INVERSE TRANSFORM SAMPLING OF $\Pr(P) \propto P^{\alpha}$

It is possible to directly sample from $\Pr(P) \propto P^{\alpha}$ using inverse transform sampling. First, it is necessary to normalize the probability density function via integration

$$\int_{P=P_{\mathrm{min}}}^{P_{\mathrm{max}}} P^{\alpha} = \frac{P_{\mathrm{max}}^{\alpha+1} - P_{\mathrm{min}}^{\alpha+1}}{\alpha+1}, \tag{1}$$

giving

$$\Pr(P) = \frac{(\alpha+1)P^{\alpha}}{P_{\mathrm{max}}^{\alpha+1} - P_{\mathrm{min}}^{\alpha+1}}. \tag{2}$$

The cumulative density function is now given by integration as

$$\begin{aligned} \Pr(P \leq P') &= \int_{P=P_{\mathrm{min}}}^{P'} \frac{(\alpha+1)P^{\alpha}}{P_{\mathrm{max}}^{\alpha+1} - P_{\mathrm{min}}^{\alpha+1}}\,\mathrm{d}P, \\ &= \frac{P'^{\alpha+1} - P_{\mathrm{min}}^{\alpha+1}}{P_{\mathrm{max}}^{\alpha+1} - P_{\mathrm{min}}^{\alpha+1}}. \end{aligned} \tag{3}$$

One may now sample from $\Pr(P)$ by setting the cumulative density function to be equal to a uniform random variate, z, on the interval $[0, 1]$, and solving for P'. This yields

$$P' = \left(z P_{\mathrm{max}}^{\alpha+1} + (1-z) P_{\mathrm{min}}^{\alpha+1}\right)^{\frac{1}{\alpha+1}}. \tag{4}$$

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