Report: A General Purpose JAX-based Controller

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Introduction

This report presents the implementation and analysis of two control approaches - Classical PID and Neural Network controllers - applied to three different systems: a bathtub water level control system, a Cournot competition economic model, and a monetary policy control system.

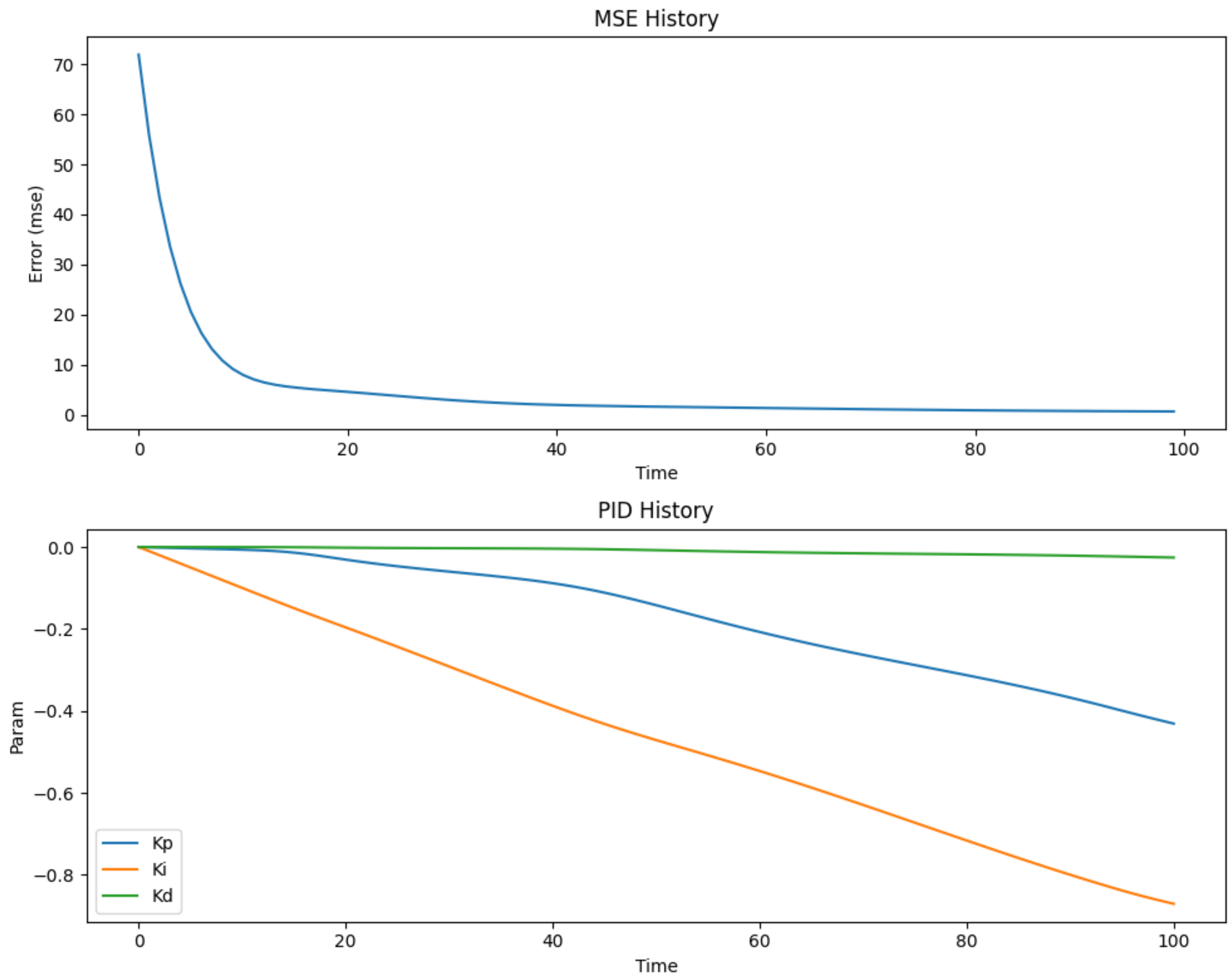
Classical PID Controller

1. Bathtub (PID)

Configuration

Parameter	Value
Controller Type	Classic PID
Initial Parameters	Kp = 0, Ki = 0, Kd = 0
Epochs	100
Timesteps	50
Learning Rate	0.01
Disturbance Range	-0.01, 0.01
Cross-sectional Area (bathtub)	20
Cross-sectional Area (drain)	0.2
Initial Water Height	50

Plot



Analysis

The MSE history plot shows rapid initial convergence from a high error value around 70, quickly dropping to below 10 within the first 20 epochs. The error continues to decrease more gradually, reaching 1.6 by epoch 50 and 0.6 by epoch 100 (not in this plot).

The PID parameter history reveals interesting dynamics in the controller's adaptation. Kp (proportional) and Ki (integral) terms show steady negative growth, with Kp reaching approximately -0.4 and Ki approaching -0.8 by the end of training. The Kd (derivative) term remains relatively stable near 0, suggesting minimal influence of the rate of change in the control strategy.

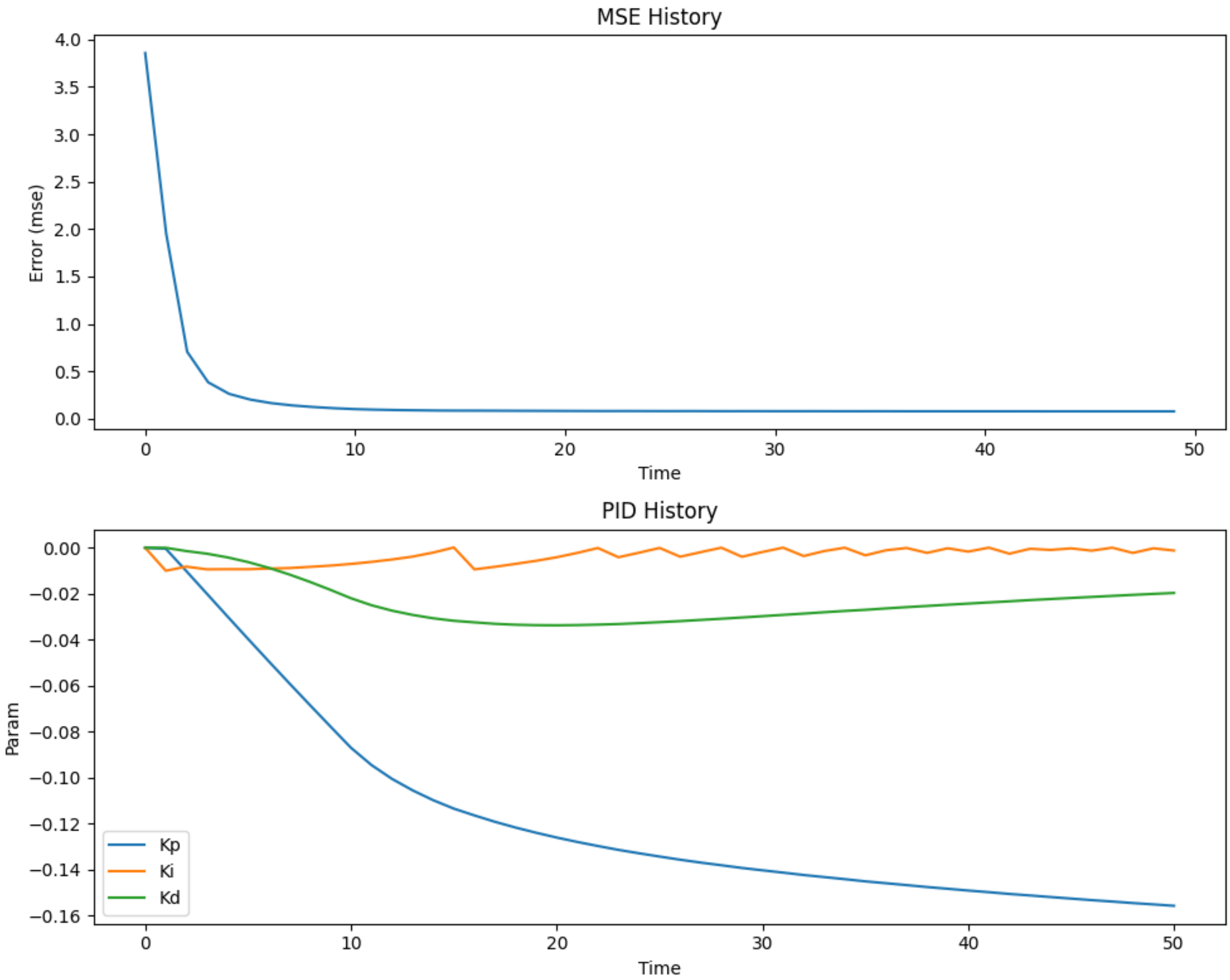
2. Cournot Competition (PID)

Configuration

Parameter	Value
Controller Type	Classic PID
Initial Parameters	Kp = 0, Ki = 0, Kd = 0
Epochs	50

Parameter	Value
Timesteps	50
Learning Rate	0.01
Disturbance Range	-0.01, 0.01
Max Price	10
Marginal Cost	2
Target Profit	4
Q1	2
Q2	2
Initial Profit p1	0

Plot



Analysis

The MSE history demonstrates grear convergence characteristics, starting from an error of approximately 4.0 and rapidly decreasing to below 0.5 within the first 10 epochs. The system maintains stable performance for the remainder of the training period, with the error asymptotically approaching a minimal value around 0.077.

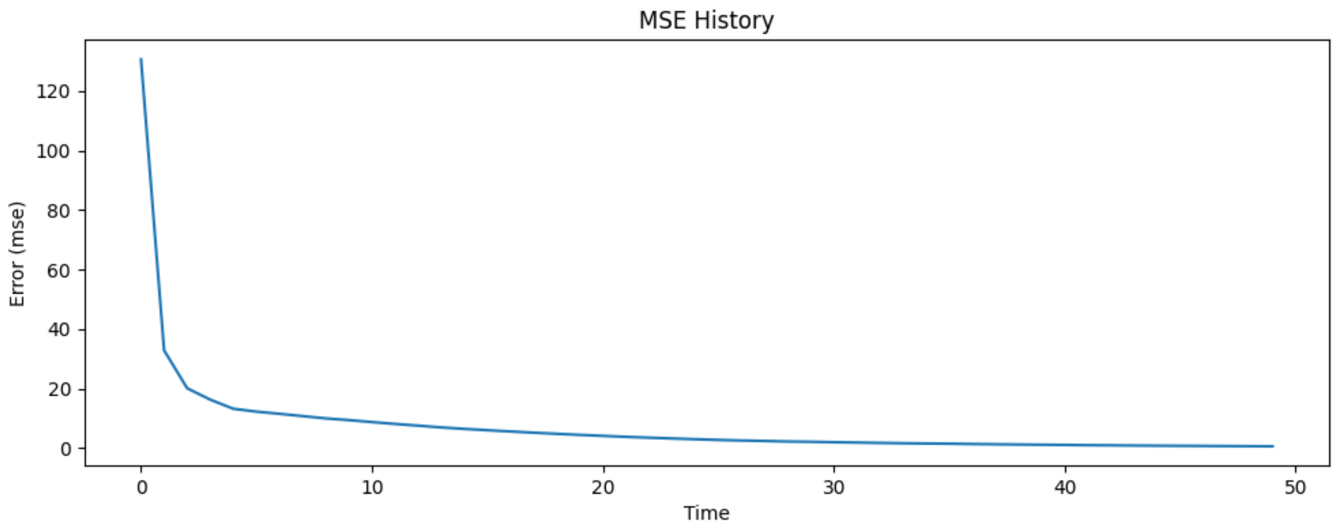
The PID parameters show distinct evolutionary patterns. The Kp term decreases steadily to approximately -0.15, while Ki and Kd terms stabilize at around 0 and -0.02 respectively, indicating a control strategy that relies primarily on proportional response to profit deviations.

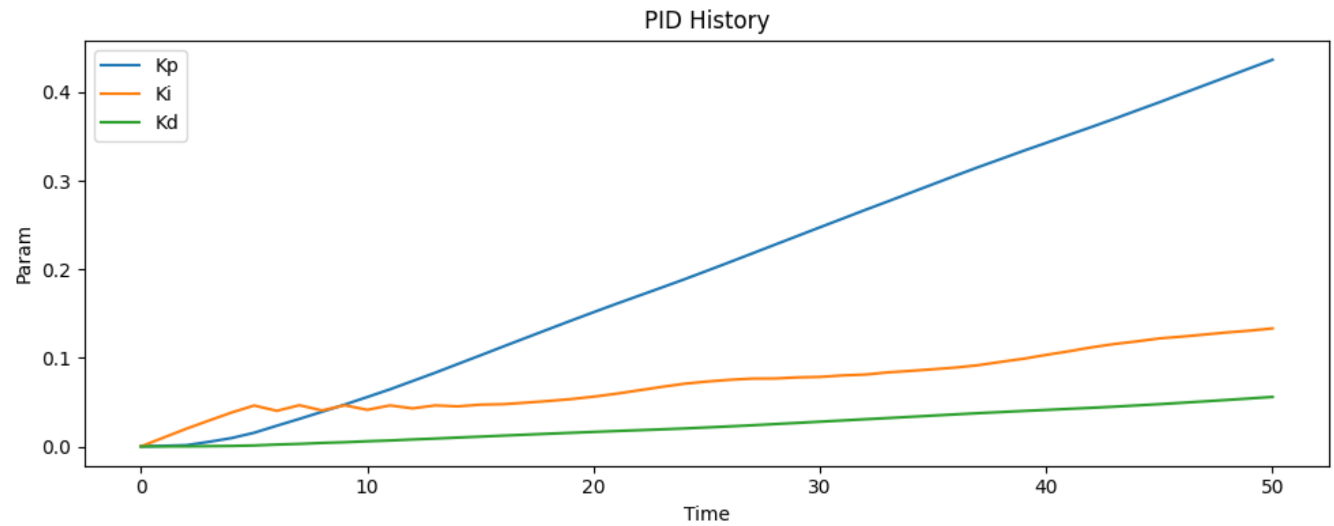
3. Monetary Policy (PID)

Configuration

Parameter	Value
Controller Type	Classic PID
Initial Parameters	Kp = 0, Ki = 0, Kd = 0
Epochs	50
Timesteps	50
Learning Rate	0.01
Disturbance Range	-0.01, 0.01
Target Interest Rate	2
Target Output Gap	0
Target Inflation	3
Initial Inflation	7
Initial Output Gap	-1
Output Gap Self-correction Rate	0.1
Interest Rate Effect on Output Gap	0.5
Output Gap Effect on Inflation	0.3
Inflation Self-correction Rate	0.2

Plot





Analysis

The MSE history plot shows a sharp initial decline from approximately 120 to below 10 within the first 10 epochs. The error continues to decrease more gradually thereafter, converging at around 0.6. This pattern indicates rapid initial learning followed by fine-tuning of the control parameters.

The PID parameter evolution shows a unique pattern compared to the other systems. All three parameters (Kp, Ki, Kd) show positive growth, with Kp reaching approximately 0.45, Ki around 0.12, and Kd about 0.05 by the end of training. This suggests that the monetary policy control system benefits from a more balanced contribution of all three control terms, with the proportional term playing the dominant role in the response to deviations from target inflation.

Neural Network Controller

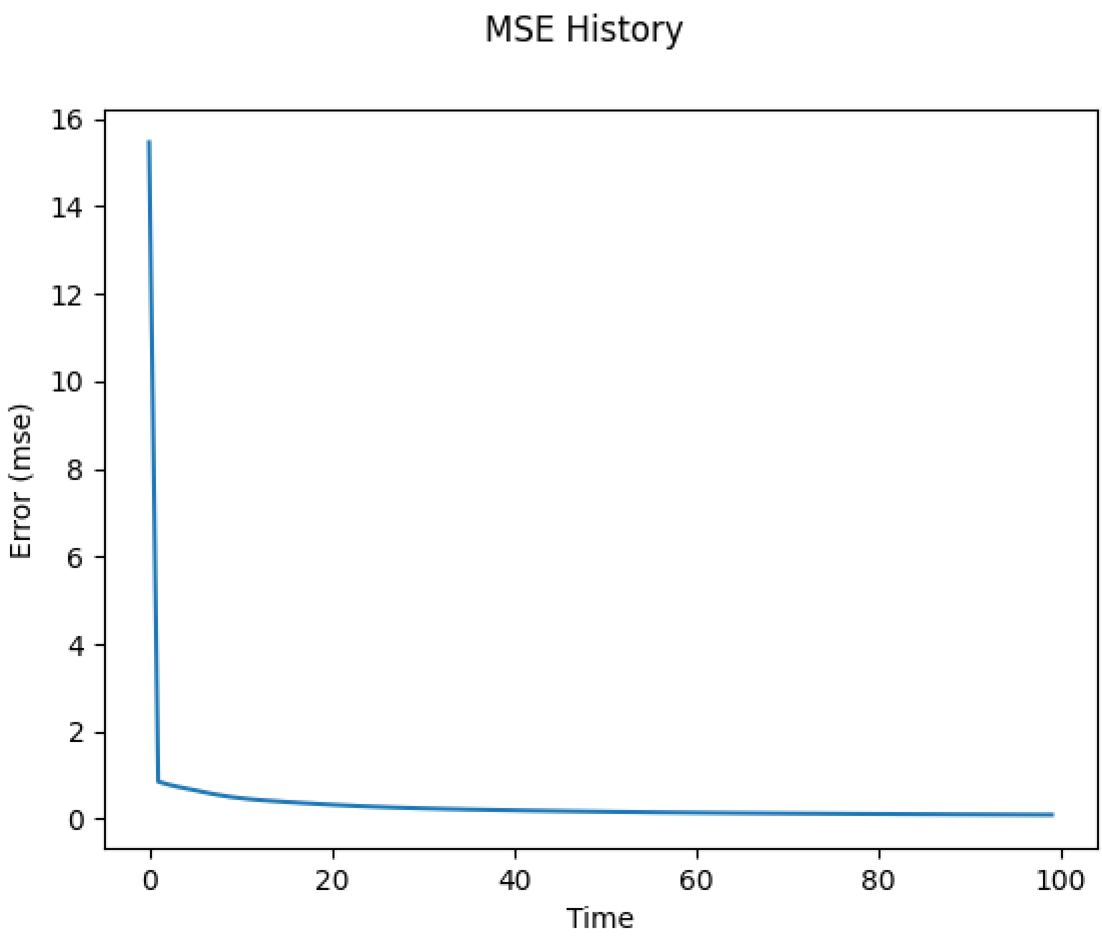
1. Bathtub (NN)

Configuration

Parameter	Value
Controller Type	Neural Network

Parameter	Value
Epochs	100
Timesteps	100
Learning Rate	0.01
Number of Layers	4
Neurons per Layer	[3, 5, 5, 1]
Weight Init Range	-0.1, 0.1
Bias Init Range	-0.05, 0.05
Disturbance Range	-0.01, 0.01
Cross-sectional Area (bathtub)	20
Cross-sectional Area (drain)	0.2
Initial Water Height	50

Plot



Analysis

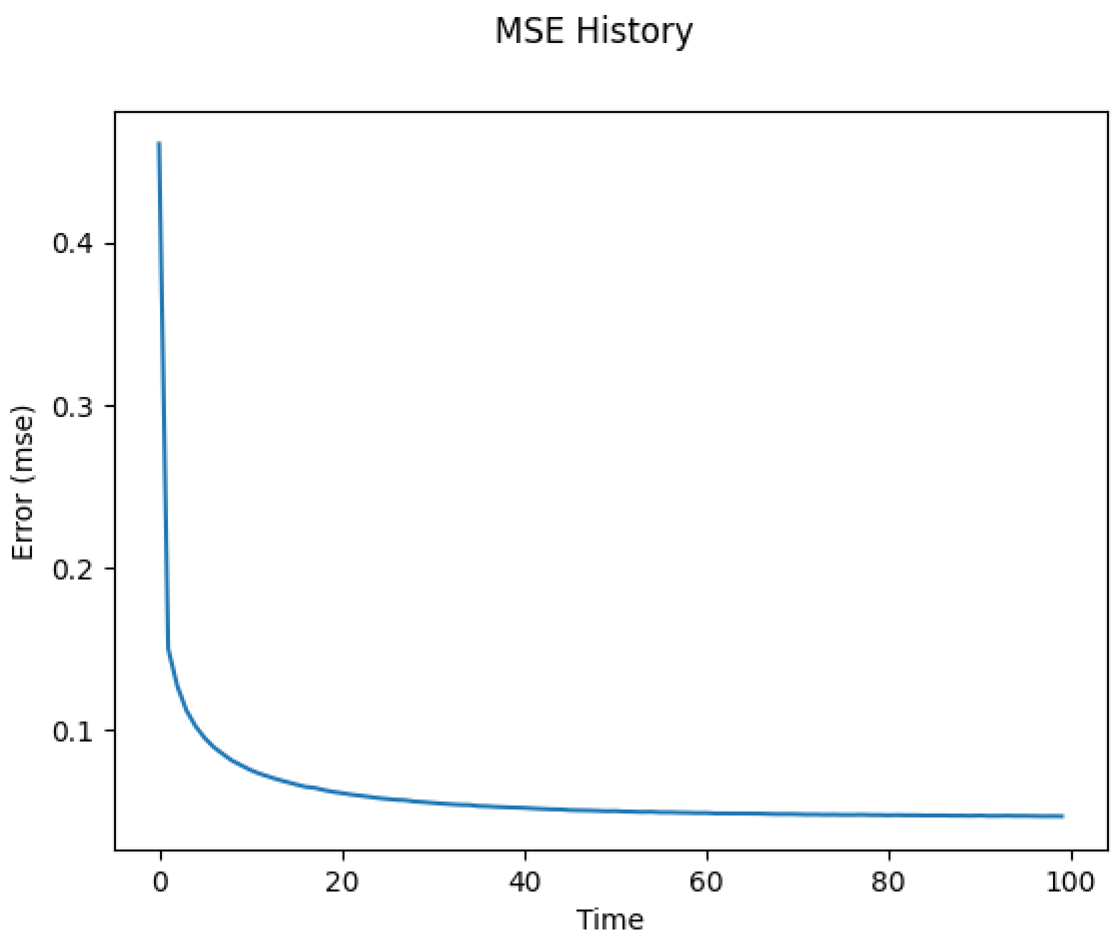
The MSE history shows quick adaptation and learning, with the error starting at approximately 16 and dropping sharply within the first few epochs to around 1. After this initial rapid decrease, the system continues to improve gradually, maintaining stable performance with an MSE decreasing to 0.12.

2. Cournot Competition (NN)

Configuration

Parameter	Value
Controller Type	Neural Network
Epochs	100
Timesteps	100
Learning Rate	0.01
Number of Layers	4
Neurons per Layer	[3, 5, 5, 1]
Weight Init Range	-0.1, 0.1
Bias Init Range	-0.05, 0.05
Disturbance Range	-0.01, 0.01
Max Price	10
Marginal Cost	2
Target Profit	4
Q1	2
Q2	2
Initial Profit p1	0

Plot



Analysis

The MSE history demonstrates robust learning behavior, with the error starting at approximately 0.45 and quickly decreasing to around 0.1 within the first 20 epochs. The learning curve shows a smooth, monotonic decrease without significant oscillations, suggesting stable optimization. The final MSE stabilizes at approximately 0.025, indicating that the neural network controller successfully learned to maintain the target profit levels despite market disturbances.

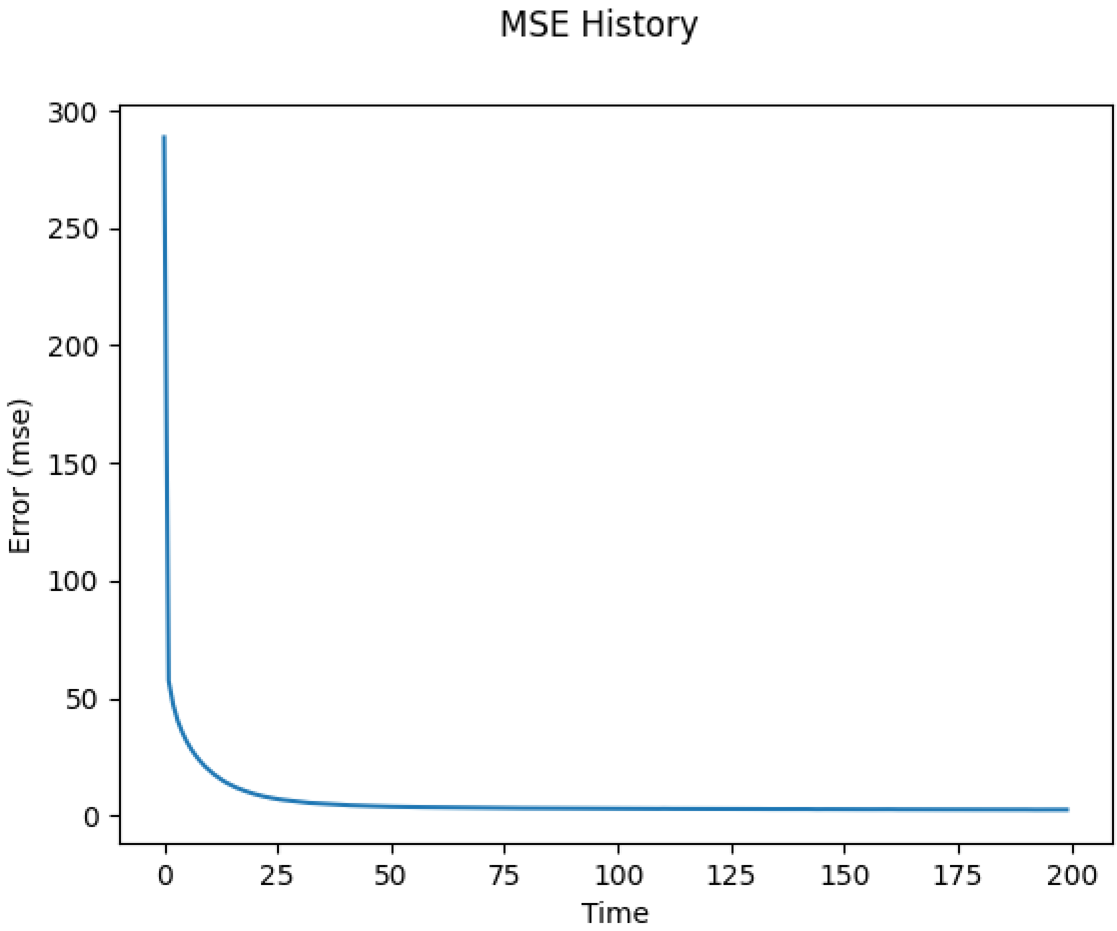
3. Monetary Policy (NN)

Configuration

Parameter	Value
Controller Type	Neural Network
Epochs	200
Timesteps	20
Learning Rate	0.0001
Number of Layers	4
Neurons per Layer	[3, 5, 5, 1]
Weight Init Range	-0.1, 0.1

Parameter	Value
Bias Init Range	-0.05, 0.05
Disturbance Range	-0.01, 0.01
Target Interest Rate	2
Target Output Gap	0
Target Inflation	3
Initial Inflation	7
Initial Output Gap	-1
Output Gap Self-correction Rate	0.1
Interest Rate Effect on Output Gap	0.5
Output Gap Effect on Inflation	0.3
Inflation Self-correction Rate	0.2

Plot



Analysis

The error begins at a relatively high value of approximately 300 and shows a sharp initial decline to around 10 within the first 25 epochs. The learning rate was set lower (0.0001) compared to the other systems, leading to a more gradual but stable convergence. After the initial rapid improvement, the system continues to optimize more slowly, eventually reaching and maintaining an MSE below 5 by epoch 200. I used shorter epochs for this run, as the gradients tended to explode with longer training periods.

Monetary Policy Model Explanation

Overview

This model simulates a simplified central banking system where a monetary authority attempts to control inflation through interest rate adjustments while considering the output gap. It represents a dynamic system with two key state variables:

- π (pi): The inflation rate
- y: The output gap (difference between actual and potential GDP)

Control Objective

The primary goal is to maintain inflation (π) at a target level (π_target) while keeping the output gap (y) close to its target (typically 0), using the interest rate (r) as the control signal.

System Dynamics

The model uses two coupled differential equations that capture the relationship between interest rates, output gap, and inflation:

Output Gap Dynamics

```
dy/dt = -a(y - y_target) - b(r - r_target)
```

Where:

- a: Output gap self-correction rate (how quickly the economy returns to potential)
- b: Interest rate effect on output gap (how much interest rates affect economic activity)
- r: Current interest rate (control signal)
- r_target: Natural interest rate

Inflation Dynamics

```
dπ/dt = c(y - y_target) - d(π - π_target) + disturbance
```

Where:

- c: Output gap effect on inflation (Phillips curve relationship)

- d: Inflation self-correction rate
- disturbance: External shocks to inflation (i.e., noise)