CONVERTING THE OSBORN ET AL. (2016) VELOCITY PRIOR TO PERIOD SPACE

From Equation 2 of Osborn et al. (2016), and using the notation as the authors, we have

$$P_{\rm circ} = \frac{8\pi^2 G}{3}\frac{\rho_\star}{v'^3}, \tag{1}$$

where all inferred periods of single transits assume circular orbits. The authors adopt a linear prior on transit velocity v' such that $\Pr(v') \propto v'$ (Osborn 2017). Here, we convert this into period-space i.e. $\Pr(P_{\rm circ})$. Let's start by writing the linear velocity prior as

$$\Pr(v'){\rm d}v' \propto v'{\rm d}v', \tag{2}$$

To convert to period space, we can write that

$$\Pr(P_{\rm circ}){\rm d}P_{\rm circ} = \Pr(v'){\rm d}v', \tag{3}$$

and using the chain rule we have

$$\Pr(P_{\rm circ}){\rm d}P_{\rm circ} = \Pr(v')\frac{{\rm d}v'}{{\rm d}P_{\rm circ}}{\rm d}P_{\rm circ}. \tag{4}$$

From Equation (1), it follows that

$$\frac{{\rm d}P_{\rm circ}}{{\rm d}v'} \propto \frac{1}{v'^4}. \tag{5}$$

Substituting Equations (2) & (5) into Equation (4) yields

$$\Pr(P_{\rm circ}){\rm d}P_{\rm circ} \propto \underbrace{v'}_{\Pr(v')}\ \underbrace{v'^{-4}}_{{\rm d}v'/{\rm d}P_{\rm circ}}\ {\rm d}P_{\rm circ}. \tag{6}$$

If we now Equation (1) to write that $v' \propto P_{\rm circ}^{-1/3}$ and substitute this into Equation (6) then we arrive at

$$\Pr(P_{\rm circ}){\rm d}P_{\rm circ} \propto P_{\rm circ}^{-5/3}{\rm d}P_{\rm circ}. \tag{7}$$

REFERENCES

Osborn, H. P., Armstrong, D. J., Brown, D. J. A., et al., 2016, MNRAS, 457, 2273
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