DD2427 Homework 6

Michal Staniaszek

April 14, 2014

1 Eigenface Computation

We compute eigenfaces differently depending on the size of the matrix X_c which contains our training data, which is a $d \times n$ matrix. The computation of the covariance C is

$$C = \frac{1}{n} X_c X_c^T \tag{1}$$

This is then a $d \times d$ matrix — computing the eigenvalues is extremely expensive. Instead, we compute

$$C_1 = \frac{1}{n} X_c^T X_c \tag{2}$$

and use

$$v_1 = X_c v \tag{3}$$

to compute an eigenvector v_1 of C using the eigenvector v of C_1. Both of these have eigenvalues of λ. This multiplication results in $d \times 1$ vector, which we would expect for an eigenvector of C. It can be shown that these eigenvectors are identical by the following method. First, we know that an eigenvector of C looks like

$$X_c X_c^T v_1 = \lambda v_1 \tag{4}$$

The eigenvectors of C_1 look like

$$X_c^T X_c v = \lambda v \tag{5}$$

If we then pre-multiply both sides of (5) by X_c we get

$$X_c X_c^T X_c v = \lambda X_c v \tag{6}$$

Then, grouping the terms,

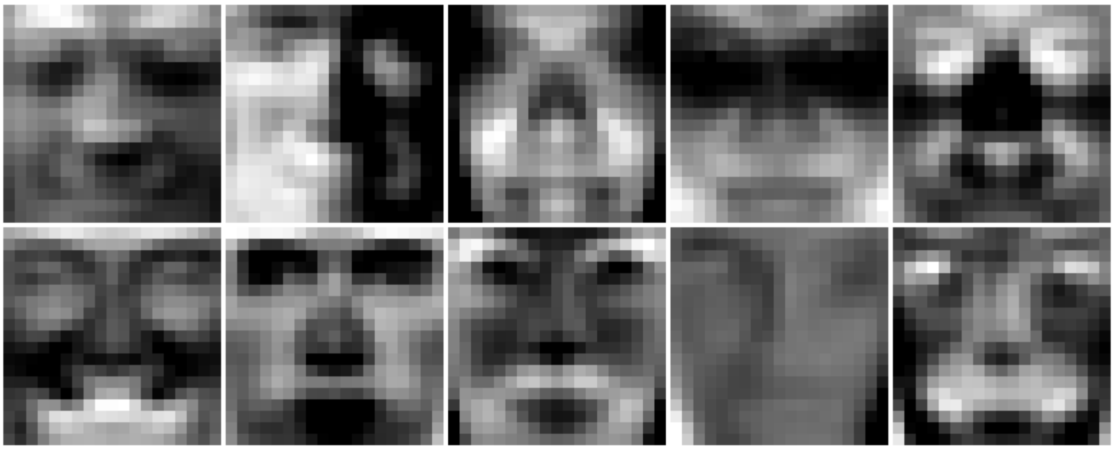
$$X_c X_c^T (X_c v) = \lambda (X_c v) \tag{7}$$

$$C (X_c v) = \lambda (X_c v) \tag{8}$$

Thus, $X_c v$ must be an eigenvector of C which has the same eigenvalue as the eigenvector of C_1.

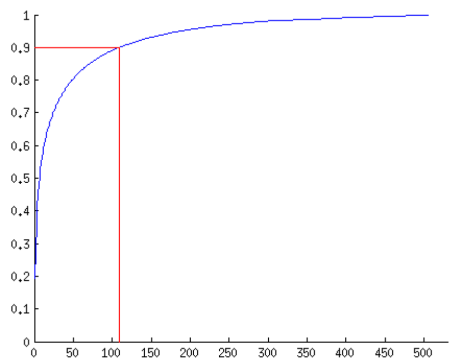
2 Eigenfaces and Reconstruction

(a) Bush Dataset

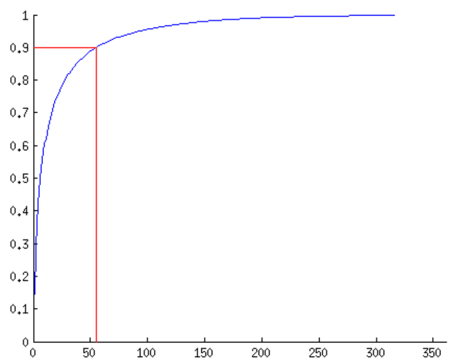


(b) ADAFACES Dataset

Figure 1: First 10 eigenfaces for the Bush and ADAFACES data sets, computed using 530 and 3000 images respectively.

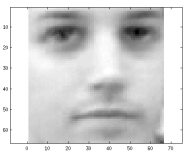


(a) Bush dataset

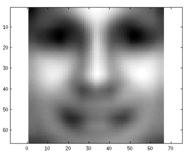


(b) ADAFACES dataset

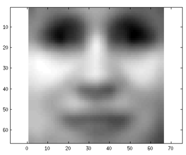
Figure 2: Cumulative sum of the eigenvalues for the Bush and ADAFACES sets using 530 and 3000 images respectively. The red line indicates the number of eigenvectors required to retain 90% of the variation in the data.



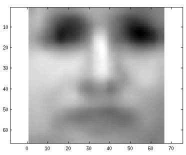
(a) Original



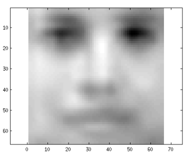
(b) 1



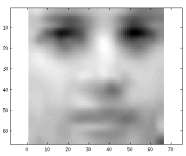
(c) 10



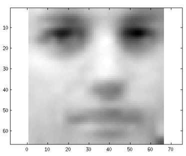
(d) 20



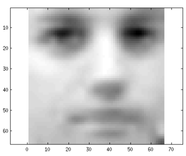
(e) 50



(f) 100



(g) 200



(h) 250

Figure 3: Reconstruction of a face using different numbers of eigenfaces computed from the 3000 examples in the ADAFACES set.