FRTN30 Network Dynamics
Hand-in 2

Due: 2023-04-29

Instructions:

- You may implement your solutions in any language you see fit, but the TAs can only guarantee you support with MATLAB/Octave. Your code should be written in a quite general manner, i.e., if a question is slightly modified, it should only require slight modifications in your code as well. Upload a PDF of your solutions together with your code in a separate, runnable file to the hand-in under *Assignments* on the Canvas course page.
- The PDF should present a detailed solution, including a description of what you are doing, a quick summary of the theory used, proper presentation of results (including readable figures with axis labels) with interpretations. Also include your name and Lucat-id on the first page.
- Comment your code well. Clarity is more important than efficiency.
- Late submission is discouraged: you get 1 points in your exam (out of 25) for each on-time submission.
- Collaboration policy: collaboration such as exchange of ideas among students is encouraged, however, every student has to submit her/his own manuscript (in pdf format) and code, and specify whom she/he has collaborated with and on what particular part of the work.
- Up to five of the best hand-ins may be rewarded with one extra point.

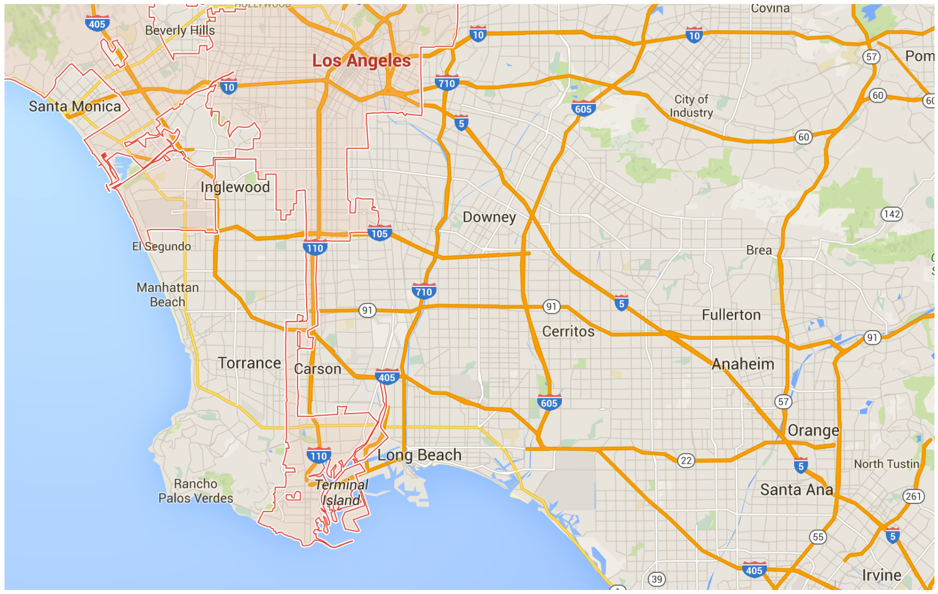

Figure 1: The highway network in Los Angeles.

Preparation:

To be well-prepared for the assignment, it is recommended to read Chapter 4 (*Network flow optimization*) in the lecture notes. Furthermore, it is helpful to look at the tasks from exercise session 6.

1 Traffic tolls in Los Angeles

In this task we will study traffic flows on the highway network in Los Angeles, see Figure 1. To simplify the problem, an approximate highway map is given in Figure 2, covering part of the real highway network. The node-link incidence matrix B, for this traffic network is given in the file `traffic.mat`. The rows of B are associated with the nodes of the network and the columns of B with the links. The i-th column of B has 1 in the row corresponding to the tail node of link e_i and (-1) in the row corresponding to the head node of link e_i. Each node represents an intersection between highways (and some of the area around).

Each link $e_i \in \{e_1, \dots, e_{28}\}$, has a maximum flow capacity C_{e_i}. The capacities are given as a vector C_e in the file `capacities.mat`. The flow capacities are retrieved from measured data as described in Appendix A. Furthermore, each link has a minimum traveling time l_{e_i}, which the drivers experience when the road is empty. In the same manner as for the capacities, the minimum traveling times are given as a vector l_e in the file `traveltime.mat`. Theses values are simply retrieved by dividing the length of the highway segment with the assumed speed limit 60 miles/hour.

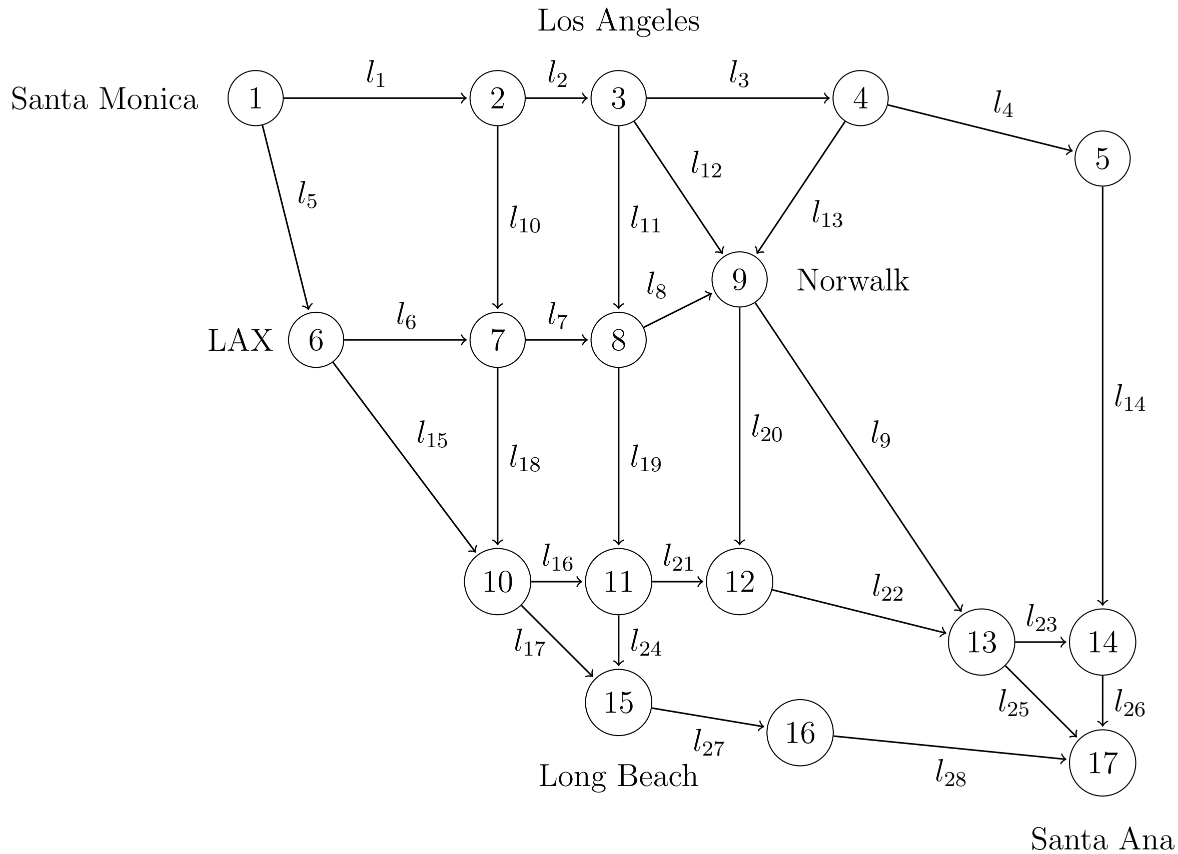

Figure 2: Some possible paths from Santa Monica (node 1) to Santa Ana (node 17).

For each link, we introduce the delay function

$$d_e(f_e) = \frac{l_e}{1 - f_e/C_e}, \quad 0 \le f_e < C_e.$$

For $f_e \ge C_e$, the value of $d_e(f_e)$ is considered as $+\infty$.

Use MATLAB to solve the following tasks.

a) Find the shortest path between node 1 and 17. This is equivalent to the fastest path (path with shortest traveling time) in an empty network. Hint: use MATLAB function `graphshortestpath`.

   Notice that, since quite a big area around an intersection is included in the node, the traveling time might be a bit shorter than one could expect.

b) Find the maximum flow between node 1 and 17.

   Hint: use MATLAB function `graphmaxflow`.

c) Given the flow vector in `flow.mat`, compute the external inflow or outflow at each node.

For the following subproblems you will use CVX which is a MATLAB-based modeling system for convex optimization. To download CVX, go to `cvxr.com/cvx/download`. You will use CVX to solve optimization problems concerning flows on the approximative traffic network of Los Angeles. As an example, the following flow optimization problem

$$\begin{aligned} \text{minimize} \ & \sum_{e=1}^{M} f_e^2 \\ \text{subject to} \ & Bf = \nu \\ & 0 \leq f \leq C \end{aligned}$$

where $M = |\mathcal{E}|$ and $\nu = \lambda - \mu$, can be written for CVX in MATLAB as

```
cvx_begin
    variable f(M)
    minimize sum(f.*f)
    subject to
            B*f == nu
            0 ≤ f ≤ c
cvx_end
```

Installation instructions can be found in CVX's users' guide `http://cvxr.com/cvx/doc/install.html`. If anything is unclear with how to download or use CVX, you can ask the TAs.

In the following, we assume that all net inflows are zero except for the one at node 1, with the same value computed from 1-c). We assume that all of the net inflow at node 1 leaves the network at node 17.

Use CVX to solve the following tasks.

d) Find the social optimum f^* with respect to the delays on the different links $d_e(f_e)$. For this, minimize the cost function

$$\sum_{e\in\mathcal{E}} f_e d_e(f_e) = \sum_{e\in\mathcal{E}} \frac{f_e l_e}{1 - f_e/C_e} = \sum_{e\in\mathcal{E}} \left(\frac{l_e C_e}{1 - f_e/C_e} - l_e C_e \right)$$

subject to the flow constraints.

Hint: use CVX function `inv_pos`.

e) Find the Wardrop equilibrium $f^{(0)}$. For this, use the cost function

$$\sum_{e\in\mathcal{E}} \int_0^{f_e} d_e(s)ds.$$

f) Introduce tolls, such that the toll on link e is $\omega_e = f_e^* d_e'(f_e^*)$, where f_e^* is the flow at the system optimum. Now the delay on link e is given by $d_e(f_e) + \omega_e$. compute the new Wardrop equilibrium $f^{(\omega)}$. What do you observe?

g) Instead of the total delay, let the cost be the total additional delay compared to the total delay in free flow be given by

$$c_e(f_e) = f_e(d_e(f_e) - l_e)$$

subject to the flow constraints. Compute the system optimum f^* for the costs above. For this, use CVX function `quad_over_lin`, and construct tolls ω_e^*, $e \in \mathcal{E}$ such that the new Wardrop equilibrium with the constructed tolls $f^{(\omega^*)}$ coincides with f^*. Compute the new Wardrop equilibrium with the constructed tolls $f^{(\omega^*)}$ to verify your result.

A PeMS data for the traffic network

Link	**Freeway**	**PM Start**	**PM End**	f_{max} [veh/h]	f [veh/h]
l_1	I10-E	3.46	12.21	8741	7524
l_2	I10-E	12.76	15.45	9864	6537
l_3	SR60-E	3.97	11.71	13350	11139
l_4	SR60-E	12.24	23.44	10926	9282
l_5	I405-S	52.93	45.14	13707	9282
l_6	I105-E	2.50	7.20	6960	6398
l_7	I105-E	7.56	13.20	7422	6728
l_8	I105-E	13.86	17.30	6678	5988
l_9	I5-S	123.21	114.71	6297	5951
l_{10}	I110-S	20.53	14.22	11102	9557
l_{11}	I710-S	17.54	11.14	8899	7423
l_{12}	I5-S	129.92	123.63	8970	7423
l_{13}	I605-S	19.15	12.41	9753	6814
l_{14}	SR57-S	15.88	5.05	9719	8536
l_{15}	I405-S	44.37	37.08	9083	7652
l_{16}	SR91-E	0.56	5.40	7416	6537
l_{17}	I405-S	36.34	31.82	13353	11924
l_{18}	I110-S	13.33	9.93	11216	9640
l_{19}	I710-S	10.29	8.33	10947	8161
l_{20}	I605-S	9.35	7.25	10019	8603
l_{21}	SR91-E	6.20	10.22	8732	7974
l_{22}	SR91-E	11.37	18.14	10763	9446
l_{23}	SR91-E	19.17	23.87	6677	5562
l_{24}	I710-S	7.54	4.29	9403	6719
l_{25}	I5-S	113.99	107.25	10355	9455
l_{26}	SR57-S	4.75	0.37	9067	6686
l_{27}	I405-S	30.55	20.65	11990	10833
l_{28}	SR22-E	2.54	11.46	8258	7403

Comments:

- All the flow data is acquired from PeMS `http://pems.dot.ca.gov/`.
- The maximum flow f_{max} is detected on one of the segments between the nodes, during the time period 12-01-2016 to 12-02-2016.
- The flow vector f is the maximum flow measured on each segment at 08-02-2016 between 17.00 and 18.00.
- The free flow traveling time was computed by assuming 60 miles/h for all links.