Exercise #5

18. September 2023

Exercises marked with a (J) should be handed in as a Jupyter notebook.

Problems marked with 4N or 4D must be submitted only by the respective course, while unmarked problems must be submitted by both courses.

Optional exercises will not be corrected.

A serious attempt in solving $70 - 75\%$ of all the tasks must be done in **both** theory and coding (if present) in order to pass the exercise.

Problem 1.

Compute the Laplace transform of the following functions.

a) $f(t) = (t-2)^4$.

b) $f(t) = te^{-t}$.

c) $f(t) = e^{-5t}\sin(t)$.

d) $f(t) = e^{-2t}\cos^2(3t) - 3t^2e^{3t}$

Problem 2.

Find the inverse Laplace transform of the following functions.

a) $F(s) = \dfrac{2s}{s^2 - 3}$.

b) $F(s) = \dfrac{s^2 + s + 1}{s^3 + s}$.

c) $F(s) = \dfrac{1}{(s-1)^2(s+1)}$.

Problem 3.

Decide for each of the following statements whether it is true or false. Explain your answer.

a) If f and g are two functions for which the Laplace transform exists, then $\mathcal{L}(f - g) = \mathcal{L}(f) - \mathcal{L}(g)$.

b) If f and g are two functions, for which the Laplace transform exists, then $\mathcal{L}(f \cdot g) = \mathcal{L}(f) \cdot \mathcal{L}(g)$.

c) If the function f satisfies $0 \leq f(t)$ for all $t \geq 0$, then $\mathcal{L}(f)(s) \geq 0$ for all s for which $\mathcal{L}(f)(s)$ exists.

d) If the function f is continuous and satisfies $0 \leq f(t) \leq 1$ for all $t \geq 0$, then $\mathcal{L}(f)(s)$ exists for all $s > 0$.

Problem 4.

Use the Laplace transform in order to solve the following initial value problems:

a) $-y'' + 2y' - 3y = 0$, $y(0) = 1$, $y'(0) = 2$.

b) $y'' - 3y' + 2y = e^{3t}$, $y(0) = 1$, $y'(0) = 0$.

c) $y'' - 10y' + 9y = 5t$, $y(0) = -1$, $y'(0) = 2$.