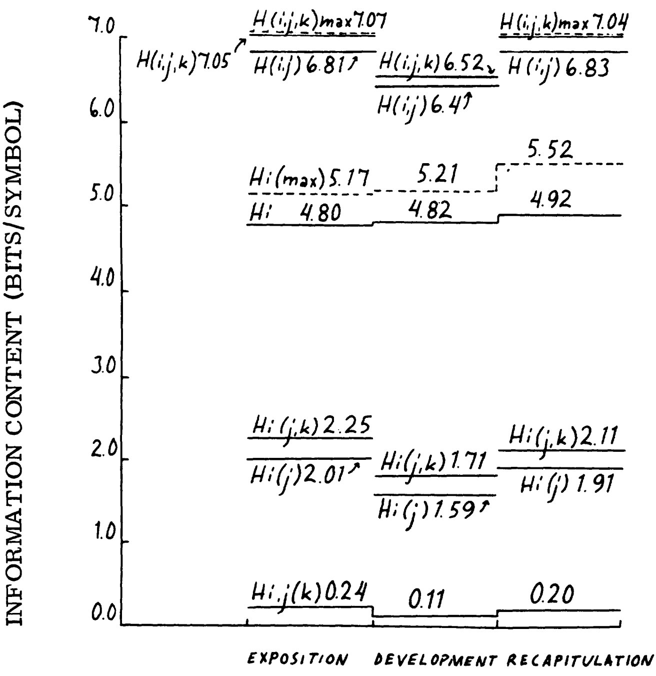
FIGURE

15 TYPE IV INTERVALS INFORMATION



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Information Content Plots

In Figures 12 through 15, the contours of the four sets of information values are similar in broad outline. There are some details worth noting, however, both in terms of what they appear to reveal about the structure of this music, and in terms of whether the results are statistically significant. Let us consider each graph in turn.

In Figure 12 all the information content plots show the exposition to have greater information content than either of the other two sections. However, chi-square tests for the H(i) plot revealed that the small difference between the exposition and the development (0.06 bits/symbol) is insignificant; on the other hand, the difference in information content of 0.26 bits/symbol between the exposition and the recapitulation is statistically significant. In general, the small differences between sections shown on the $H_{i,j}(k)$ plot are probably also insignificant, as are those between the development and the recapitulation on the $H_i(j)$ and H(i, j) plots. All the other differences are probably significant statistically. This means that, in general, the exposition has a more complex interval structure than the other two sections.

In Figure 13 the three sections are quite similar in the H(i) plot. Chi-square tests confirm that the difference between the identical H(i) values for the exposition and development and the H(i) value for the recapitulation is not statistically significant. In all the other plots, the development is somewhat lower in information content than the other sections. We cannot, however, trust these plots completely. Furthermore, chi-square tests showed that the difference between $H(i)_{max}$ and H(i), which is 0.13 bits/symbol in both the exposition and development, is significant statistically for the exposition, but barely if at all significant for the development. Since the two alphabets are exactly the same size for both sections, the only other factor that could cause this disparity is the relative shortness of the development. We therefore cannot measure any significant differences among the three sections, as far as Type II intervals are concerned.

In the Type III interval plots shown in Figure 14, the exposition and recapitulation are nearly equal in all plots but two, while the development is appreciably lower. In the H(i) and $H_{i,j}(k)$ plots, the three sections are quite close, and chi-square tests show that their differences are not statistically significant. The nearness of H(i, j, k) to its maximum value indicates that

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the data become rapidly exhausted as the analysis proceeds from lower to higher-order calculations, indicating a complex interval structure (although the existing first-order redundancy is high enough in all sections to be statistically significant).

Finally, the graph of Type IV interval information contents shown in Figure 15 appears to be quite similar to the Type III intervals plot. This graph is not really significant, because in these calculations the monogram alphabets are quite large (36 symbols in the exposition, 37 in the development and 46 in the recapitulation) in comparison to the respective lengths of the sections of 137, 94 and 134 symbol occurrences, respectively. (The length of the exposition has been adjusted to 137 symbol occurrences to eliminate the repetition and allow for the slight differences between the first and second endings.)

Redundancy Plots

In general, we observe above all that these plots (Figures 16 through 19) compensate for differences in alphabet size among the different computations and directly present measures of order or predictability for the various intervals in question. All these plots must, of course, be considered significant or not significant in line with the tests applied to the information content values presented above. In Figure 16, for the Type I intervals, the plot of $R_i(j)$ has the highest redundancy values for all sections, as expected, with the development being highest in redundancy on this plot. In the non-conditional plots the exposition is lowest in redundancy, the development next, and the recapitulation highest. Moreover, in both redundancy and information content the development is less differentiated from the other sections in Type I interval structure than it is in pitch structure. This is expected because the greater number of possible pitch choices in the development does not add any more Type I intervals to the alphabet.

In Figure 17 redundancy is in general lower for Type II interval structures than it is for Type I interval structures. Thus, although the larger Type II alphabet yields more information, it is used with lesser predictibility than the Type I alphabet.

In Figure 18 the differences between the sections are slight in all three plots. The monograms feature small, statistically insignificant rises from section to section; the digrams show the development to be least redundant, and the trigrams have zero (or negligible) redundancy in all three sections. Probably none of the differences between sections is statistically signifi-

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FIGURE

16 TYPE I INTERVALS REDUNDANCY

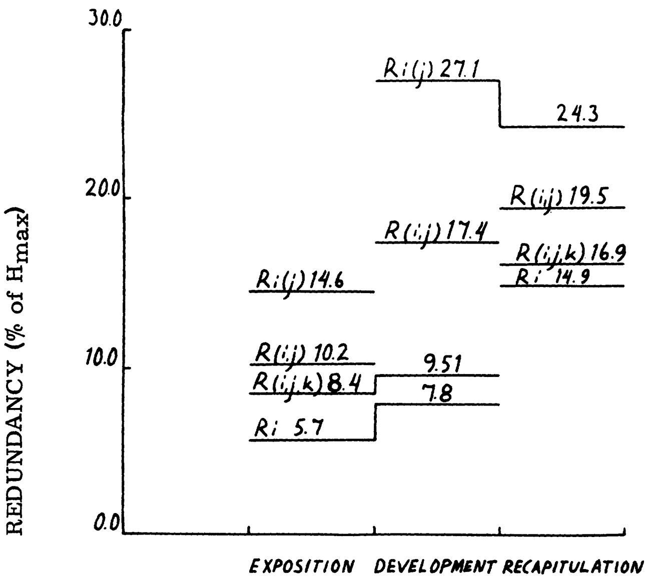

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FIGURE

17 TYPE II INTERVALS REDUNDANCY

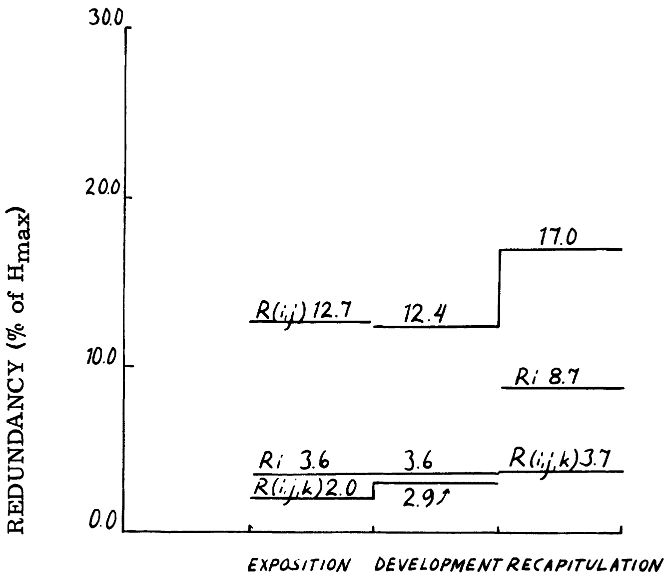

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FIGURE

18 TYPE III INTERVALS REDUNDANCY

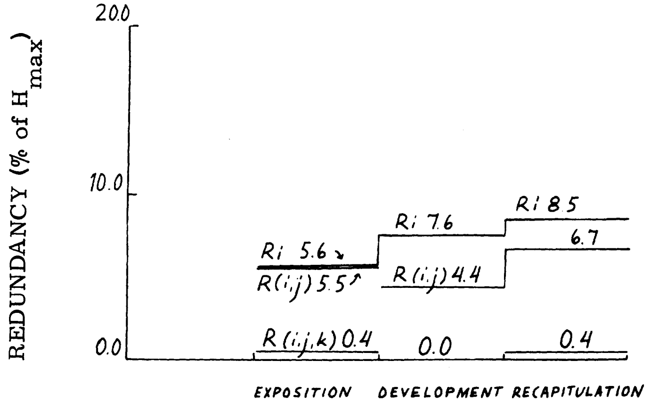

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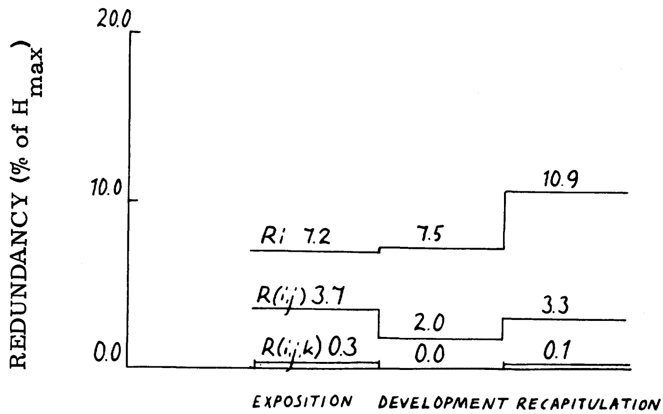
FIGURE

19 TYPE IV INTERVALS REDUNDANCY



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cant; consequently, the three sections, as far as can be determined, have very similar interval structures.

Finally, in Figure 19, all the redundancies appear to be quite low, as might be expected. The one thing we can conclude with safety here is that, in any given portion of the movement, a maximum of previously unheard Type IV intervals and interval combinations occurs. Other conclusions, such as comparisons of sections, would be unreliable.

INFORMATION CONTENT OF ATTACK INTERVALS

The term "attack interval" refers to the time span (measured in eighth notes) between attacks. For instance, in 2/2 meter, when one tone is attacked on the first beat and the next tone is attacked on the second beat – one half-note later – the attack interval was considered to have a duration of four eighth notes. A grace note was arbitrarily counted as half an eighth note and two tones attacked simultaneously were recorded as an attack interval of zero. The method of counting the order in which attack intervals appear was the same as for pitch or interval calculations. The durations of the tones were ignored; therefore, the alphabet for these calculations is made up of the time spans existing between attacks. While this procedure is obviously limited, it should reflect fairly accurately relative degrees of rhythmic complexity in the music. Higher-order information theory analysis is valid only for one-event-at-a-time sequences; therefore, it would have been impossible to include other rhythmic factors such as duration (which often overlap) in these calculations.

Information Content Plot

In general, the exposition has the simplest attack interval structure, and the recapitulation has the most complex attack interval structure according to this set of calculations (Figure 20). The only exceptions to this observation are that the development is lowest in $H_{i,j}(k)$ and very slightly higher in H(i) values than the recapitulation. This general pattern differs significantly from pitch and interval calculations. The similarity of the development and recapitulation is not surprising, since they both contain eighth notes which are not found in the exposition; they also have larger attack interval alphabets than the exposition – 10 for the development and 8 for the recapitulation, as opposed to only 5 for the exposition. The attack interval structure is in general less complex than the pitch structure. Chi-square tests reveal that in all sections H(i) is significantly lower than

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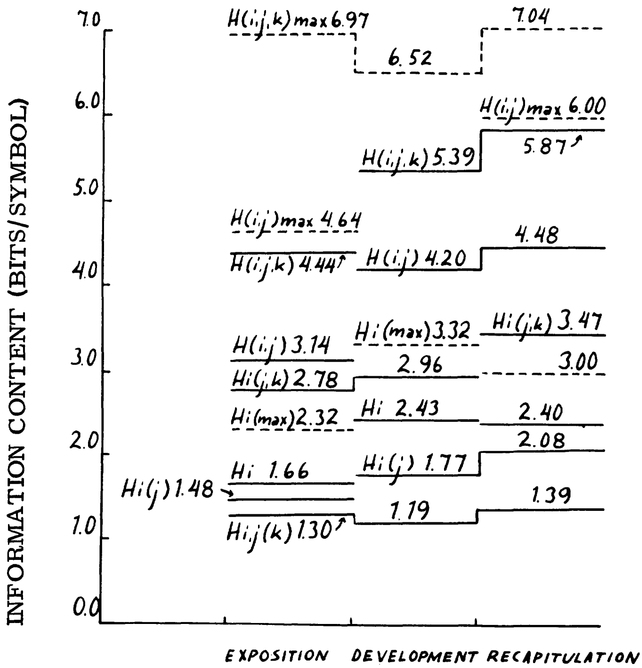
FIGURE

20 ATTACK INTERVAL INFORMATION



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96

FIGURE

21 ATTACK-INTERVAL REDUNDANCY

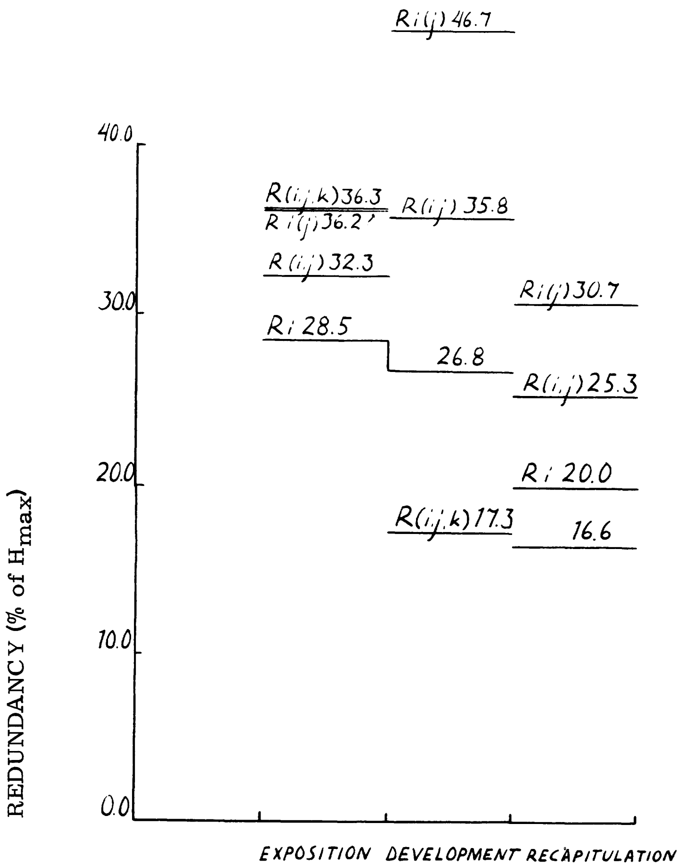

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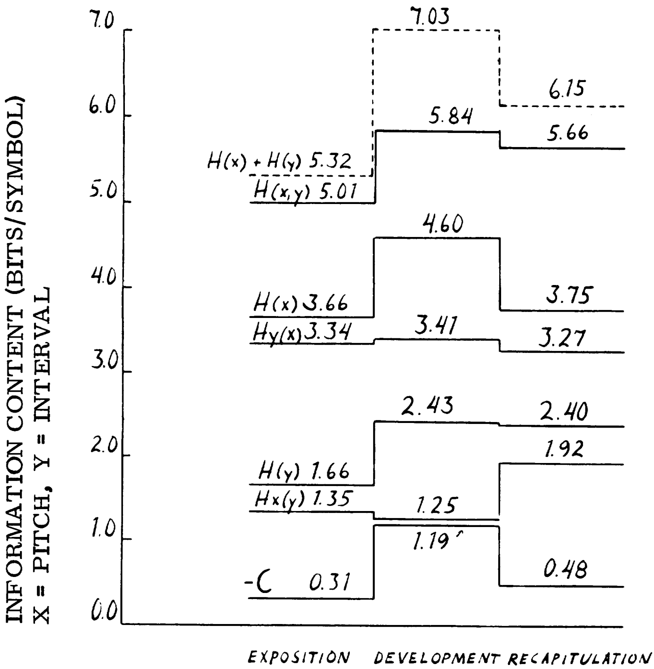
FIGURE

22 PITCH-ATTACK INTERVAL INFORMATION



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$H(i)_{max}$. However, sample sizes of the development and recapitulation become somewhat critical in relation to first-order calculations. The difference of only 0.18 bits/symbol between H_i and $H_i(j)$ is quite significant in the exposition, where P is less than 0.001.*14 In the development, where this difference is much larger, i.e., 0.66 bits/symbol, P lies between 0.01 and 0.001. This is considered significant but the fact that the larger difference produces a smaller value for P indicates that there are constraints imposed by the shortness of the development in addition to probability constraints. In the recapitulation, the difference of 0.32 bits/symbol between H(i) and $H_i(j)$ gives a value for P which is surprisingly large – between 0.98 and 0.95. Therefore, the short lengths of the development and recapitulation can exert a statistically significant effect upon first-order information values calculated from alphabets containing as few as 8 or 10 symbols. This is ample justification for not calculating conditional redundancies for large alphabets.

Redundancy Plot

The most striking feature of this graph (Figure 21) is that the exposition redundancies are much more closely spaced than the redundancies of the other sections. This is probably due to the smallness of the alphabet used in the exposition. The exposition is highest in R(i) and R(i, j, k); the development is highest in R(i, j). The recapitulation is lowest in all four kinds of redundancy.

COMBINED PITCH ATTACK INTERVAL INFORMATION CONTENTS

If we designate pitch structure by x and attack interval structure by y, p(x) is then the same as p(i) for pitch, and p(y) is the same as p(i) for attack intervals. Each pitch occurrence is associated with some attack interval because even pitches struck simultaneously produce an attack interval of zero.

The Combined Probabilities

Three sets of probabilities, with their corresponding information contents and redundancies, are used to describe the combination of these two parameters:

(1) p(x, y) – probabilities for pitch and attack interval combinations. Each individual p(x, y) is calculated from the equation

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$$p(x, y) = x, y/S, \tag{6}$$

where x, y is the number of occurrences of some particular combination, and S is the number of attacks in the section.

(2) $p_x(y)$ – the set containing a subset of attack interval probabilities for each pitch used in the section. Each individual probability is calculated by the equation

$$p_x(y) = x, y/x \tag{7}$$

where x, y is the number of occurrences of a particular attack interval with a particular pitch, and x is the number of occurrences of this particular pitch in the section.

(3) $p_y(x)$ – the set containing a subset of pitch probabilities for each attack interval used in the section. Each of these subsets describes how often each pitch occurs with a particular attack interval. The equation for calculating each probability is

$$p_y(x) = x, y/y \tag{8}$$

where x, y is the number of occurrences of a particular pitch with a particular attack interval, and y is the number of occurrences of the particular attack interval in the section. It is important to remember that the set of $p_y(x)$ is different from the set of $p_x(y)$.

Information and redundancy values for each of the three above sets of probabilities are calculated by the same equations used for the first-order digram and second-order calculations for single parameters. The results of these calculations are shown in Figure 22. Because this is a plot of combined probabilities, a few remarks concerning the properties of such combined probabilities are in order. If the two structures, pitch and attack interval, were independent, H(x, y) would equal H(x) + H(y), a function which is shown in the plot in dotted lines. Moreover, $H_y(x)$ would then be identical with H(y), because the probability of any given attack interval occurring with any given pitch would be the same as the probability of occurrence of the attack interval without reference to its occurrence with any pitch. The converse would also be true: the probability of any given pitch occurring with any given attack interval would be the same as the first-order probability of the pitch when no other parameter is being considered. Since H(x, y) is equal to $[H(x) + H_x(y)]$ and since H(y) would be the same as $H_x(y)$ in the independent case, H(x, y) is equal to H(x) + H(y) in the independent case.

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The opposite of the independent case is that in which constraints are absolute, i.e., in which a particular pitch always occurs with a particular attack interval. Knowledge of the one symbol would then be the key to knowledge of the other, which would supply zero information to persons knowing the appropriate probabilities. If the constraints were not absolute, but still significantly high, knowledge of one symbol would merely enable us to make a better guess about the second symbol, thus lowering, but not cancelling, the information content of the second symbol.

Except in very long messages, the two sets of probabilities cannot possibly match, whereby $H(x)$ equals $H_y(x)$ and $H(y)$ equals $H_x(y)$ unless

$$S = K(xy) \tag{9}$$

where K is a positive whole integer, x is the number of symbols in the x alphabet, y is the number of symbols in the y alphabet, and S is the number of symbols in the message (i.e., the length of the message). As S increases in magnitude, it becomes less important that K be a whole number.

Information Content Plot

It is obvious from Figure 22 that the pitch and attack interval structures in this music are neither independent nor subject to absolute constraints, since the actual information values in every case lie between zero and the values for the independent case. The plot labeled -C at the bottom of the graph shows by how much, in bits/symbol, each of the information plots is lowered by the interdependence of the parameters and the shortness of the sections. The constraints exerted by the pitch structure on the attack interval structure are equal to those exerted by the attack interval structure on the pitch structure; moreover, these constraints are in turn equal to the quantity $[H(x) + H(y) - H(x, y)]$. The quantity -C which reflects all of these constraints, is thus a measurement of the interdependence of the pitch and attack interval structures.

Chi-square tests on the $H_y(x)$ probability distribution of each section were performed. These indicated that the constraints of 0.31 bits/symbol in the exposition, 1.19 bits/symbol in the development, and 0.48 bits/symbol in the recapitulation are all significant statistically. P is less than 0.001 in the exposition, between 0.01 and 0.02 in the development, and about 0.01 in the recapitulation. We can now make the following com-

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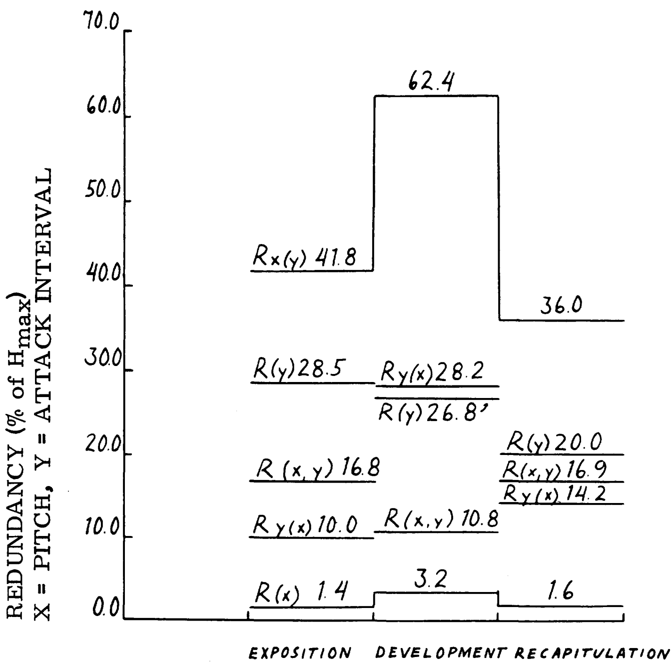
FIGURE

23 PITCH-ATTACK INTERVAL REDUNDANCY



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102

FIGURE

24 FIRST ORDER INTERVAL INFORMATION RATES

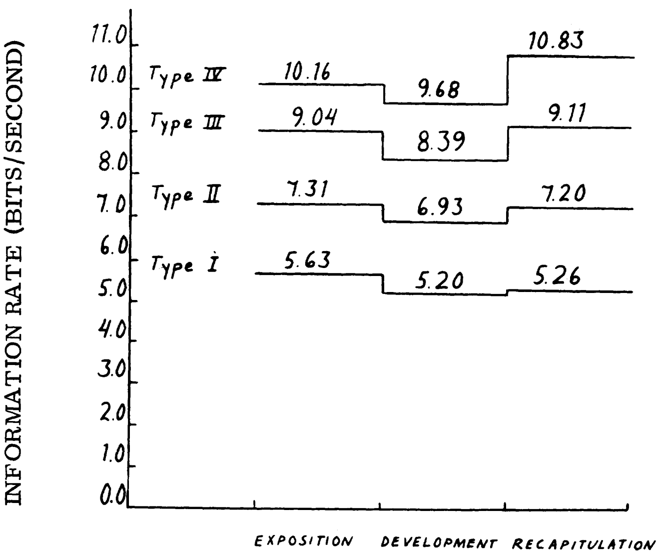

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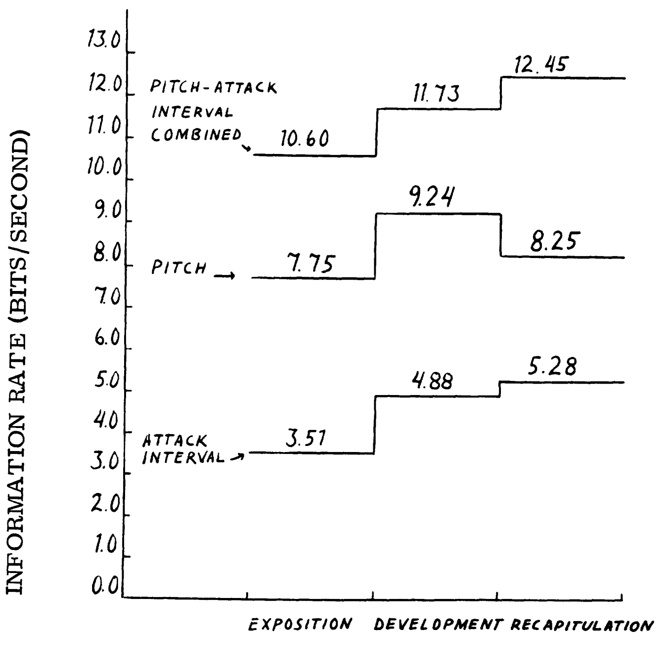
FIGURE

25 FIRST ORDER PITCH AND ATTACK INTERVAL INFORMATION RATES



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parisons of the three sections:

(1) In the $H(x, y)$ plot, the development and recapitulation are both appreciably higher in information content than the exposition, with the development being highest of all three. Both the development and recapitulation seem significantly higher in information content per symbol pair than the exposition; i. e., the differences of 0. 83 and 0. 65 bits/symbol compare favorably with the degree of constraint found to be significant in chi-square tests. On the other hand, the difference of 0. 18 bits/symbol between the development and recapitulation seems not to be statistically significant.

(2) The slight differences among sections in the $H_y(x)$ plot are probably not significant statistically.

(3) In the $H_x(y)$ plot, the recapitulation seems to be significantly higher in information content than the other sections.

Redundancy Plot

The $R(x, y)$ plot given in Figure 23 shows that the exposition and recapitulation are very nearly equal in redundancy, while the development is appreciably lower than these two sections. The development is much higher than the other two sections in both the $R_x(y)$ and the $R_y(x)$ plots.

INFORMATION RATES AND TOTALS

Thus far in this study, information contents have been expressed in bits per symbol. Also of interest are information rates in bits per second and total information in bits per section for a given parameter.

Information Rates

From the metronome markings furnished in the score and the same note counts used for calculating information contents, rates of 2. 12 tones/second in the exposition, 2. 01 tones/second in the development, and 2. 20 tones/second in the recapitulation were obtained (see Figures 24 and 25). Because there is not much variation in average rate of attacks among the sections, the first-order time rate information plots (except for the Type III and Type IV intervals and the pitch-attack interval plots)

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TABLE

5 TOTAL BITS PER SECTION

	Exposition	Development	Recapitulation
Pitch	464.82	432.40	502.50
Type I Interval	337.55	243.46	320.26
Type II Interval	438.35	324.30	438.18
Type III Interval	542.29	392.92	554.76
Type IV Interval	609.60	453.08	659.28
Attack Interval	210.82	228.42	321.60
Pitch Attack-Interval	636.27	548.96	758.44

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are similar in shape to the first-order plots of information content for a given parameter. Thus, the variations of information rate among the sections are due not so much to differences in average rate of symbol occurrence as to differences in the probability distributions.

Information Totals

Each of the figures in Table 5 represents the total number of binary digits required to represent a parameter for a given section, assuming an ideally accurate and efficient coding procedure. The information totals shown in the table were obtained by multiplying the number of symbols in a section by the first-order information content in bits/symbol for that section. Repetitions of the sections were disregarded in these calculations.

SOME COMPARISONS OF THE ANALYTICAL METHODS

In computing information contents, we have treated the musical message in Webern's composition as a one-at-a-time sequence of events. Consequently, each calculation analyzes a reduced form of Webern's music that is not necessarily identical with Webern's pre-compositional scheme. With this in mind, let us propose three compositional stages, each having a characteristic information content:

> Stage 1 – the tone row and all of its derivatives in a completely abstract form.
>
> Stage 2 – a total pitch structure, including intervals and attack intervals. The canons and tone rows are present, but disguised at this stage, because there are no clues in the form of instrumentation, etc. The attack intervals have to be specified, of course, for the canons to come out right. This is the stage most conveniently analyzed with information theory.
>
> Stage 3 – the total, final form of the composition, in which all nuances, durations, timbres, phrasing, etc., are evident.

Information theory as used herein is not too suitable for analyzing Stage 3 because:

> (a) Combining several parameters would greatly increase the size of samples needed for valid results. The alpha-

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bet of symbols would be very large if pitch, attack interval, instrument, Type I interval, and dynamic markings were all combined in one grand calculation. A potential alphabet of $13 \times 5 \times 9 \times 7 \times 6$ (i.e., 24,570) symbols would result for the exposition alone.

(b) The contrapuntal organization of this compositional stage is not accounted for in a simple application of information theory analysis. In an analysis purporting to account for all aspects of a polyphonic composition, the use of an arbitrary one-at-a-time sequence would have to be modified.

Table 6 summarizes results comparing Stages 1 and 2 at the first-order level by providing an estimate of the information content of parts of the process of composition. For instance, the first-order pitch information content of the tone row is 3.58 bits/symbol, which is the same as the information content of random music since all symbols are equiprobable on an average when taken over a large enough sample of tone rows. The reduction in information content achieved by recognizing the constraint imposed by tone-row usage is $1/12 \log_2 12! = (3.32/12) \log_{10} (4.79 \times 10^8) = 2.40$ bits/symbol. However, this computation is in a sense a twelfth-order calculation and consequently is difficult to use in a direct comparison to data of much lower order. Since our information content calculations take no account of serial processes as such, a comparison to the first-order value is more reasonable. We have seen that $H(x, y)$ of the combined pitch-attack interval structures is 5.01 bits/symbol in the exposition. Therefore, in proceeding from Stage 1 to Stage 2 the composer added information to the composition in the amount of 1.43 bits/symbol in the exposition, 2.26 bits/symbol in the development, and 2.08 bits/symbol in the recapitulation. This result suggests a new analytical method, namely, that of comparing the information potential of the musical materials with that of shaping processes. Accordingly, Webern may have created more compositional information in the development and recapitulation than in the exposition, with the exception that the recapitulation interval structure seems somewhat less complex than the exposition and development interval structures.

Let us now compare the interval structures of the tone row and Stage 2 from another point of view. We see in Table 7 that one tone row playing at a time would have a less rich intervallic structure than actually exists in the three four-part canons. Of interest is the treatment of the minor second which consti-

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108

TABLE 6

A. INFORMATION CONTENTS OF STAGES 1 AND 2

	Stage 1		Stage 2		
	Row H_{max}	Row H(i)	Exposition H(i)	Development H(i)	Recapitulation H(i)
Pitch	3.58	3.58	3.66	4.60	3.75
Type I Interval	2.81	1.69	2.65	2.59	2.39
Type II Interval	3.46	2.59	3.45	3.45	3.27
Attack Interval			1.66	2.43	2.40
			H(x,y)	H(x,y)	H(x,y)
Pitch-Attack Interval			5.01	5.85	5.66

B. DIFFERENCE BETWEEN STAGE 2 H(i) AND ROW H(i)

	Exposition	Development	Recapitulation
Pitch	0.08	1.02	0.17
Type I Interval	0.96	0.90	0.70
Type II Interval	0.86	0.86	0.68

C. DIFFERENCES BETWEEN PITCH-ATTACK INTERVAL H(x,y) AND ROW AND STAGE 2 PITCH H(i)

	Exposition	Development	Recapitulation
H(x,y) minus Row Pitch H(i)	1.43	2.26	2.08
H(x,y) minus Stage 2 Pitch H(i)	1.35	1.24	1.91

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TABLE

7 TYPE I INTERVAL PROBABILITIES

	Stage 1	Stage 2		
	Tone Row	Exposition	Development	Recapitulation
Unison	0.000	0.039	0.085	0.045
Minor Second	0.546	0.281	0.319	0.433
Major Second	0.000	0.152	0.149	0.112
Minor Third	0.182	0.113	0.170	0.112
Major Third	0.182	0.117	0.085	0.157
Perfect Fourth	0.000	0.148	0.138	0.075
Tritone	0.091	0.148	0.053	0.067

8 TYPE II INTERVAL PROBABILITIES

	Stage 1	Stage 2		
	Tone Row	Exposition	Development	Recapitulation
Unison	0.000	0.039	0.085	0.046
Minor Second	0.273	0.129	0.149	0.202
Major Second	0.000	0.082	0.074	0.052
Minor Third	0.091	0.051	0.096	0.052
Major Third	0.091	0.051	0.032	0.067
Perfect Fourth	0.000	0.078	0.074	0.052
Tritone	0.091	0.148	0.053	0.067
Perfect Fifth	0.000	0.070	0.064	0.022
Minor Sixth	0.091	0.066	0.053	0.090
Major Sixth	0.091	0.063	0.074	0.060
Minor Seventh	0.000	0.070	0.074	0.060
Major Seventh	0.273	0.152	0.170	0.231

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tutes over half of the intervals of the tone row. Although remaining the most heavily weighted interval in all three sections, its probability is considerably lowered in the canons – less so in the recapitulation than in the other sections. All the intervals except the minor third and unison are in fact closer to the tone-row probabilities in the recapitulation than in the other sections. Thus the tone row interval structure seems least affected by contrapuntal interference in the recapitulation. This is also reflected in the Type I interval first-order information values, since the recapitulation has a significantly lower information content than the other two sections (see above).

Finally, in Table 8, we see that in the recapitulation Type II interval probabilities are also often nearer to the tone-row probabilities than are the corresponding exposition and development probabilities.

Our data demonstrate two practical limitations of information theory analysis that must always be recognized by users of the method. The first is the sample-size problem. Information theory analysis is much more valid when applied to large samples than to small ones. However, it is not always possible or even desirable to choose a very long musical passage for analysis. The analyst must therefore decide whether the passage he is interested in is long enough for information theory analysis to provide worthwhile results. To this end, such tests of statistical significance as the chi-square test can be applied, as was done in this study. Obviously, the larger the sample, the greater will be the mathematical validity of the results. A logical next step in the analysis of Webern's music might, therefore, be to include his whole output in a statistical-information theory study. However, such a calculation would not discriminate stylistic differences among Webern's compositions. This could best be accomplished by analyzing each composition separately, then analyzing them all together to obtain the average information content of his whole output.

The second limitation is the alphabet size problem. As shown in the interval analyses, the analyst sometimes has the choice of using a larger or a smaller alphabet in a calculation. The larger interval alphabets yielded doubtful results simply because, with a small sample and a large alphabet, the individual symbols do not occur enough times to produce statistically reliable results. A situation of diminishing returns exists here. If the analyst works with large samples and small alphabets, he obtains reliable but generalized results; if he works with smaller samples and/or larger alphabets, he obtains more

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specific but less reliable results. In summary, however, as long as reasonable precautions are taken, information theory analysis can provide a valuable summary of the effect of the number of symbols used, their relative frequency, and their combinatorial arrangements upon the structural complexity of a musical composition.

EXAMPLES OF FIRST-ORDER INFORMATION AND REDUNDANCY CALCULATIONS

Exposition Pitch Information

The exposition pitches were numbered from 1 to 13, beginning with the lowest pitch. The exposition was then translated from musical notation into a sequence of numbers for processing by ILLIAC I, which was programmed to yield the results shown in Table 9. These were printed out in the format shown, except that the pitches were not named. The second column shows the number of pitch attacks preceding the first occurrence of each pitch in the exposition. The third, fourth, and fifth columns show the probability, the logarithm to the base 2 of the probability, and the probability times $\log_2$ of the probability, respectively, for each pitch. Adding all the figures in the fifth column yields the average information content in bits/symbol for the exposition pitch structure: 3.659128, or rounded off, 3.66 bits/symbol. The minus sign is cancelled by the minus sign appearing in Eq. (1). The sixth column gives the number of times each pitch occurs in the exposition. The last column shows the size of the sample from which each probability was calculated. In this case, since the repetition of the exposition was counted, the total sample was 256. In a first-order calculation such as this, the sample is the same for each probability. In higher-order calculations, sample sizes depend upon the occurrences of symbols and n-grams; therefore, several different sample sizes may be involved.

EXAMPLE OF CHI-SQUARE CALCULATION

Certain hypotheses concerning such statistical data as the exposition pitch probabilities may be tested with the chi-square formula*17:

$$\chi^2 = \Sigma(f_o - f_e)^2 / f_e \qquad (10)$$

where f_o is the frequency observed, and f_e is the frequency

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TABLE 9 ILLIAC PRINTOUT FOR THE EXPOSITION PITCH STRUCTURE

Pitch	First Occurs	p(i)	$-\log_2 p(i)$	$-p(i)\log_2 p(i)$	f	Total
A	0	.093750	-3.415037	-.320160	24	256
F♯	1	.093750	-3.415037	-.320160	24	256
F	2	.082031	-3.607682	-.295943	21	256
G	4	.078125	-3.678071	-.287350	20	256
G♯	6	.082031	-3.607682	-.295943	21	256
C	7	.093750	-3.415037	-.320160	24	256
C♯	10	.082031	-3.607682	-.295943	21	256
B	12	.078125	-3.678071	-.287350	20	256
B♭	14	.082031	-3.607682	-.295943	21	256
E	16	.078125	-3.678071	-.287350	20	256
E♭	23	.039062	-4.678071	-.182738	10	256
D	24	.078125	-3.678071	-.287350	20	256
E♭	32	.039062	-4.678071	-.182738	10	256
				-3.659128		

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TABLE

10 A CHI-SQUARE CALCULATION FOR THE EXPOSITION PITCH STRUCTURE

Pitch	f_o	f_e	$f_o - f_e$	$(f_o - f_e)2$	$(f_o - f_e)^2 / f_e$
A	24	19.7	4.3	18.5	0.9
F♯	24	19.7	4.3	18.5	0.9
C	24	19.7	4.3	18.5	0.9
F	21	19.7	1.3	1.7	0.1
G♯	21	19.7	1.3	1.7	0.1
C♯	21	19.7	1.3	1.7	0.1
B♭	21	19.7	1.3	1.7	0.1
G	20	19.7	0.3	0.1	0.0
B	20	19.7	0.3	0.1	0.0
E	20	19.7	0.3	0.1	0.0
D	20	19.7	0.3	0.1	0.0
E♭	10	19.7	-9.7	94.0	4.8
E♭	10	19.7	-9.7	94.0	4.8
		256.1			$\chi^2 = 12.7$

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expected if the hypothesis being tested were true. A maximum of information is created when all symbols of the alphabet occur equally often (or in other words, when all symbols occur an average number of times); a statistical test of deviation from the mean will then also be a test of the statistical significance of deviation from H_{max}, or of the redundancy calculated for the message. Table 10 shows a sample format for calculation of chi-square. To interpret the value of chi-square, the analyst refers to a statistical table such as the one in the reference cited. This table lists values of chi-square for 1 to 30 degrees of freedom, and certain values of P (the probability of obtaining a higher value of chi-square than was actually obtained), ranging from 0.001 to 0.99. With 13 different symbols, there are only 12 degrees of freedom, because, in constructing a message of fixed length from 13 different symbols, one can freely decide how many times each of the first 12 symbols will occur, but then the occurrences of the remaining symbol will be fixed in number. Hence, this one symbol does not represent a degree of freedom. Referring to the row for 12 degrees of freedom in the table cited, we see that when chi-square equals 11.340, P equals 0.50; likewise, when chi-square equals 14.011, P equals 0.30. Since the value of chi-square shown in Table 10 is 12.7 and therefore lies between 11.340 and 14.011, the value of P in this example lies between 0.50 and 0.30. Chi-square is not considered significantly high unless P has a value less than 0.05. The hypothesis that the observed deviations from the mean and first-order redundancy are statistically significant is therefore disproved. Thus, this test shows the exposition pitch structure to be in a random state at the first-order level.

R E F E R E N C E S

1 René Leibowitz, Schoenberg and his School (trans. Dika Newlin), New York, 1949, p. 212. This and the other published analyses cited are intended as illustrations of compositional principles rather than complete descriptions.

2 George Perle, Serial Composition and Atonality, Berkeley and Los Angeles, 1962, p. 133. Perle gives a reduction of the first movement through half of the development.

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11 L. Hiller and L. M. Isaacson, Experimental Music, New York, 1959, pp. 22-35.

12 L. Hiller, "Informationstheorie und Computermusik", Darmstädter Beiträge zur neuen Musik, Vol. 8, B. Schott's Söhne, Mainz, 1964. A more detailed discussion of how information theory is applied to musical analysis and a summary of previous work in this area is given here.

13 L. Hiller and C. Bean, "Information Theory Analyses of Four Sonata Expositions", Journal of Music Theory, 10 (1966), p. 96.

14 For more details concerning these equations, see Cherry, On Human Communications, pp. 167-184.

15 L. Hiller and R.A. Baker, "Computer Music", in H. Borko, ed., Computer Applications in the Behavioral Sciences, Englewood Cliffs, N. J. 1962, p. 424. See also R. Baker, A Statistical Analysis of the Harmonic Practice of the 18th and Early 19th Centuries, Unpublished doctoral dissertation, University of Illinois, Urbana, 1963 (see especially pp. 21-28).

16 That is, the probability of obtaining a higher value of chi-square than was actually obtained. This is more fully explained below.

17 See, for example, H. Arkin and R. Colton, Tables for Statisticians, New York, 1962, pp. 15-16 and 121.

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