THE ORBITAL PERIOD PRIOR FOR SINGLE TRANSITS

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The finite baseline of any photometric survey means that transiting planets (or stars) can have a sufficiently long period to impart just a single transit in our observations. With multiple consecutive transits, the determination of the object's orbital period, P, is trivial - given by the time difference between events. In such cases, the photometric data, $\mathcal{D}$, provides a highly precise and unambiguous period, which means that choice of period prior, $\mathrm{Pr}(P)$, has negligible influence on the results. With a single transit, however, the period is only weakly constrained by the data, via the transit duration, stellar density and (unknown) eccentricity (Kipping 2010). Accordingly, the choice of period prior has a large influence on the *a-posteriori* period.

Whilst there are a only few dozen examples of single transits observed by *Kepler* amongst the 4000+ planetary candidates (Wang et al. 2015; Uehara et al. 2016; Foreman-Mackey et al. 2016), *TESS* is expected to find over a thousand (Villanueva et al. 2018) amongst a similar sized total (Barclay et al. 2018), bringing greater attention to this issue.

The prior on P should reflect the expected distribution of orbital periods as seen by the survey in question. This does not equal the intrinsic distribution of periods, $\mathrm{Pr}(P)$, because the survey's own observational effects sculpt the distribution. For example, it would be impossible to ever record a single transit in a continuous photometric time series of baseline W with $P < W/2$ (else multiple transits would occur), even though the intrinsic prior $\mathrm{Pr}(P)$ can have support here. Thus, the fact that only a single transit occurred represents a piece of information which we can condition our prior upon, which we represent by the symbol $\hat{n}_1$.

In addition to $\hat{n}_1$, we also know the observational window, W, and the fact the planet has the correct geometry to undergo transits, $\hat{b}$, which biases us towards short periods more likely to display transits. Finally, we know the mid-transit time minus the start of the observational baseline, L, which we might expect to provide a useful lower limit on the period. Ignorning detection bias, which is only relevant for low signal-to-noise transits (Kipping & Sandford 2016), the overall prior on a single transit period may be expressed as

$$\mathrm{Pr}(P|\hat{n}_1, W, \hat{b}, L) \propto \mathrm{Pr}(L|\hat{n}_1, W, \hat{b}, P)\mathrm{Pr}(P|\hat{n}_1, W, \hat{b}) \tag{1}$$

where the right-hand side expands the left using Bayes' theorem. The final term can expanded in a similar way as

$$\mathrm{Pr}(P|\hat{n}_1, W, \hat{b}) \propto \mathrm{Pr}(\hat{n}_1|P, W, \hat{b})\mathrm{Pr}(P|W, \hat{b}) \tag{2}$$

where $\mathrm{Pr}(P|W, \hat{b})$ has no dependency on W and thus becomes $\mathrm{Pr}(P|\hat{b})$. Expanding $\mathrm{Pr}(P|\hat{b})$ once more with Bayes' theorem and then combining all the terms we have

$$\underbrace{\mathrm{Pr}(P|\hat{n}_1, W, \hat{b}, L)}_{\text{overall prior}} \propto \underbrace{\mathrm{Pr}(L|\hat{n}_1, W, \hat{b}, P)}_{\text{phase constraint}} \underbrace{\mathrm{Pr}(\hat{n}_1|P, W, \hat{b})}_{\text{window effect}} \underbrace{\mathrm{Pr}(\hat{b}|P)}_{\text{geometric bias}} \underbrace{\mathrm{Pr}(P)}_{\text{intrinsic prior}} \tag{3}$$

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Let us write the intrinsic prior as P^{α}, where $\alpha = -1$ returns a log-uniform distribution. The geometric bias is well-understood and scales as $P^{-2/3}$ after applying Kepler's Third Law to Equation 2 of Kipping (2014). The window bias is more subtle and most easily calculated by defining it as

$$\Pr(\hat{n}_1|P,W,\hat{b}) = \Pr(n \geq 1|P,W,\hat{b})(1 - \Pr(n \geq 2|P,W,\hat{b})), \tag{4}$$

where n is the number of transits occuring in the window W. The first term is easy to compute by considering that $P < W$ is guarenteed to provide at least one transit and $P > W$ will drop off thereafter as $1/P$:

$$\Pr(n \geq 1|P,W,\hat{b}) \propto \begin{cases} 1 & \text{if } P \leq W, \\ \frac{W}{P} & \text{if } P > W. \end{cases} \tag{5}$$

Similarly, $n \geq 2$ is guarenteed if $P \leq W/2$ but impossible if $P > W$. Between these two extremes one should expect a P^{-1} scaling as before, and so normalizing this intermediate regime to connect to the cases into a continuous function yields:

$$\Pr(n \geq 2|P,W,\hat{b}) = \begin{cases} 1 & \text{if } P \leq \frac{W}{2}, \\ \frac{W-P}{P} & \text{if } \frac{W}{2} < P \leq W, \\ 0 & \text{if } P > W. \end{cases} \tag{6}$$

Finally, turning to the phase constraint, for $P > W$ we expect that L is uniformly distributed between 0 and W - no phase is more likely than any other. But, if P was slightly larger than $W/2$, then L could only take on a narrow range around $W/2$ in order for the transit to avoid multiple transits. This can be encoded by writing that

$$\Pr(L|P,W,\hat{b}) = \begin{cases} \frac{\mathbb{H}[L+P-W]-\mathbb{H}[L-P]}{2P-W} & \text{if } \frac{W}{2} < P \leq W, \\ \frac{\mathbb{H}[L]-\mathbb{H}[L-W]}{W} & \text{if } P > W, \end{cases} \tag{7}$$

where $\mathbb{H}$ is the Heaviside Theta function. Combining all of these components leads to the simplified result that

$$\Pr(P|\hat{n}_1,W,\hat{b},L) \propto \begin{cases} 0 & \text{if } P \leq \max(W-L,L), \\ P^{\alpha-5/3} & \text{if } \max(W-L,L) < P \leq P_{\text{max}}, \\ 0 & \text{if } P > P_{\text{max}}. \end{cases} \tag{8}$$

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REFERENCES

Barclay, T., Pepper, J. & Quintana, E. V., 2018, ApJS, 239, 2

Foreman-Mackey, D., Hogg, D. W., Agol, E. & Schölkopf, B., 2016, AJ, 152, 206

Kipping, D. M., 2010, MNRAS, 407, 301

Kipping, D. M., 2014, MNRAS, 444, 2263

Kipping, D. M. & Sandford, E., 2016, MNRAS, 463, 1323

Osborn, H. P., Armstrong, D. J., Brown, D. J. A., et al., 2016, MNRAS, 457, 2273

Uehara, S., Kawahara, H., Masuda, K., Yamada, S. & Aizawa, M., 2016, ApJ, 822, 2

Villanueva, S., Dragomir, D. & Gaudi, B. S., 2018, arXiv e-prints:1805.00956

Wang, J., Fischer, D. A., Barclay, T., et al., 2015, ApJ, 815, 127

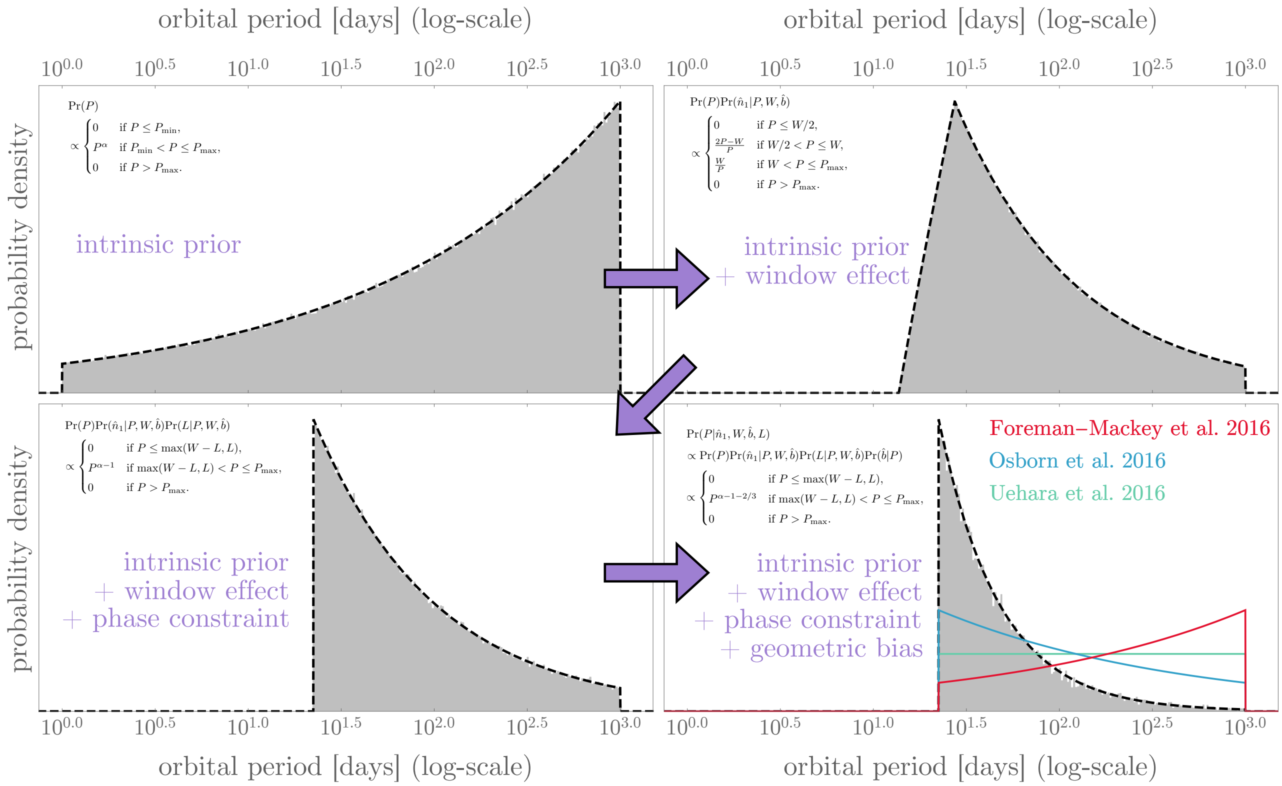

Figure 1. *Comparison of* 10^6 *Monte Carlo periods (gray histograms) and the analytic formulae presented here (black dashed). Starting from the intrinsic prior (top-left), where we adopt* $\alpha = -2/3$ *(log-uniform in semi-major axis) as an example, we add each component contributing to the single transit period prior one by one, assuming* $W = 27.4$ *days. Initial periods are drawn using inverse transform sampling from the intrinsic distribution and then pushed through a set of filters representing each component. All histograms are plotted in log-scale and the formulae are transformed accordingly. The bottom-left panel uses* $L = 5$ *days in the Monte Carlo experiments. We compare our final distribution to three previously adopted priors for single transits in the literature (*$\sim P^{-2/3}$ *by Foreman-Mackey et al. 2016;* $\sim P^{-5/3}$ *by Osborn et al. 2016;* $\sim P^{-1}$ *by Uehara et al. 2016).*