COURSE SUMMARY NOTES

METHODS UNIT 2

Contents

1 Indices

Index Laws

Multiplication

$$a^m \times a^n = a^{m+n}$$

Division

$$a^m \div a^n = a^{m-n}$$

Power of a Power

$$(a^m)^n = a^{mn}$$

Power of a Product

$$(ab)^m = a^m b^n$$

Negative Indices

$$a^{-1} = \frac{1}{a}$$

Fractional Indices

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

Powers of Primes

1	2	3	4	5	6	7
2	4	8	16	32	64	128
3	9	27	91	243		
5	25	125	625			
7	49	343				

Note:

1. Recognising these powers is very helpful in solving questions involving indices.

Solving Equation Involving Indices

$$a^m = a^n \implies m = n \tag{1.1}$$

$$m^a = n^a \implies m = n \tag{1.2}$$

2 Exponential Equations

Base Function

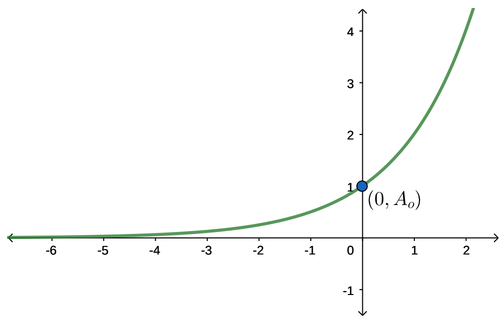

Figure 2.1: Exponential Growth $k > 1$.

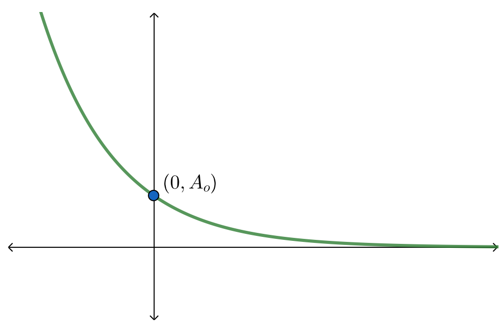

Figure 2.2: Exponential Growth $0 < k < 1$.

$$A = A_o \cdot k^x \tag{2.1}$$

Transformations of exponential Functions

$$f(x) = a \cdot k^{bx+c} + d \tag{2.2}$$

3 Sequences & Series

Sequence

A sequence is an ordered set of **terms**.

Series

A series is the sum of the terms in a sequence.

Arithmetic Progression

An arithmetic progression (**AP**), also known as an arithmetic sequence, is a sequence of n numbers such that the differences between successive terms is a constant (d).

Recursive Definition

$$T_n = T_{n-1} + d \tag{3.1}$$

$$T_1 = a \tag{3.2}$$

Explicit Definition

$$T_n = a + (n-1)d$$

Sum to n terms.

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right) \tag{3.3}$$

$$S_n = \frac{n}{2}\left(a + l\right) \tag{3.4}$$

Geometric Progression

An geometric progression (**GP**), also known as an arithmetic sequence, is a sequence of n numbers such that the ratios between successive terms is a constant (r).

Recursive Definition

$$T_n = T_{n-1} \times r \tag{3.5}$$

$$T_1 = a \tag{3.6}$$

Explicit Definition

$$T_n = a \times r^{(n-1)} \tag{3.7}$$

Sum to n terms.

$$S_n = \frac{a(1 - r^n)}{1 - r} \tag{3.8}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \tag{3.9}$$

Sum to ∞ terms.

$$S_\infty = \frac{a}{1 - r} \text{ for } -1 < r < 1 \tag{3.10}$$

4 Differentiation

Limiting Chord

To find the tangent to a curve we could start by finding the slope of secant as an estimation and then reduce the distance between the two points of intersection.

First Principles of Differentiation

From the limiting chord process we can establish the first principle of differentiation.

$$\text{Gradient Function} = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} \tag{4.1}$$

Notation

$$y \to \frac{\mathrm{d}\, y}{\mathrm{d}\, x} \tag{4.2}$$

$$f(x) \to f'(x) \tag{4.3}$$

Power Rule

Note:

1. Proving 4.5 using first principals is strongly encouraged.

$$f(x) = a \cdot x^n \tag{4.4}$$

$$f'(x) = a \cdot n \cdot x^{n-1} \tag{4.5}$$

Sum Rule

$$y = f(x) \pm g(x) \tag{4.6}$$

$$\frac{dy}{dx} = f'(x) \pm g'(x) \tag{4.7}$$

5 Applications of Differentiation

Sketching Functions

1. Find x and y intercepts.

$$y - \text{intercept } (x = 0)$$
$$x - \text{intercept } (y = 0)$$

2. Find turning points and teset nature.

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 0$$

3. Test nature of function at extremes.

$$\text{as } x \to \pm\infty \quad y \to ?$$

4. Determine gradient at other x values as required.

Note:

1. To avoid possible error of the second derivative test in identifying the nature of a turning point (i.e. $f(x) = x^4$) check $\frac{\mathrm{d}y}{\mathrm{d}x}$ "sufficiently close" to either side of the stationary point.

Global Max/Min

To determine the maximum or minimum value for a function, $f(x)$, over the the domain $a \leq x \leq b$.

1. Find all solutions to $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. Let the solutions be $x_1, x_2, \cdots$ etc.
2. Compare $f(a),\ f(x_1),\ f(x_2), \cdots,\ f(b)$ to determine max/min

6 Rectilinear Motion

Rectilinear Motion

$$x(t) \rightarrow \text{Displacement of a body as a function of time.}$$

$$\frac{\mathrm{d}\, x(t)}{\mathrm{d}\, t} = v(t) \rightarrow \text{Velocity of a body as a function of time.}$$

$$\frac{\mathrm{d}^2\, x(t)}{\mathrm{d}\, t^2} = \frac{\mathrm{d}\, v(t)}{\mathrm{d}\, t} = a(t) \rightarrow \text{Acceleration of body as a function of time.}$$

Note:

1. $x(0)$ is initial displacement.
2. $v(0)$ is initial velocity
3. $a(0)$ is initial acceleration.
4. If the displacement is on the positive side of the origin then the displacement is positive.
5. If the direction of travel is in the positive direction then the velocity is positive.
6. Visualise acceleration as a force. If the force is acting in the positive direction then the acceleration is positive.

7 Anit-Differentiation

Anti-Differentiation Power Rule

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$$

Note:

1. The constant of integration can found by substituting in a point and solving for c.