FRTN30 Network Dynamics

Assignment 2

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1 Traffic tolls in Los Angeles

In this task, we will study traffic flows from Santa Monica (node 1) to Santa Ana (node 17) in Los Angeles.

a) Find the shortest path between node 1 and 17

I use Matlab function *shortestpath*. The shortest path between node 1 and 17 is $1 \rightarrow 2 \rightarrow 3 \rightarrow 9 \rightarrow 13 \rightarrow 17$ with the shortest traveling time is 0.533 hours.

b) Find the maximum flow between node 1 and 17

I use Matlab function $maxflow$ in this task. The maximum flow between node 1 and 17 is 22448.

c) Compute the external inflow or outflow at each node

Multiplying matrix B by matrix $flow$, we have the inflow and outflow matrix.

Type	**Node**	**Values**
Outflow	1	16806
	2	8570
	3	19448
	4	4957
	6	4768
	7	413
	10	1169
Inflow	5	-746
	8	-2
	9	-5671
	11	-5
	12	-7131
	13	-380
	14	-7412
	15	-7810
	16	-3430
	17	-23544

Table 1: External inflow and outflow values

d) Find the social optimum f^* with respect to the delays on the different links de(fe)

We use CVX to minimize the cost function

$$\sum_{e \in \epsilon} f_e d_e(f_e) = \sum_{e \in \epsilon} (\frac{l_e C_e}{1 - f_e / C_e} - l_e C_e)$$

subject to the flow constraints.

The optimal value is 25943.6. The flow vector can be found from the below table.

Link	Flow value	Link	Flow value
1	6642	15	5525
2	6059	16	2854
3	3132	17	4886
4	3132	18	2215
5	10164	19	464
6	4638	20	2338
7	3006	21	3318
8	2543	22	5656
9	3132	23	2373
10	583	24	0
11	0	25	6414
12	2927	26	5505
13	0	27	4886
14	3132	28	4886

Table 2: The social optimum f^*

e) Find the Wardrop equilibrium $f^{(0)}$

We use the cost function:

$$\sum_{e\in\epsilon}\int_0^{f_e} d_e(s)ds = \sum_{e\in\epsilon}\int_0^{f_e}\frac{l_e}{1-s/C_e}ds = \sum_{e\in\epsilon} -l_eC_elog(1-f_e/C_e)$$

Using CVX as the previous task, the optimal value is 15729.6. The flow vector can be found from the below table.

Link	Flow value	Link	Flow value
1	6716	15	5445
2	6716	16	2353
3	2367	17	4933
4	2367	18	1842
5	10090	19	697
6	4645	20	3036
7	2804	21	3050
8	2284	22	6087
9	3418	23	2587
10	0	24	0
11	177	25	6919
12	4171	26	4954
13	0	27	4933
14	2367	28	4933

Table 3: The Wardrop equilibrium $f^{(0)}$

f) Compute the new Wardrop equilibrium $f^{(\omega)}$

The toll on link e is:

$$\omega_e = f_e^* d_e^{'}(f_e^*) = f_e^* \frac{l_e C_e}{(C_e - f_e^*)^2}$$

We calculate the cost function with the toll:

$$\sum_{e \in \epsilon} \int_0^{f_e} d_e(s)ds = \sum_{e \in \epsilon} \int_0^{f_e} (\frac{l_e}{1 - s/C_e} + \omega_e)ds = \sum_{e \in \epsilon} (-l_e C_e log(1 - f_e/C_e) + f_e \omega_e)$$

Using CVX, the optimal value is 61886. As I can observe, the new Wardrop equilibrium $f^{(\omega)}$ is the same as the social optimum f^*.

Link	**Flow value**	**Link**	**Flow value**
1	6642	15	5525
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6	4638	20	2338
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8	2543	22	5656
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11	0	25	6414
12	2927	26	5505
13	0	27	4886
14	3132	28	4886

Table 4: The new Wardrop equilibrium $f^{(\omega)}$

g) Compute the new Wardrop equilibrium with the constructed tolls $f^{(\omega^*)}$

We use the new cost function

$$\sum_{e \in \epsilon} f_e(d_e(f_e) - l_e) = \sum_{e \in \epsilon} (\frac{l_e C_e}{1 - f_e/C_e} - l_e C_e - l_e f_e)$$

The optimal value is 15095.5. The system optimum f^* can be found from the below table.

Link	**Flow value**	**Link**	**Flow value**
1	6653	15	5510
2	5775	16	3044
3	3420	17	4882
4	3420	18	2415
5	10153	19	444
6	4643	20	2008
7	3106	21	3487
8	2662	22	5495
9	3009	23	2204
10	878	24	0
11	0	25	6301
12	2355	26	5624
13	0	27	4882
14	3420	28	4882

Table 5: The system optimum f^*

Using the equation [4.29] from chapter [4.5] in the lecture notes and the new cost function, we have:

$$\omega_e^* = c_e^{'}(f_e^*) - d_e(f_e^*) = f_e^* d_e^{'}(f_e^*) - l_e$$

Similar to the previous task, we compute the new Wardrop equilibrium with the constructed tolls $f^{(\omega^*)}$. The optimal value is 50795.9 and the new Wardrop equilibrium coincides with f^*.