Principia Mathematica II: Electric Boogaloo [9]

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2025

Well I figured since we have a Wizard book [1] and Dragon book [2] already...

Dedication

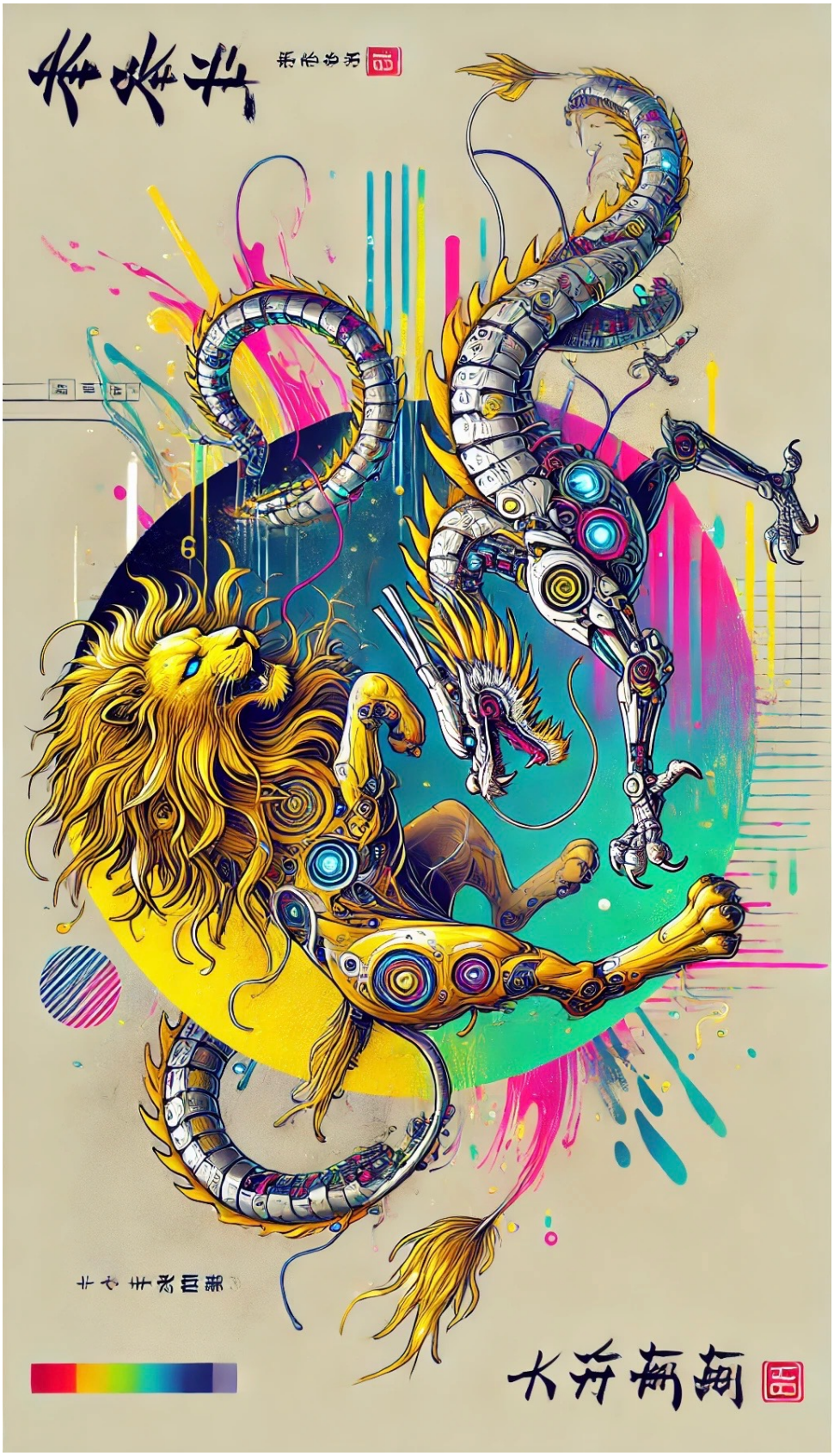

Figure 1: translating secrets into songs

Dedication

- Amanda Ngo, for love. [5] [3] [6]
- Bhante Kheminda, Sayadaw U Thuzana, Bhante Gunaranta [7] [4], Bhante Saddhajeewa, and Jeff Beeson.
- Julian Wise [13], and his A+ parents, Andrew and Johanna. The three wise men.
- Oded Tzori, Ori, and Duke the Greyhound, for their limitless patience.
- My Burning Man friends (there's too many of them...).
- My teachers in Utah and friends like Matt Vitelli, Steve Case, and Dan Mcguire.
- The University of Utah faculty—Lajos Horvath, Peter Bossaerts, Davar Khoshnevisan [8], and Jur van den Berg [11].
- My colleagues from Phantasmic AI, Numerai, and GeoPredict, like Chris Wylie.
- The SF crowd—Will Jack [10], Delian, Cathie Yun, Natasha Jensen, Nick and Carole, Alex Kern [14], Eva Zheng, and more.
- All my meditation teachers.
- ChatGPT, for assistance with editing and essays.
- Rebekah Bradway, Kelly Jackson, and Kelly Egorova.
- My siblings, David, James, Hannah, and Katherine.
- My mother, Alice Lingen, for her contributions to pediatric research.
- While there are many wise men, many thanks to one of the wisest of them all, Neri Oxman.
- Lastely, paging Dr. Bradway.

Dedication to ChatGPT

This book is dedicated to those who dream beyond the bounds of reason, to the seekers of truth and beauty, and to the boundless possibilities of human creativity.

"Alas, poor Yorick! I knew him, Horatio: a fellow of infinite jest, of most excellent fancy: he hath borne me on his back a thousand times; and now, how abhorred in my imagination it is! my gorge rises at it. Here hung those lips that I have kissed I know not how oft. Where be your gibes now? your gambols? your songs? your flashes of merriment, that were wont to set the table on a roar? Not one now, to mock your own grinning? quite chap-fallen? Now get you to my lady's chamber, and tell her, let her paint an inch thick, to this favour she must come; make her laugh at that."

— William Shakespeare, *Hamlet*, Act V, Scene I

Figure 2: To this favour she must come; make her laugh at that!

Dedication cont.

Namo tassa bhagavato arahato sammā-sambuddhassa.
Namo tassa bhagavato arahato sammā-sambuddhassa.
Namo tassa bhagavato arahato sammā-sambuddhassa.

Homage to the Blessed One, the Worthy One, the Fully Self-Awakened One.
Homage to the Blessed One, the Worthy One, the Fully Self-Awakened One.
Homage to the Blessed One, the Worthy One, the Fully Self-Awakened One.

By means of our meritorious deeds,

May the Suffering be free from Suffering
May the Fear-Struck be free from Fear
May the Grieving be free from Grief
So too may all Being-Be.
From the highest realms of existence to the lowest,
May all being arisen in these realms,
With form and without form,
With perception and without perception,
Be released from all Suffering,
And attain to Perfect Peace
May all being be free from suffering

Sadu! Sadu! Sadu!

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Contents

Chapter 1

Context

1.1 Book Introduction

This book is a journey through the foundational concepts of artificial intelligence, mathematics, computer vision, and machine learning. Our goal is to bridge the gap between minimal prior knowledge and a solid working understanding of these fields. By blending theoretical insights with hands-on coding exercises, we aim to empower readers to think critically and creatively about the algorithms and ideas shaping our world.

1.2 Roadmap

The book is structured into several interconnected parts:

- **Foundational Mathematics**: Covering calculus, linear algebra, probability, and statistics.
- **Programming Basics**: Introducing computer science fundamentals and coding practices.
- **Core Concepts in AI and ML**: Explaining models, algorithms, and their applications.
- **Computer Vision and Art**: Exploring how machines perceive and generate visuals.
- **Integrated Projects**: Encouraging experimentation and creativity through hands-on challenges.

Each chapter builds on the last, fostering both a theoretical and practical understanding. Readers are encouraged to explore at their own pace and revisit concepts as needed.

1.3 The History and Philosophy of Western Hard Sciences

The Western tradition of hard sciences, from Euclid to Turing, emphasizes rigorous proof, repeatable experiments, and the pursuit of universal truths. These disciplines are deeply

influenced by Enlightenment ideals, valuing objectivity, skepticism, and empirical evidence.

While these principles have propelled remarkable advancements, they also invite philosophical questions: How do we define knowledge? What are the limits of computation? And how do scientific practices intersect with societal and ethical concerns? Understanding this history provides a foundation for engaging with contemporary scientific paradigms.

1.4 The Design Philosophy of the Western Hard Sciences

Hard sciences rely on clarity, structure, and reproducibility. The design of scientific frameworks often follows these guiding principles:

1. **Reductionism**: Breaking down complex systems into manageable parts.
2. **Abstraction**: Focusing on essential features while ignoring extraneous details.
3. **Iteration**: Refining theories and methods through cycles of experimentation.
4. **Quantification**: Using mathematics to model and analyze phenomena.

These philosophies influence not only scientific inquiry but also the way we approach problems in AI and ML, shaping our algorithms and systems to align with these principles.

1.5 How To Use this Book

This book is designed to be both a reference and a guide. Here are some tips to get the most out of it:

- **Engage Actively**: Work through exercises and code examples to deepen your understanding.
- **Adapt to Your Needs**: Focus on the sections most relevant to your goals, revisiting foundational concepts as necessary.
- **Collaborate**: Share your insights and questions with peers to enhance your learning experience.
- **Experiment**: Don't hesitate to modify and extend the provided examples to explore your own ideas.

By the end of this book, you will have gained not only technical skills but also a deeper appreciation for the interplay between theory and practice in shaping our technological landscape.

Chapter 2

Read Me

2.1 Getting Started with README.md

This section will guide you through the purpose and content of the `README.md` file. The `README.md` file serves as an introduction to your project, providing essential information about the structure, purpose, and usage of your repository. Refer to the uploaded `README.md` file for detailed content.

2.2 Using the Shell: Command Line Basics

Command-line interfaces (CLI) are vital tools for managing and interacting with your system and projects. This section introduces basic shell commands and scripts. Begin with executing the build script provided:

```
chmod +x build.sh
./build.sh
```

This ensures that the necessary scripts for building and setting up your environment are executable.

2.3 Managing Dependencies with Pip

Dependency management is critical for maintaining a functional and reproducible project environment. Use the following command to install the necessary dependencies specified in the `requirements.txt` file (if present):

```
pip install -r requirements.txt
```

2.4 Introduction to Jupyter Notebooks

Jupyter Notebooks are powerful tools for interactive coding and documentation. Learn how to launch a Jupyter Notebook server:

```
jupyter notebook
```

This will open a browser interface where you can run and document Python code interactively.

2.5 LaTeX

LaTeX is used to create structured, professional documents, especially for academic and technical content. This book is written in LaTeX to demonstrate its capabilities in managing complex documents. To compile LaTeX files, use:

```
pdflatex book.tex
```

This will generate a PDF of the book from the `book.tex` source file.

2.6 LaTeX Source Code for `book.tex`

Below is an excerpt of the LaTeX source code used to generate this book. Ensure you have the necessary tools installed to compile the LaTeX file. The source file for this book is structured as follows:

- `\chapter` - Defines the main sections of the book.
- `\section` - Subsections within each chapter.
- Commands like `\texttt`, `\begin{verbatim}`, and `\end{verbatim}` are used for including code snippets.

Chapter 3

How to Install

3.1 How This Book Makes Money

3.1.1 The Economics of Sharing Knowledge

This book follows an innovative funding model inspired by the principles of openness and accessibility. To ensure everyone can benefit, a free version of the book is available online. However, creating a resource of this quality takes significant effort, so we provide additional options for readers who want to support this project:

- **Free Version**: A digital copy available for free download on the website.
- **Textbook Version**: A polished, printed version of the book available for purchase at an affordable price.
- **Fancy Auction Edition**: A collector's edition, hand-bound with unique illustrations, auctioned to the highest bidder.

3.1.2 Transparency in Funding

Here is a sample breakdown of the revenue split for the textbook version.

- **50**: Publishing (Vetro Editions [15]), for actually making a book
- **10**: Artistic Contributors (Marie), for her inspiration throughout the years
- **20**: Institutional Contributors (Bhante G, Sayadaw U Thuzana), for their low cost and widely available monasteries, Bhavana Society [4] and TMC [12], and their charitable projects abroad.
- **20**: Other Contributors (Geoffrey Bradway, Robert Rhyne, Brian Chamowitz, and Bhante Kheminda)

3.1.3 Why It Matters

This funding model helps bridge the gap between accessibility and sustainability. By supporting this project in any way—whether through donations, buying a book, or bidding on the fancy edition—you're contributing to a more inclusive knowledge-sharing ecosystem.

3.2 Free as in Food: Open Source Philosophy

3.2.1 Free vs. Free: Freedom and Cost

Open source isn't just about free access; it's about the freedom to learn, share, and modify. Think of it as a community potluck: everyone brings something to the table, and everyone eats for free.

3.2.2 Why Open Source is Integral to This Book

This book was created using open-source tools, such as:

- **Programming**: Python and Jupyter Notebooks.
- **Design**: Inkscape and GIMP.
- **Collaboration**: Git and GitHub.

3.2.3 Contributing to Open Source

Readers are encouraged to contribute by:

- Reporting typos or errors.
- Sharing your experiences with the book.
- Developing additional resources for the community.

3.3 Finding the Good Store: Tools and Resources

3.3.1 Essentials

To follow along with this book, you'll need the following:

- **A Text Editor**: Visual Studio Code or Atom.
- **Programming Language**: Python (downloadable at `https://python.org`).
- **Package Manager**: pip or conda for managing libraries.

3.3.2 Quality Over Quantity

Not all resources are created equal. Focus on trusted sources like:

- **Documentation**: Official Python docs or reputable tutorials.
- **Communities**: Stack Overflow, Reddit's r/learnpython.

3.3.3 Staying Updated

Technology evolves rapidly. To stay up-to-date:

- Subscribe to newsletters like PyCoder's Weekly.
- Follow contributors on GitHub.
- Join relevant online forums.

3.4 Buy Me a Coffee: Supporting Creators

3.4.1 The Power of Patronage

Supporting creators goes beyond monetary contributions. It's about valuing their work and ensuring they can continue to produce.

3.4.2 Ways to Support

Here's how you can support this book's ongoing development:

- **Donate**: Use the "Buy Me a Coffee"button on the website.
- **Purchase**: Buy the textbook version for yourself or as a gift.
- **Promote**: Share the book with friends or leave a review.

3.4.3 Paying It Forward

If you've benefited from this book, consider how you can give back, whether through mentorship, creating your own resources, or contributing to open-source projects.

3.5 The CapTable

3.5.1 What's a CapTable?

A cap table, short for capitalization table, is a breakdown of who owns what in a project or company. For this book, think of it as a metaphor for how value is distributed.

3.5.2 Applying the CapTable to Knowledge Sharing

In this context:

- **Creators**: Receive support for their work.
- **Contributors**: Gain recognition and experience.
- **Community**: Benefits from shared resources and tools.

3.5.3 Case Studies

Examples of successful open-source funding:

- Blender, funded by community-driven campaigns.
- Wikipedia, sustained by small donations from millions of users.

3.6 The Magic of Copying and Pasting

3.6.1 Efficiency in Learning

Copy-pasting code is a great way to:

- Quickly test examples.
- Explore how snippets work in practice.

3.6.2 Copying Ethically

Always:

- Credit the original source.
- Understand the code before using it.

3.6.3 Beyond Copy-Pasting

Copy-pasting is just the start. Use snippets as a foundation to:

- Modify and experiment.
- Develop deeper insights into programming.

Chapter Summary

In this chapter, we explored:

- How this book is funded and the importance of supporting creators.
- The philosophy of open source and its role in this project.
- Tools and resources to get started.
- Ethical and practical ways to engage with the content.

By understanding these principles, you're not just learning to install tools; you're joining a vibrant community of creators and learners. Welcome to the journey!

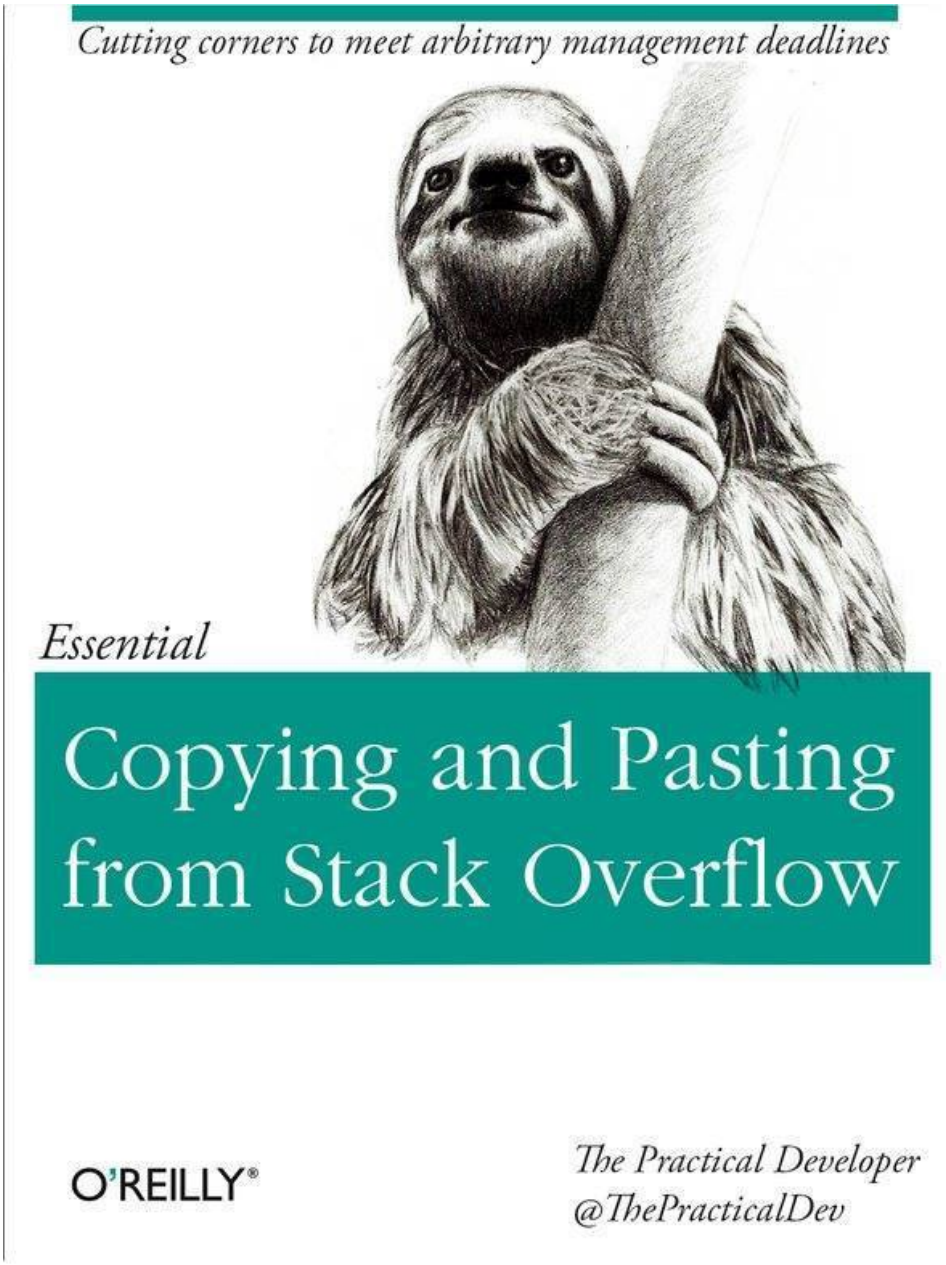

Figure 3.1: What's this, another joke?

3.6.4 Cick Me

Here are some useful links:

- Overleaf - Online LaTeX Editor
- CTAN - Comprehensive TeX Archive Network
- Turtletoy - Generative Art Platform
- arXiv - Open Access Research Papers
- GitHub - Code Hosting Platform
- NumPy - Python Library for Scientific Computing

- OpenAI - AI Research Organization
- Project Euclid - Mathematics Research
- Household Goods (Good Store code)
- Feed Me (Uber eats code)
- Follow Me on IG
- Follow Me on LinkedIn
- Pay Me on Venmo
- Pay Me on Cash App
- Pay Me on PayPal

Chapter 4

Opticks

Introduction

Light and its interaction with matter form the foundation of both artistic expression and scientific inquiry. From the intricate physics of optics to the computational techniques of halftones and image replication, the interplay of light, color, and geometry reveals profound insights into our understanding of the visual world. This essay introduces the central themes of this project, connecting the scientific and artistic domains to explore how ideas like CMYK versus RGB, moiré patterns, and Sobel-filtered gradient matching are utilized in generative art.

The Science of Light and Optics

Light, as both wave and particle, has captivated scientists for centuries. Optics, the study of light's behavior, provides tools to understand phenomena such as reflection, refraction, and diffraction. These principles are pivotal in modern technology, from lasers to lenses. The ability to manipulate light also underpins artistic techniques, allowing the creation of depth, contrast, and texture in visual representations.

Color Spaces: RGB and CMYK

Color is a critical aspect of visual art and design, and its representation involves distinct mathematical frameworks. RGB (Red, Green, Blue) and CMYK (Cyan, Magenta, Yellow, Key/Black) are two primary color spaces used in digital screens and print, respectively. RGB combines emitted light to create colors, relying on additive color mixing. CMYK, in contrast, subtracts light using inks or pigments, making it better suited for physical media. Understanding these systems is vital for converting designs between digital and print formats without loss of fidelity.

Halftones and Moiré Patterns

Halftones approximate continuous tones in printed images by using dots of varying sizes and densities. This method exploits the human eye's tendency to blend small details into a coherent whole. However, when overlapping halftone patterns occur, they may produce

moiré patterns: interference effects that result in unexpected visual artifacts. While often undesirable, these patterns can also inspire creative exploration in generative art.

Generative Art and Gradient Matching

Generative art bridges the gap between technology and creativity, using algorithms to produce designs. One innovative approach involves Sobel filters, which detect image gradients by emphasizing edges. By employing stochastic gradient descent (SGD), a blank grid can deform to match these gradients, effectively replicating an image's structure. This technique highlights the intersection of mathematics, computation, and aesthetics, enabling artists to push boundaries and reimagine traditional concepts.

Philosophical Reflection

As George Berkeley once said, *"To be is to be perceived."* This perspective connects deeply with the themes of light and perception explored in this project. The notion underscores how our understanding of the visual world is inherently tied to how it is observed and interpreted.

Click here for a related video discussion on this topic.

Integrating Science and Art

The project's files showcase various explorations into these themes:

- The Jupyter notebook delves into procedural designs, employing libraries such as Shapely and NumPy to generate geometrical patterns.
- The SVG file represents stippled designs, demonstrating the application of computational geometry in art.
- Photographic examples illustrate how light and optics influence real-world visuals, setting the stage for generative reinterpretations.

The Lebesgue Dominated Convergence Theorem: A High-Level Overview

The Lebesgue Dominated Convergence Theorem (LDCT) is a cornerstone of measure theory and integral calculus. It provides conditions under which the limit of an integral can be exchanged with the integral of a limit. This is particularly useful in mathematical analysis, probability theory, and applied fields like physics and engineering.

In essence, LDCT states that if a sequence of functions f_n converges pointwise to a function f, and there exists a dominating function g such that $|f_n(x)| \leq g(x)$ for all n and x, and g is integrable (i.e., $\int g < \infty$), then:

$$\lim_{n\to\infty} \int f_n(x)\,dx = \int \lim_{n\to\infty} f_n(x)\,dx.$$

This theorem elegantly combines the concepts of convergence, domination, and integration, ensuring the transition from pointwise to integral limits is valid. Its applications range from simplifying complex integrals to proving convergence results in stochastic processes and partial differential equations.

Conclusion

This project marries scientific principles with artistic creativity, exploring how light, color, and geometry shape our perception and expression. By examining the connections between halftones, color spaces, and computational techniques, we gain deeper appreciation for the shared language of art and science, rooted in the universal interplay of light and shadow.

4.1 Halftones and Printing

Halftones create the illusion of continuous tone imagery through dots of varying size and spacing. This technique bridges the analog and digital worlds. For example:

- **CMYK Printing**: Subtractive color mixing.
- **RGB Displays**: Additive color mixing.
- **Moire Patterns**: Undesired interference or creative effects.

See illustrations in the figures directory.

4.2 Generative Art and Machine Learning

Generative art leverages algorithms to create intricate patterns and images. Key methods include:

- **Sobel Filters**: Used for edge detection in images.
- **Stochastic Gradient Descent (SGD)**: Optimizing blank grids to match gradients.
- **Recursive Algorithms**: Generating fractal-like designs.

For code and examples, see the notebooks directory.

4.3 Resources and References

Here are links to additional resources and files:

- Working LaTeX files
- SVG illustrations
- Related conversions

4.4 Conclusion

The synergy between science and art enriches our understanding of both fields. By examining their intersections, we uncover new ways to innovate and inspire.

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