

Hypothesis Testing

Presented by David John Baker
August 2019

Hypothesis Testing



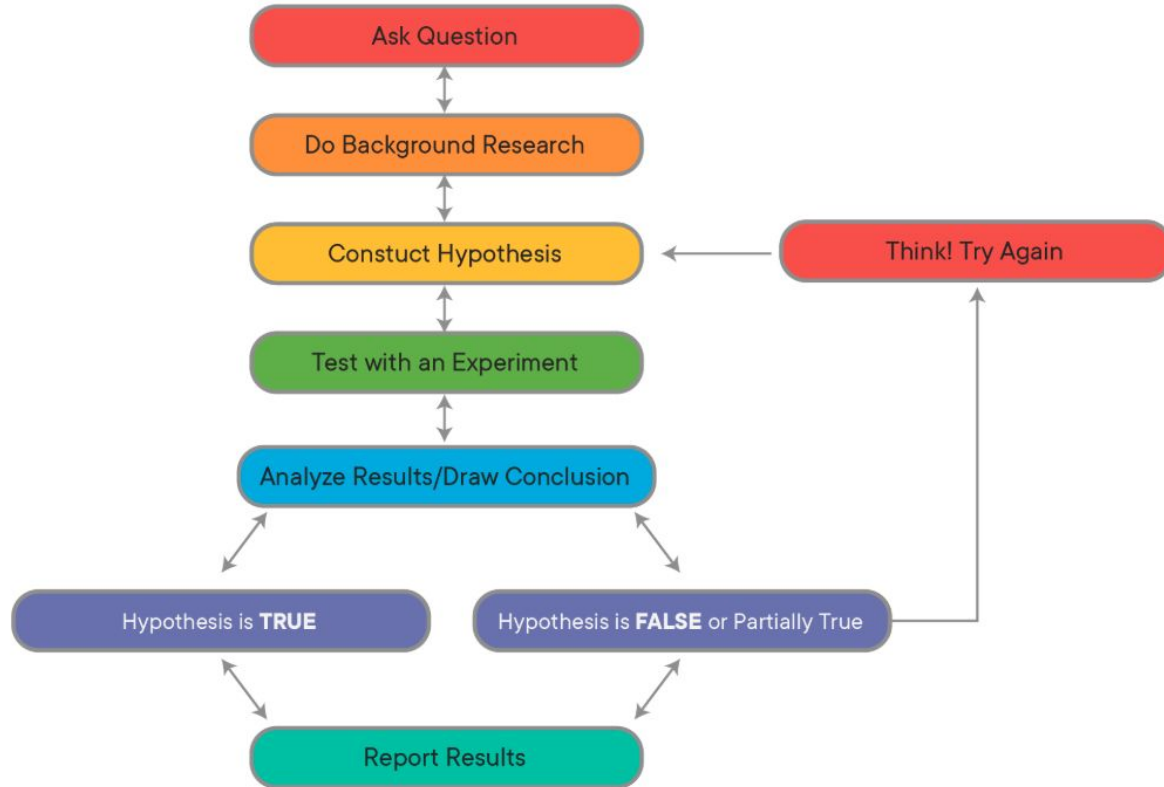
- Knowing stuff about the world is hard
- The Scientific Method is there to help us out
- Today we make our first pass at understanding one way of thinking about how we can do science
- Dave's Opinion: Time invested in thinking about how we know what we know / philosophy of science will develop your critical thinking skills better than any other investment.

Lesson goals

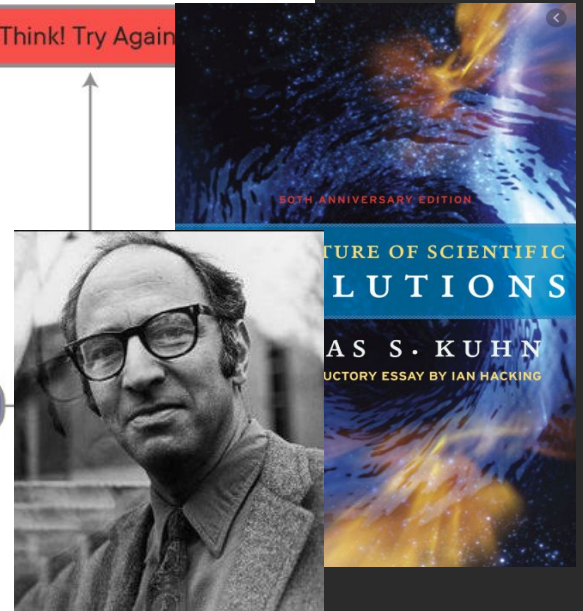
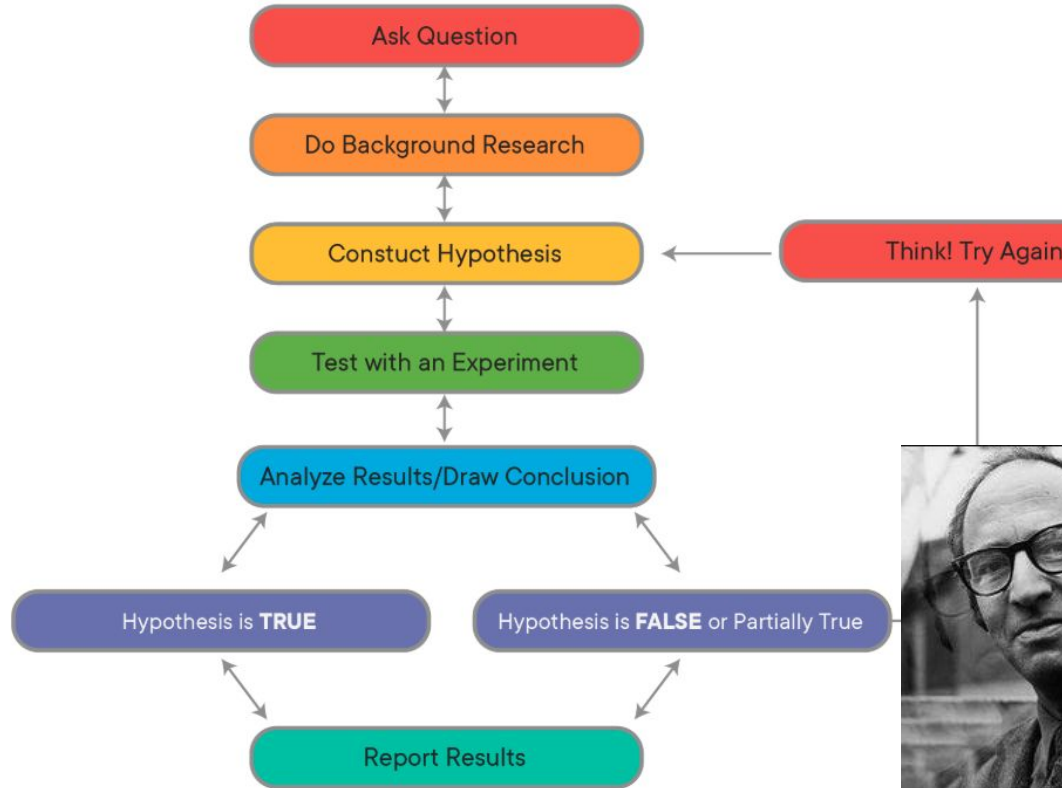
- Scientific Method (Theory vs Practice)
- The Problem of Induction
- Popper, Falsifiability, Demarcation
- Logic of Null Hypothesis Significance Testing
- Four Types of Outcomes in NHST
- Introduction to p values
- Run through a single statistical test

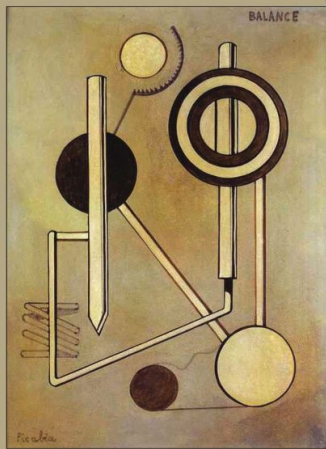


The Scientific Method



The Scientific Method

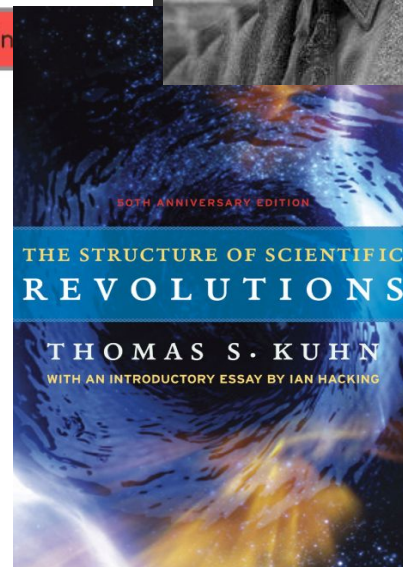
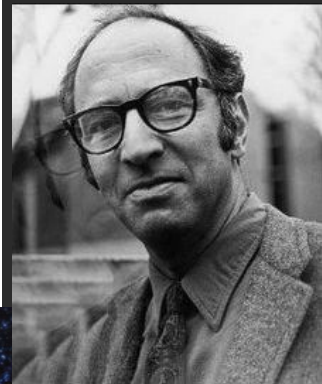
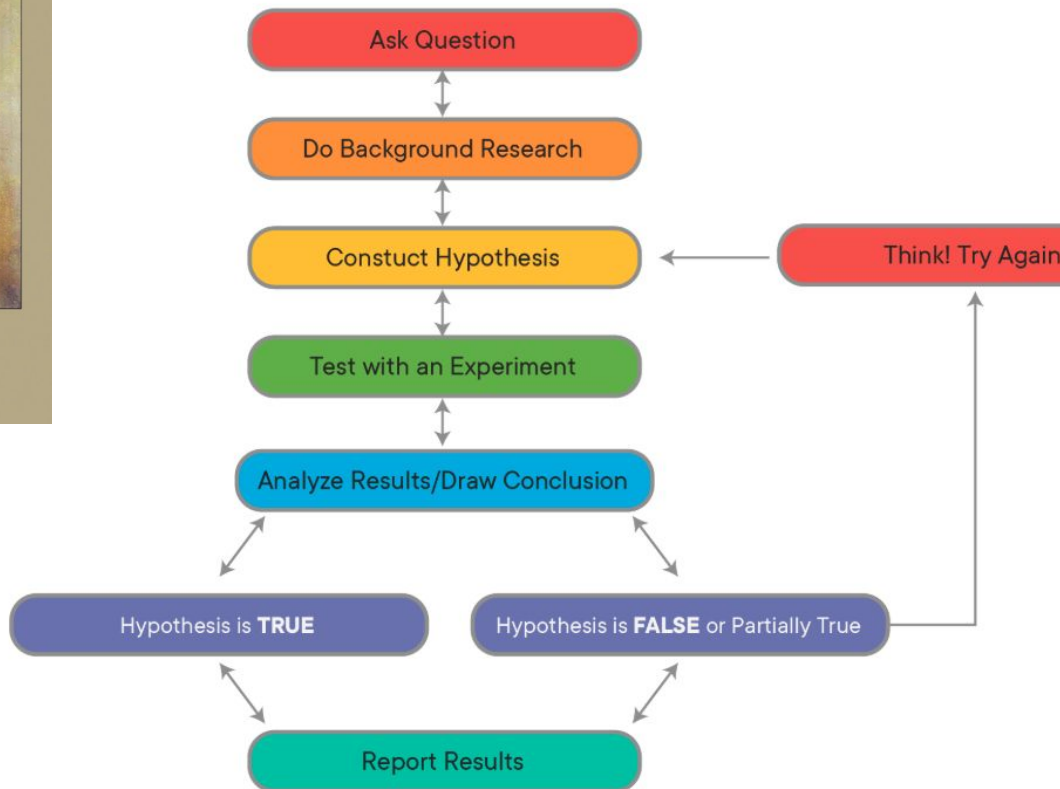




New Edition
AGAINST METHOD
Paul Feyerabend
Introduced by Ian Hacking



The Scientific Method



What makes a question scientific?

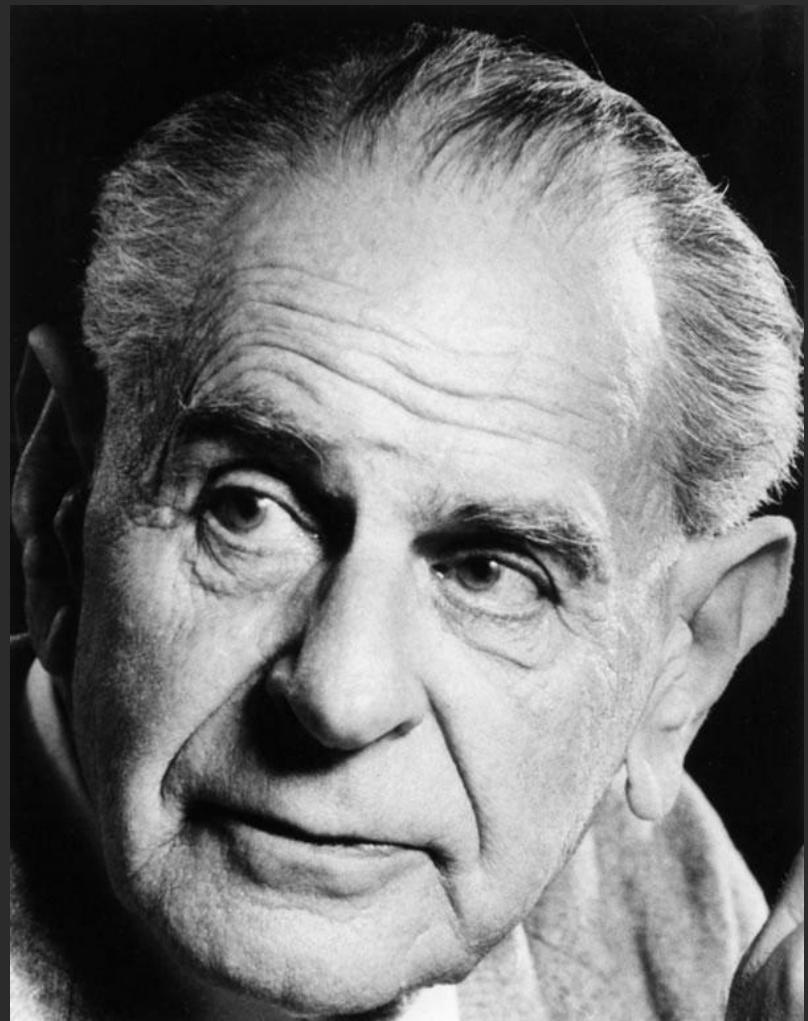
For Example...

Are questions about astrology (horoscopes) and astronomy (theory of relativity) equally scientific?



Karl Popper

- Problem of demarcation
- Falsifiability
- Problem of Induction

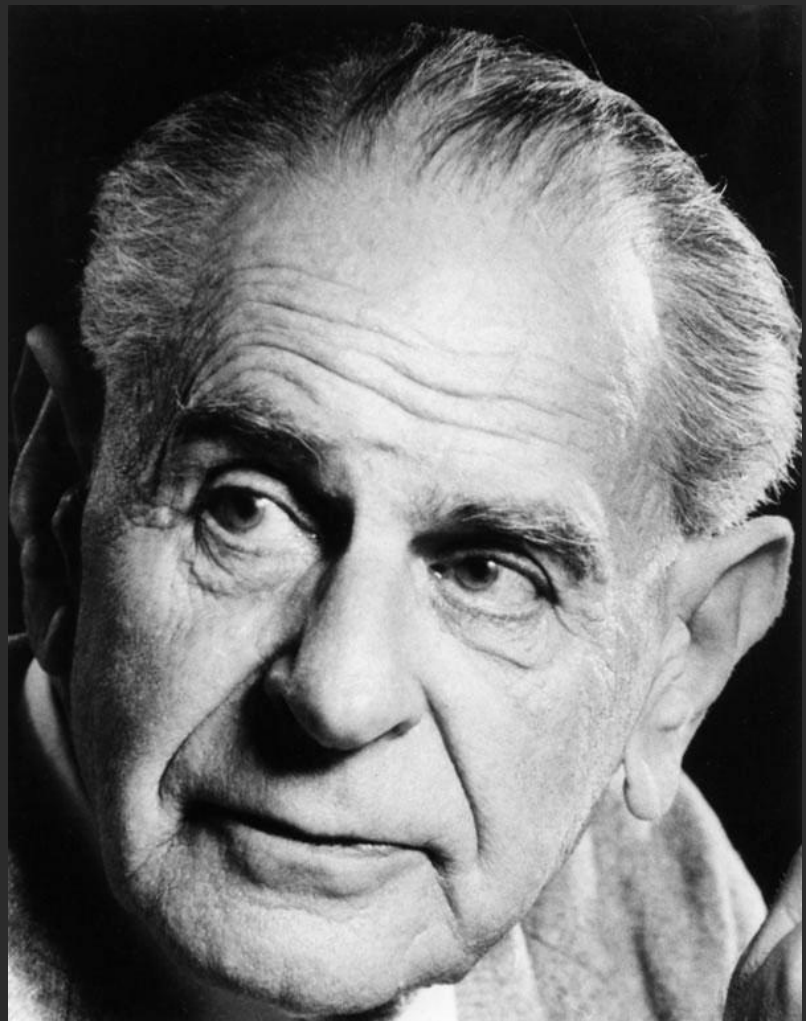


Karl Popper

- Problem of demarcation
- Falsifiability
- Problem of Induction

It's harder to prove wrong that mercury being in retrograde will affect someone's mood.

It's easier to prove wrong that items, when dropped from similar heights will fall at different rates

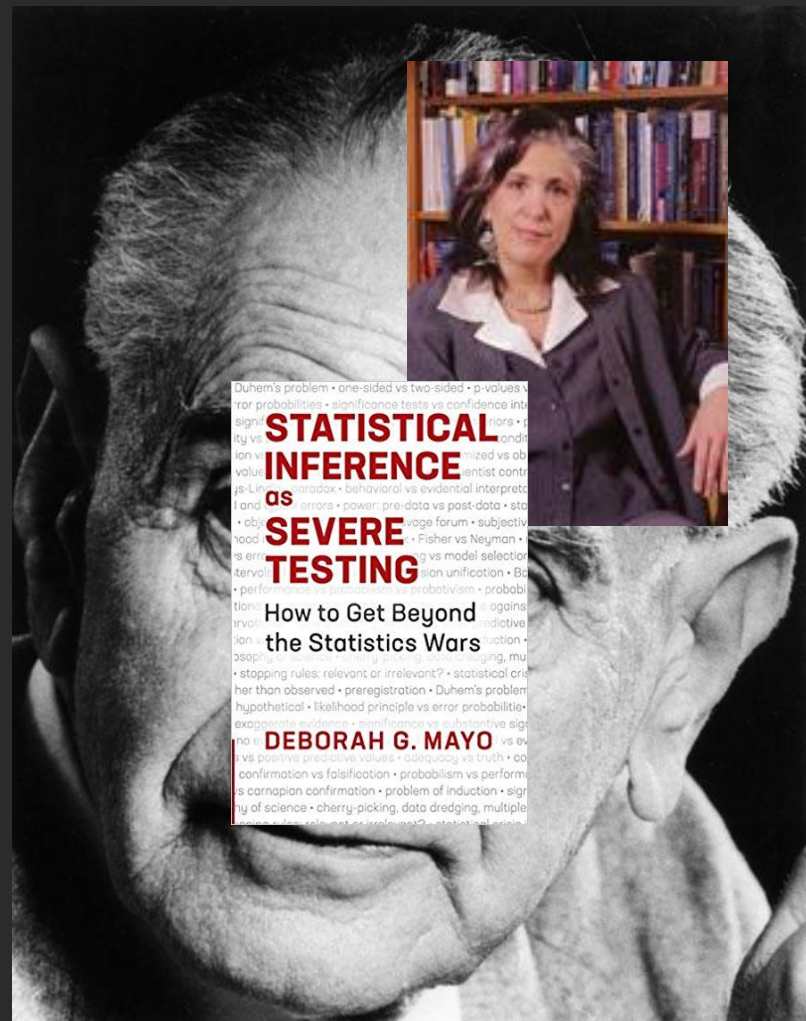


Karl Popper

- Problem of demarcation
- Falsifiability
- Problem of Induction

It's harder to prove wrong that mercury being in retrograde will affect someone's mood.

It's easier to prove wrong that items, when dropped from similar heights will fall at different rates



Deduction vs Induction

All swans are white.

Roger is a swan.

Roger is white.

Melvin the swan is white.

Gary the swan is white.

Mary the swan is white.

Terry the swan is white.

Cherry the swan is white.

All swans are white (?)

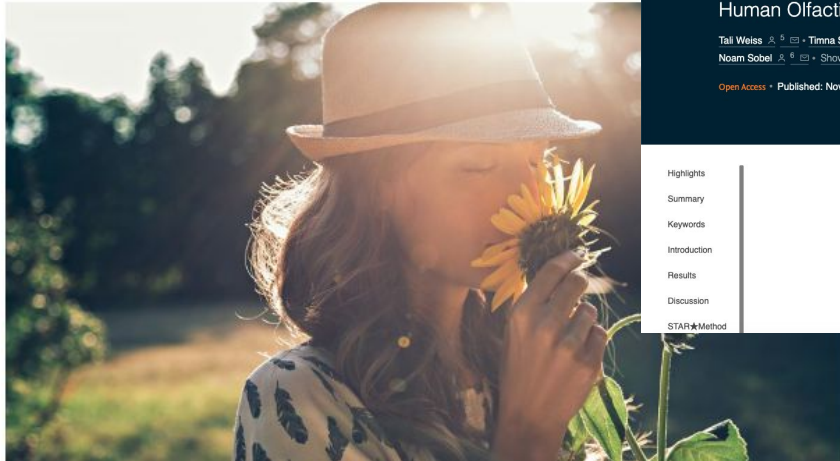


Problem of Induction

Women Missing Brain's Olfactory Bulb Can Still Smell, Puzzling Scientists

By Yasemin Saplakoglu - Staff Writer 13 hours ago Health

Researchers have discovered a small group of people that seem to defy medical science.



Neuron

Log in Register Subscribe Claim

CASE STUDY | ONLINE NOW

Human Olfaction without Apparent Olfactory Bulbs

Tali Weiss ¹ ² • Timna Soroka ³ • Lior Gorodisky • ... Edna Furman-Haran • Thijs Dhollander • Noam Sobel ¹ ² • Show all authors • Show footnotes

Open Access • Published: November 06, 2019 • DOI: <https://doi.org/10.1016/j.neuron.2019.10.006>

Highlights

Summary

Keywords

Introduction

Results

Discussion

STAR+Method

Highlights

- Humans can have normal olfaction without apparent olfactory bulbs
- Olfaction without apparent bulbs is seen in 0.6% of women, but not in men
- Olfaction without apparent bulbs is associated with left-handedness

Summary

PDF Figures Save Share Reprints Request

PlumX Metrics

Cell
Career Network

The best jobs
in life science

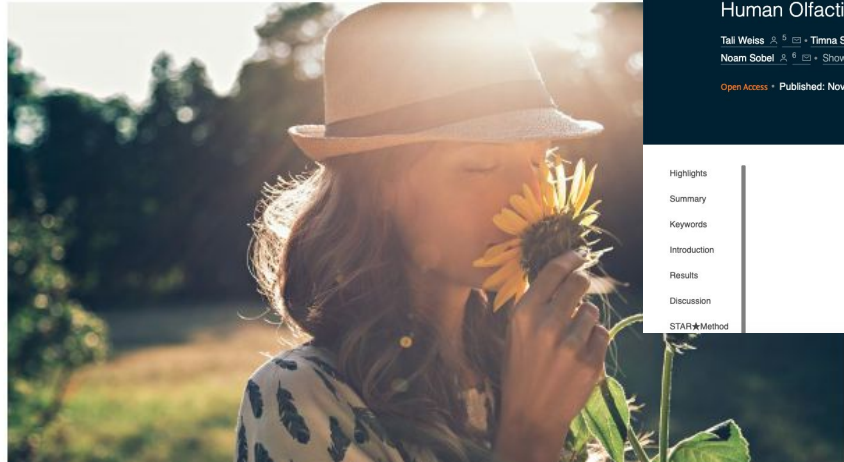
Feedback

Problem of Induction

Women Missing Brain's Olfactory Bulb Can Still Smell, Puzzling Scientists

By Yasemin Saplakoglu - Staff Writer 13 hours ago Health

Researchers have discovered a small group of people that seem to defy medical science.



Is every day you live more evidence that you will continue to live?!?

Neuron

Log inRegisterSubscribeClaim

CASE STUDY | ONLINE NOW

Human Olfaction without Apparent Olfactory Bulbs

Tali Weiss · Timna Soroka · Lior Gorodisky · ... Edna Furman-Haran · Thijs Dhollander · Noam Sobel · Show all authors · Show footnotes

[Open Access](#) · Published: November 06, 2019 · DOI: <https://doi.org/10.1016/j.neuron.2019.10.006>

Highlights

Summary

Keywords

Introduction

Results

Discussion

STAR Method

Highlights

- Humans can have normal olfaction without apparent olfactory bulbs
- Olfaction without apparent bulbs is seen in 0.6% of women, but not in men
- Olfaction without apparent bulbs is associated with left-handedness

Summary

PDFFiguresSaveShareReprintsRequest

PlumX Metrics

The best jobs in life science

Feedback

Problem of Induction

Formulation of the problem [\[edit\]](#)

In [inductive reasoning](#), one makes a series of observations and [infers](#) a new claim based on them. For instance, from a series of observations that a woman walks her dog by the market at 8 am on Monday, it seems valid to infer that next Monday she will do the same, or that, in general, the woman walks her dog by the market every Monday. That next Monday the woman walks by the market merely adds to the series of observations, it does not prove she will walk by the market every Monday. First of all, it is not certain, regardless of the number of observations, that the woman always walks by the market at 8 am on Monday. In fact, [David Hume](#) would even argue that we cannot claim it is "more probable", since this still requires the assumption that the past predicts the future.

Second, the observations themselves do not establish the validity of inductive reasoning, except inductively. [Bertrand Russell](#) illustrated this point in *The Problems of Philosophy*:

Domestic animals expect food when they see the person who usually feeds them. We know that all these rather crude expectations of uniformity are liable to be misleading. The man who has fed the chicken every day throughout its life at last wrings its neck instead, showing that more refined views as to the uniformity of nature would have been useful to the chicken.

In several publications it is presented as a story about a turkey, fed every morning without fail, who following the laws of induction concludes this will continue, but then his throat is cut on Thanksgiving Day.^{[\[3\]](#)}



Usually inferred from repeated observations: "The sun always rises in the east."



Usually not inferred from repeated observations: "If someone dies, it's never me."

We can't accumulate evidence for a theory by just adding more data since the addition of one piece of contrary evidence (the appearance of a black swan) has the potential to destroy our theory.

This has happened over and over again, see the history of science.

So how do we get around this problem?

// Exploit the asymmetry between getting data to establish a theory and finding data to be critical of it. Enter NHST.

Null Hypothesis Significance Testing

- Instead of accumulating evidence FOR a theory. We instead set up TWO competing hypotheses.
- H_0 : Null Hypothesis: Assumes nothing is happening.
- H_1 : Alternative Hypothesis: Assumes something is happening.



Null Hypothesis Significance Testing Examples

- I am interested in theory that people who have plant based diet will have lower cholesterol than those who eat a mixed diet.
- How do I begin to build support for this theory?
- Can't just go around asking people, "Well I know one guy who eats just meat and HIS cholesterol is just fine" (Induction problem)
- Need to be able to generalize this theory...



Null Hypothesis Significance Testing Examples

- So instead of building FOR theory, we instead say “I have a (null) hypothesis that there is no difference in cholesterol levels between plant only eaters and those who eat a mixed diet.”
- If this hypothesis were to be proven wrong, what are we left with?
- A competing (alternative) hypothesis noting there IS a difference between these two groups (hopefully in the direction we thought!)



Types of Errors

	H0 True	H1 True
Significant Finding	False Positive	True Positive
Non-Significant Finding	True Negative	False Negative

How to Remember Types of Errors

Never confuse Type I and II errors again:

Just remember that the Boy Who Cried Wolf caused both Type I & II errors, in that order.

First everyone believed there was a wolf, when there wasn't. Next they believed there was no wolf, when there was.

Substitute "effect" for "wolf" and you're done.

Kudos to @danolner for the thought. Illustration by Francis Barlow
"De pastoris puero et agricolis" (1687). Public Domain. Via [wikimedia.org](https://commons.wikimedia.org/wiki/File:De_pastoris_puero_et_agricolis.jpg)



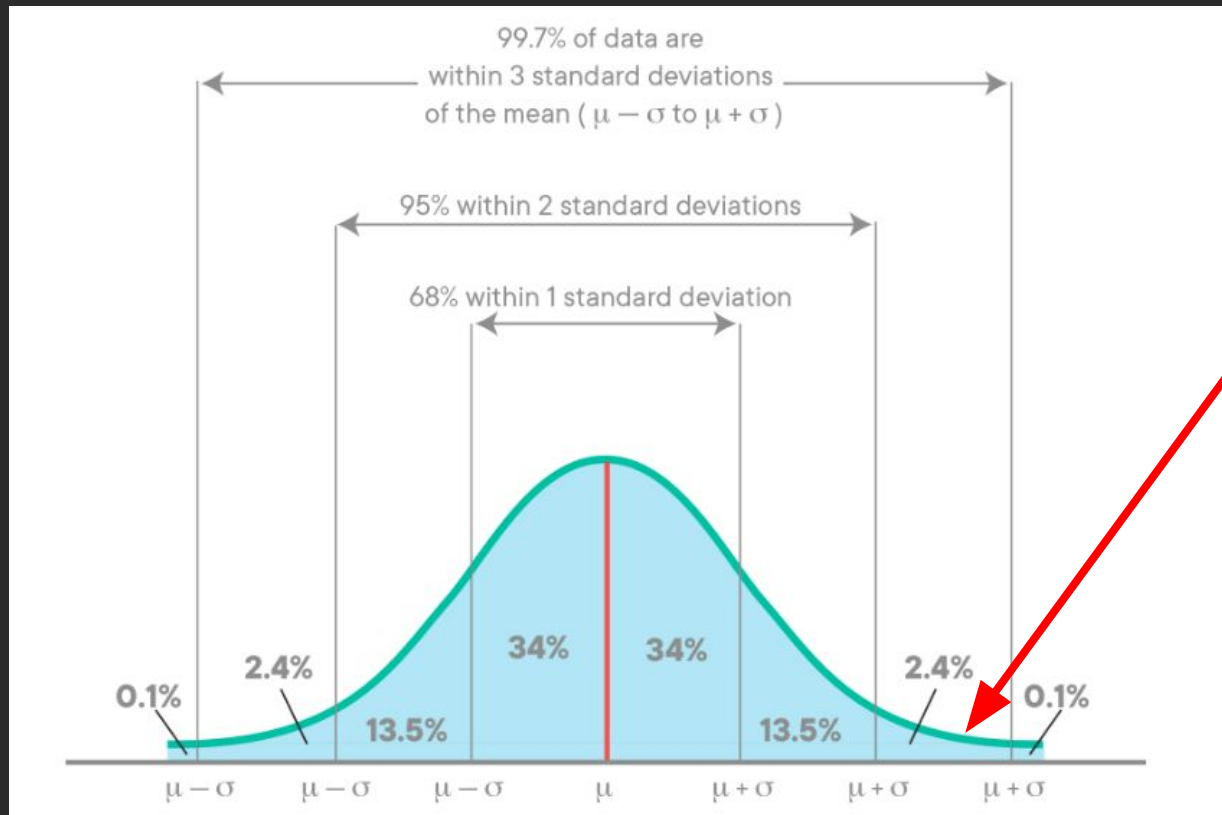
Types of Errors

	H0 True	H1 True
Significant Finding	False Positive	True Positive
Non-Significant Finding	True Negative	False Negative

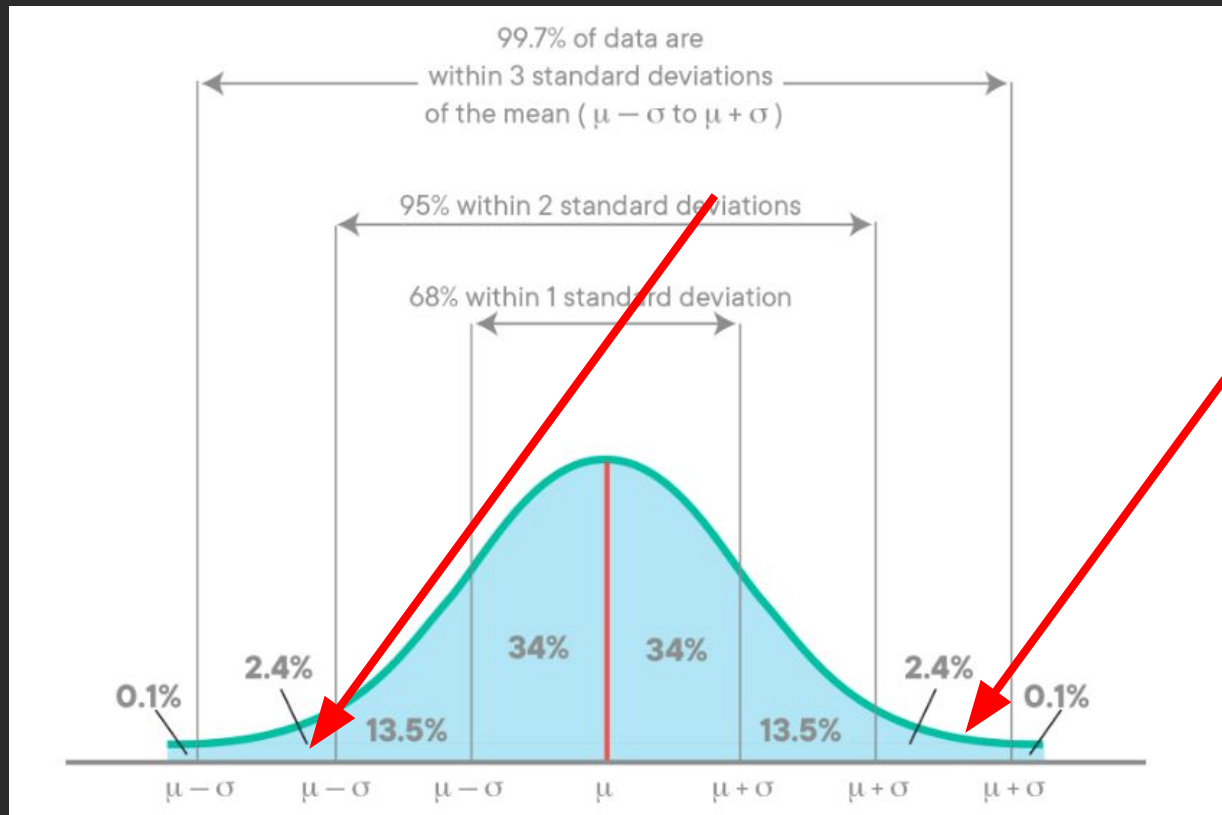
Types of Errors

	H0 True (50%)	H1 True (50%)
Significant Finding $\alpha = 5\%$, $1-\beta=80\%$	False Positive $5\%*50\%=2.5\%$	True Positive $80\%*50\%=40\%$
Non-Significant Finding $1-\alpha = 95\%$, $\beta=20\%$	True Negative $95\%*50\%=47.5\%$	False Negative $20\%*50\%=10\%$

Rejection Region (One Tailed)



Rejection Region (Two Tailed)



Settings

Solve for? ☐ Power ☐ Alpha ☐ n ☒ d

Power ($1 - \beta = 0.8$)



Significance level ($\alpha = 0.05$)



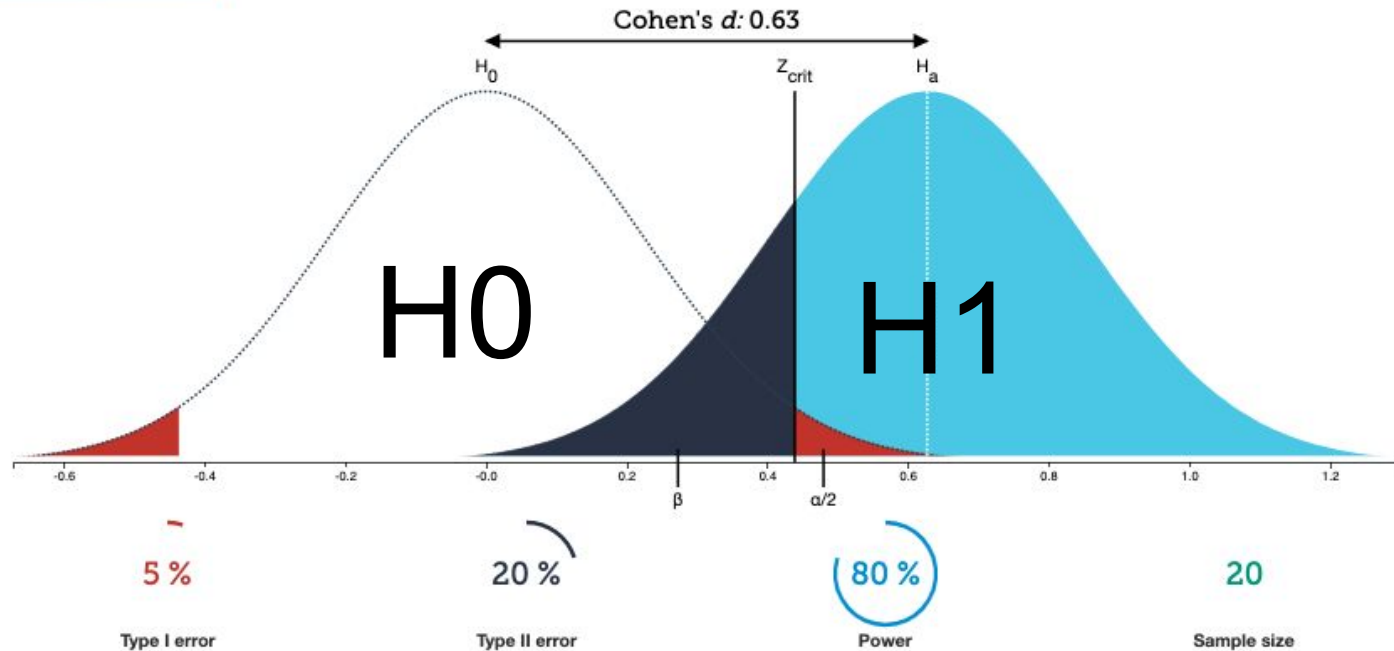
Sample size ($n = 20$)



One-tailed

Two-tailed

Reset zoom



Calculating a Test Statistic

- The general formula for a test statistic is ...

$$\frac{\text{Statistic} - \text{Parameter}}{\text{Standard error}}$$



If we know standard deviation, we use z test

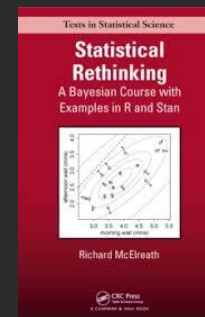
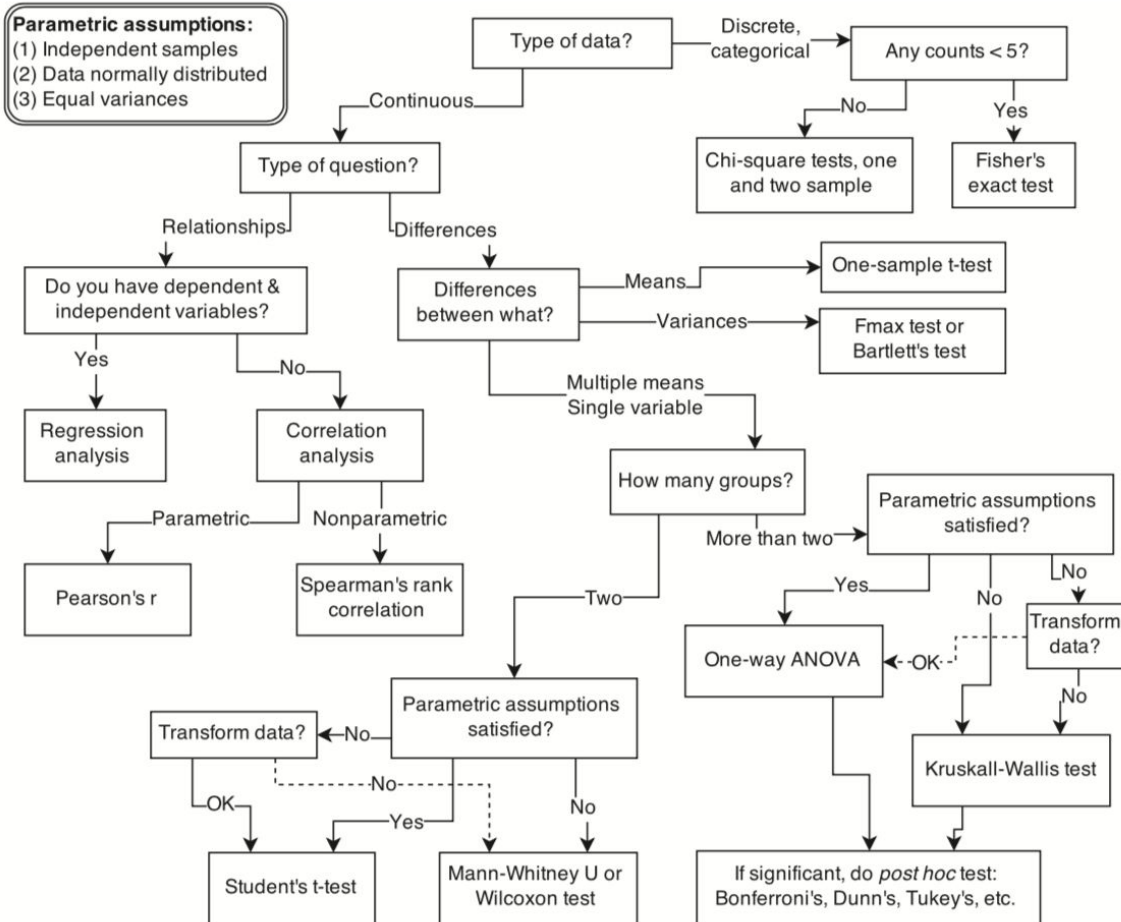
$$z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

$$\frac{\text{Statistic} - \text{Parameter}}{\text{Standard error}}$$

If we don't know standard deviation, we used t test

$$t = \frac{\bar{X} - \mu}{s_{\bar{X}}} \quad s_{\bar{X}} = \frac{s}{\sqrt{n}}$$





Common statistical tests are linear models

Last updated: 02 April, 2019

See worked examples and more details at the accompanying notebook: <https://lindeloef.github.io/tests-as-linear>

	Common name	Built-in function in R	Equivalent linear model in R	Exact?	The linear model in words	Icon
Simple regression: $\text{lm}(y \sim 1 + x)$	y is independent of x P: One-sample t-test N: Wilcoxon signed-rank	t.test(y) wilcox.test(y)	$\text{lm}(y \sim 1)$ $\text{lm}(\text{signed_rank}(y) \sim 1)$	✓ for N > 14	One number (intercept, i.e., the mean) predicts y. - (Same, but it predicts the <i>signed rank</i> of y.)	
	P: Paired-sample t-test N: Wilcoxon matched pairs	t.test(y1, y2, paired=TRUE) wilcox.test(y1, y2, paired=TRUE)	$\text{lm}(y_2 - y_1 \sim 1)$ $\text{lm}(\text{signed_rank}(y_2 - y_1) \sim 1)$	✓ for N > 14	One intercept predicts the pairwise $y_2 - y_1$ differences. - (Same, but it predicts the <i>signed rank</i> of $y_2 - y_1$.)	
	y ~ continuous x P: Pearson correlation N: Spearman correlation	cor.test(x, y, method='Pearson') cor.test(x, y, method='Spearman')	$\text{lm}(y \sim 1 + x)$ $\text{lm}(\text{rank}(y) \sim 1 + \text{rank}(x))$	✓ for N > 10	One intercept plus x multiplied by a number (slope) predicts y. - (Same, but with <i>ranked x</i> and y)	
	y ~ discrete x P: Two-sample t-test P: Welch's t-test N: Mann-Whitney U	t.test(y1, y2, var.equal=TRUE) t.test(y1, y2, var.equal=FALSE) wilcox.test(y1, y2)	$\text{lm}(y \sim 1 + G_2)^A$ $\text{glm}(y \sim 1 + G_2, \text{weights}=\dots)^A$ $\text{lm}(\text{signed_rank}(y) \sim 1 + G_2)^A$	✓ ✓ for N > 11	An intercept for group 1 (plus a difference if group 2) predicts y. - (Same, but with one variance <i>per group</i> instead of one common.) - (Same, but it predicts the <i>signed rank</i> of y.)	
Multiple regression: $\text{lm}(y \sim 1 + x_1 + x_2 + \dots)$	P: One-way ANOVA N: Kruskal-Wallis	aov(y ~ group) kruskal.test(y ~ group)	$\text{lm}(y \sim 1 + G_2 + G_3 + \dots + G_N)^A$ $\text{lm}(\text{rank}(y) \sim 1 + G_2 + G_3 + \dots + G_N)^A$	✓ for N > 11	An intercept for group 1 (plus a difference if group $\neq 1$) predicts y. - (Same, but it predicts the <i>rank</i> of y.)	
	P: One-way ANCOVA	aov(y ~ group + x)	$\text{lm}(y \sim 1 + G_2 + G_3 + \dots + G_N + x)^A$	✓	- (Same, but plus a slope on x.) <i>Note: this is discrete AND continuous. ANCOVAs are ANOVAs with a continuous x.</i>	
	P: Two-way ANOVA	aov(y ~ group * sex)	$\text{lm}(y \sim 1 + G_2 + G_3 + \dots + G_N + S_2 + S_3 + \dots + S_K + G_2 * S_2 + G_3 * S_3 + \dots + G_N * S_K)$	✓	Interaction term: changing sex changes the y ~ group parameters. <i>Note: $G_{2:N,N}$ is an indicator (0 or 1) for each non-intercept levels of the group variable. Similarly for $S_{2:N,K}$ for sex. The first line (with G_i) is main effect of group, the second (with S_i) for sex and the third is the group * sex interaction. For two levels (e.g. male/female), line 2 would just be "S_2" and line 3 would be S_2 multiplied with each G_i.</i>	[Coming]
	Counts ~ discrete x N: Chi-square test	chisq.test(groupXsex_table)	Equivalent log-linear model $\text{glm}(y \sim 1 + G_2 + G_3 + \dots + G_N + S_2 + S_3 + \dots + S_K + G_2 * S_2 + G_3 * S_3 + \dots + G_N * S_K, \text{family}=\dots)^A$	✓	Interaction term: (Same as Two-way ANOVA.) <i>Note: Run glm using the following arguments: glm(model, family=poisson())</i> As linear-model, the Chi-square test is $\log(y) = \log(N) + \log(a) + \log(\beta_i) + \log(a\beta_i)$ where a_i and β_i are proportions. See more info in the accompanying notebook .	Same as Two-way ANOVA
	N: Goodness of fit	chisq.test(y)	$\text{glm}(y \sim 1 + G_2 + G_3 + \dots + G_N, \text{family}=\dots)^A$	✓	(Same as One-way ANOVA and see Chi-Square note.)	1W-ANOVA

List of common parametric (P) non-parametric (N) tests and equivalent linear models. The notation $y \sim 1 + x$ is R shorthand for $y = 1 \cdot b + a \cdot x$ which most of us learned in school. Models in similar colors are highly similar, but really, notice how similar they *all* are across colors! For non-parametric models, the linear models are reasonable approximations for non-small sample sizes (see "Exact" column and click links to see simulations). Other less accurate approximations exist, e.g., Wilcoxon for the sign test and Goodness-of-fit for the binomial test. The signed rank function is `signed_rank = function(x) sign(x) * rank(abs(x))`. The variables G_i and S_i are "[dumm coded](#)" [indicator variables](#) (either 0 or 1) exploiting the fact that when $\Delta x = 1$ between categories the difference equals the slope. Subscripts (e.g., G_2 or y_1) indicate different columns in data. `lm` requires long-format data for all non-continuous models. All of this is exposed in greater detail and worked examples at <https://lindeloef.github.io/tests-as-linear>.

^A See the note to the two-way ANOVA for explanation of the notation.

^B Same model, but with one variance per group: `glm(value ~ 1 + G2, weights = varIdent(form = ~1|group), method="ML")`.



Steps of Hypothesis Testing(ish)

- State Null and Alternative Hypotheses
- Set Your Alpha Level (and pick direction of your test)
- Select Sample and Collect
- Locate Region of Rejection / Critical Values
- Compute Your Test Statistic
- Decide if you reject null hypothesis!



Step One

Scenario: On a standardized anagram task, $\mu = 26$ anagrams solved with a $\sigma = 4$. A researcher tests whether the arousal from anxiety is distracting and will decrease performance. A sample of $n = 14$ anxiety patients is tested on the task. Their average performance is 23.36 anagrams.

- a. **Step one:** State the null and alternative hypotheses.

$$H_0 : \mu = 26 \qquad H_A : \mu < 26$$

Always consider directionality in this step!!!



Settings

Solve for?

☐ Power

☐ Alpha

☐ n

☒ d

Power ($1-\beta = 0.8$)

Significance level ($\alpha = 0.05$)

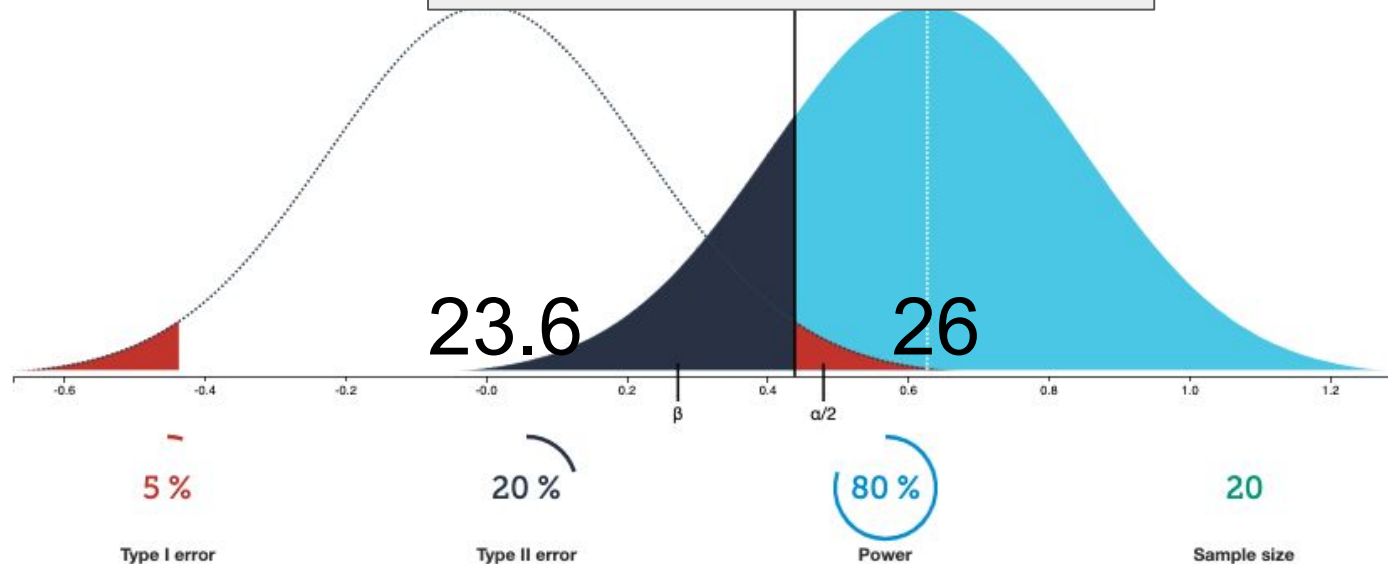
Sample size ($n = 20$)

One-tailed

Two-tailed

Reset zoom

← Is this Difference Significant? →



Step Two

Scenario: On a standardized anagram task, $\mu = 26$ anagrams solved with a $\sigma = 4$. A researcher tests whether the arousal from anxiety is distracting and will decrease performance. A sample of $n = 14$ anxiety patients is tested on the task. Their average performance is 23.36 anagrams.

- b. **Step two:** Set the criterion for rejecting H_0 . Alpha is usually set to .05, but could be other values depending on the research context. Again, directionality is important to consider.



Step Three and Four

Scenario: On a standardized anagram task, $\mu = 26$ anagrams solved with a $\sigma = 4$. A researcher tests whether the arousal from anxiety is distracting and will decrease performance. A sample of $n = 14$ anxiety patients is tested on the task. Their average performance is 23.36 anagrams.

- c. **Step three:** Select the sample and collect your data.
- d. **Step four:** Locate the region of rejection and the critical value(s) of your test statistic. Again, directionality is important to consider.

Step Five

Scenario: On a standardized anagram task, $\mu = 26$ anagrams solved with a $\sigma = 4$. A researcher tests whether the arousal from anxiety is distracting and will decrease performance. A sample of $n = 14$ anxiety patients is tested on the task. Their average performance is 23.36 anagrams.

- e. **Step five:** Compute the appropriate test statistic. σ is known, so we use the z test.

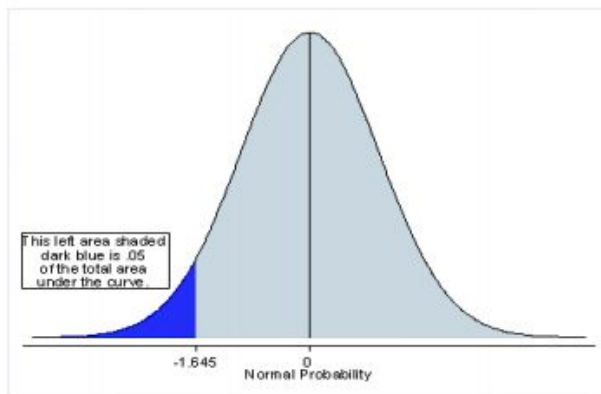
$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{14}} = 1.07 \qquad z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{23.36 - 26}{4/\sqrt{14}}$$

$$z = \frac{-2.64}{1.07} = -2.47$$

Step Six

Scenario: On a standardized anagram task, $\mu = 26$ anagrams solved with a $\sigma = 4$. A researcher tests whether the arousal from anxiety is distracting and will decrease performance. A sample of $n = 14$ anxiety patients is tested on the task. Their average performance is 23.36 anagrams.

- f. **Step six:** Decide whether to reject H_0 . Is -2.47 more extreme than the critical value?



Step Six (Only using positive critical test table...*)

Scenario: On a standardized anagram task, $\mu = 26$ anagrams solved with a

$\sigma = 4$. A research

and will decrease
tested on the task

f. Step six: Decid
than the critical

Degrees of freedom	Significance level						
	One-tailed test:	5%	2.5%	1%	0.5%	0.2%	0.1%
1		3.078	4.015	6.385	8.163	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15			2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725

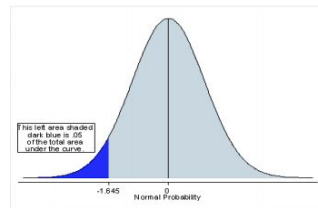
See that our
CRITICAL
VALUE of
-2.47 is
GREATER
THAN our ONE
tailed test of
(-)1.7771



Step Six

Scenario: On a standardized anagram task, $\mu = 26$ anagrams solved with a $\sigma = 4$. A researcher tests whether the arousal from anxiety is distracting and will decrease performance. A sample of $n = 14$ anxiety patients is tested on the task. Their average performance is 23.36 anagrams.

- f. **Step six:** Decide whether to reject H_0 . Is -2.47 more extreme than the critical value?

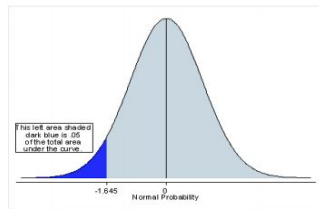


Since our TEST STATISTIC (2.47) is greater than our CRITICAL VALUE (1.77) we have reason to believe that the sample of data we gathered comes from a population so different from the original we can declare it to be significantly different (in the *statistical sense*).

Step Six

Scenario: On a standardized anagram task, $\mu = 26$ anagrams solved with a $\sigma = 4$. A researcher tests whether the arousal from anxiety is distracting and will decrease performance. A sample of $n = 14$ anxiety patients is tested on the task. Their average performance is 23.36 anagrams.

- f. **Step six:** Decide whether to reject H_0 . Is -2.47 more extreme than the critical value?

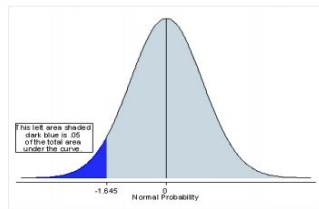


The blue region also corresponds to a very small p value since it's so far out in the tail. Here we did not compute the actual p value of the TEST STATISTIC, but know it's less than .05 (set earlier) since it is larger than our CRITICAL VALUE.

Step Six

Scenario: On a standardized anagram task, $\mu = 26$ anagrams solved with a $\sigma = 4$. A researcher tests whether the arousal from anxiety is distracting and will decrease performance. A sample of $n = 14$ anxiety patients is tested on the task. Their average performance is 23.36 anagrams.

- f. **Step six:** Decide whether to reject H_0 . Is -2.47 more extreme than the critical value?



Computers will calculate your p value in the future!! All it tells you is “the probability of the observed, or more extreme, data, under the assumption that the null-hypothesis is true”.

NO MORE. NO LESS. IT IS NOT THE PROBABILITY OF YOUR HYPOTHEIS BEING CORRECT! YOU NEED BAYESIAN STATS FOR THAT!!

<http://daniellakens.blogspot.com/2017/12/understanding-common-misconceptions.html>



NHST Visualization



Settings

Solve for?

☐ Power

☐ Alpha

☐ n

☒ d

Power ($1-\beta = 0.8$)

Significance level ($\alpha = 0.05$)

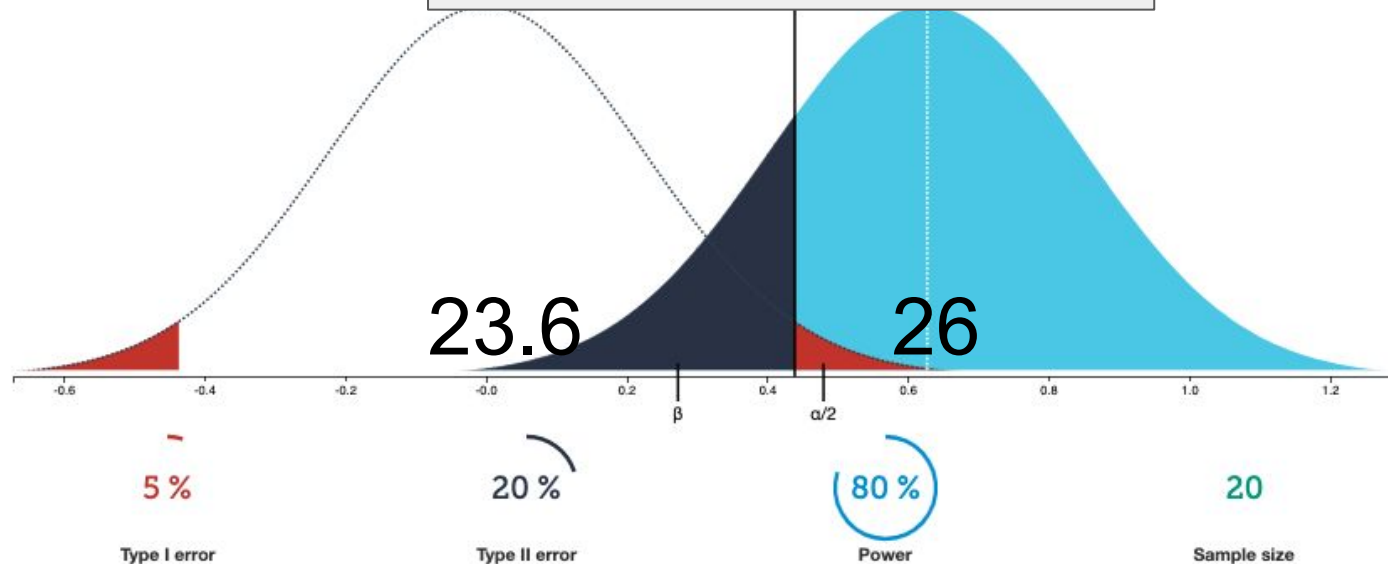
Sample size ($n = 20$)

One-tailed

Two-tailed

Reset zoom

← Is this Difference Significant? →



Settings

Solve for?

☐ Power

☐ Alpha

☐ n

☒ d

Power ($1-\beta = 0.8$)

Significance level ($\alpha = 0.05$)

Sample size ($n = 20$)

One-tailed

Two-tailed

Reset zoom

← Is this Difference Significant? →

YES!!!

23.6

26

5 %

Type I error

20 %

Type II error

80 %

Power

20

Sample size



Is it worth
writing home
about?

That depends on your theoretical framework, reliability of your tools, if the samples were representative of your larger population and didn't have any biases, repeats again under different conditions... and all that. Need to also consider your alpha, power, size of the effect, (how big the difference is in a standardized measure that incorporates errors) and how many people were in your sample!

Just because it's SIGNIFICANT does not mean it's MEANINGFUL.

Statistical significant JUST gives us a sanity check when we investigate this paradigm again and again and again to make sure we are on the right track!

Good statistics will never make up for bad theory!!



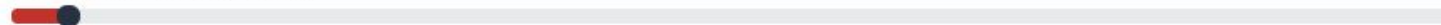
Settings

Solve for? ☐ Power ☐ Alpha ☐ n ☒ d

Power ($1-\beta = 0.8$)



Significance level ($\alpha = 0.05$)



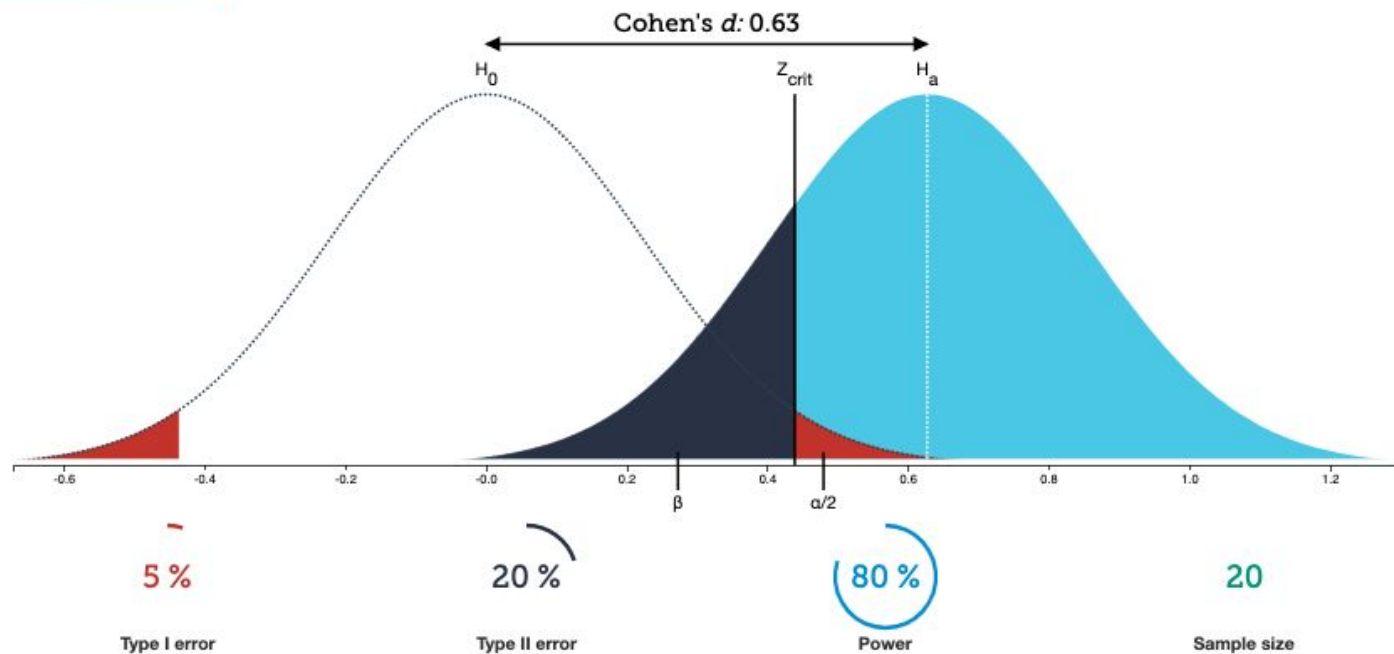
Sample size ($n = 20$)



One-tailed

Two-tailed

Reset zoom



<https://rpsychologist.com/d3/NHST/>



**Turn to your
partner, explain
one of the
following to them,
write one question
down about
something you
don't understand**

//

- Scientific Method (Theory vs Practice)
- The Problem of Induction
- Popper, Falsifiability, Demarcation
- Logic of Null Hypothesis Significance Testing
- Four Types of Outcomes in NHST
- p values
- Run through a single statistical test