Exercise 7

Problem 2

a) $y' = -2ty^2 \qquad y(0) = 1$

$$\frac{y'}{y^2} = -2t$$

$$\int \frac{y'}{y^2}\, dy = \int -2t\, dt$$

$$-\left(\frac{1}{y} + c\right) = -\frac{2t^2}{2}$$

$$\frac{1}{y} = t^2 - C$$

$$y = \frac{1}{t^2 - C} \quad , \quad y(0) = 1$$

$$y(0) = \frac{1}{0 - C} = 1 \implies \underline{C = -1}$$

$$y(t) = \frac{1}{t^2+1}$$

$$y(0{,}4) = \frac{1}{(0{,}4)^2+1} = \frac{1}{1{,}16} = \underline{\underline{\frac{25}{29}}}$$

4-step Eulers method $h=0,1$

b)

$y_{i+1} \approx y_i + hf(t_i, y_i)$

$f(t_i, y_i) = y'$ $\quad y(0)=1 \Rightarrow y_0 = 1$

$y_1 = y_0 + h\cdot(-2t_0 y_0^2) = 1 + 0,1\cdot 0 = \underline{1}$

$y_2 = 1 + h(-2\cdot 0,1\cdot 1^2) = 1 + 0,1\cdot(-0,2) = 1 - 0,02$
$= \underline{0,98}$

$y_3 = 0,98 + 0,1(-2\cdot 0,2\cdot 0,98^2) = \underline{0,9416}\ *$

$y_4 = * + 0,1(-2\cdot 0,3\cdot *) = \underline{\underline{0,8884}}$

$e_4 \approx |0,8884 - 0,8621| = \underline{\underline{0,0263}}$

c) Heuns method $h=0,2$

$\left.\begin{array}{l} k_1 = f(x_n, y_n) \\ k_2 = f(x_n+h,\ y_n + h\cdot k) \end{array}\right\} \Rightarrow y_{n+1} = y_n + \frac{h}{2}(k_1+k_2)$

$x_0 = 0$
$y_0 = 1$
$f(x,y) = -2xy^2$

y_1: $\left.\begin{array}{l} k_{1,0} = -2\cdot 0\cdot 1^2 = 0 \\ k_{2,0} = -2\cdot 0,2\cdot 1 + 0,2\cdot 0 = -0,4 \end{array}\right\}$ $y_1 = 1 + 0,1\cdot(0-0,4)$
$= \underline{0,96}$

y_2: $\left.\begin{array}{l} k_{1,1} = -2\cdot 0,2\cdot 0,96^2 = -0,36864 \\ k_{2,1} = -2\cdot 0,4\cdot(0,96+0,2\cdot -0,36864)^2 = -0,62838 \end{array}\right\}$ $y_2 = \underline{\underline{0,8603}}$

$$e_2 = |y(0,4) - y_2| = \underline{\underline{1,77 \cdot 10^{-3}}}$$

d) RK4 1 step

$k_1 = \underline{0}$

$k_2 = \underline{\underline{-0,4}}$

$k_3 = \underline{-0,33856}$

$k_4 = \underline{\underline{-0,59799}}$

$$y_1 = y_0 + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 0,8617$$

$$e_1 = |y_1 - y(0,4)| = \underline{\underline{4,1 \cdot 10^{-4}}}$$

<u>Smallest error with RK4</u>

Problem 3

a) $m_1 \cdot u'' = -k_1 \cdot u + k_2(v-u)$, $u(0)=a$

$u'(0)=b$

$m_2 \cdot v'' = -k_2(v-u)$

$v(0)=c$

$v'(0)=d$

Skriv om til 4 1.ordens ODE'er:

$$\left.\begin{aligned} y_1 &= u \\ y_2 &= v \\ y_3 &= u' \\ y_4 &= v' \end{aligned}\right\} \quad \begin{aligned} y_1' &= u' = y_3 \\ y_2' &= v' = y_4 \\ y_3' &= u'' = \frac{1}{m_1}(-k \cdot u + k_2(v-u)) \\ y_4' &= v'' = \frac{1}{m_2}(-k(v-u)) \end{aligned}$$

$y_1(0)=a$

$y_2(0)=c$

$y_3(0)=b$

$y_4(0)=d$

b) 1 step m Heuns metode

$a=0$, $b=1$, $c=0$, $d=1$

$u_{n+1} = y_n + hf(x_n, y_n)$

$y_{n+1} = y_n + \frac{h}{2}\left(f(x_n, y_n) + f(x_{n+1}, u_{n+1})\right)$

$$\vec{u}_1 = \begin{bmatrix} u(0) \\ v(0) \\ u'(0) \\ v'(0) \end{bmatrix} + h \cdot \begin{bmatrix} y_1'(x_0) \\ y_2'(x_0) \\ y_3'(x_0) \\ y_4'(x_0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} + 0,1 \cdot \begin{bmatrix} 1 \\ 1 \\ \frac{1}{10}(-100 \cdot 0 + 200(0)) \\ -\frac{200}{5}(0-0) \end{bmatrix} = \underline{\underline{\begin{bmatrix} 0,1 \\ 0,1 \\ 1 \\ 1 \end{bmatrix}}}$$

$$\vec{y}_1 = \begin{bmatrix} u(0) \\ v(0) \\ u'(0) \\ v'(0) \end{bmatrix} + \frac{h}{2}\left(\begin{bmatrix} y_1'(x_1) \\ y_2'(x_1) \\ y_3'(x_1) \\ y_4'(x_1) \end{bmatrix} + f(x_1, u_1)\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} + 0{,}05\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + 0{,}05\begin{bmatrix} 1 \\ 1 \\ \frac{1}{10}(-100 \cdot 0{,}1 + 200 \cdot 0) \\ -\frac{200}{5}(0) \end{bmatrix}$$

$$= \begin{bmatrix} 0{,}1 \\ 0{,}1 \\ 1 - 0{,}05 \\ 1 \end{bmatrix} = \begin{bmatrix} 0{,}1 \\ 0{,}1 \\ 0{,}95 \\ 1 \end{bmatrix}$$