2a the joint posterior

$\pi(\lambda, \mu \mid y)$

$\propto \pi(y \mid \lambda, \mu)\, \pi(\lambda, \mu)$

$\propto \pi(y \mid \lambda, \mu)\, \pi(\lambda \mid \mu)\, \pi(\mu)$ — $(*)$

where $\pi(y \mid \lambda, \mu) = \prod_{i=1}^{4} \prod_{j=1}^{5} \pi(y_{ij} \mid \lambda_i, \mu)$

$= \prod_{i=1}^{4} \prod_{j=1}^{5} \frac{e^{-\lambda_i} \lambda_i^{y_{ij}}}{(y_{ij})!}$ $\quad (y_{ij} \mid \lambda_i, \mu \sim Po(\lambda_i))$

$\propto \prod_{i=1}^{4} \prod_{j=1}^{5} e^{-\lambda_i} \lambda_i^{y_{ij}}$

$\propto \prod_{i=1}^{4} e^{-5\lambda_i} \lambda_i^{\sum_{j=1}^{5} y_{ij}}$

$\propto e^{-5 \sum_{i=1}^{4} \lambda_i} \prod_{i=1}^{4} \lambda_i^{\sum_{j=1}^{5} y_{ij}}$ — (1)

and $\pi(\lambda \mid \mu) = \prod_{i=1}^{4} (\mu e^{-\lambda_i \mu})$ $\quad (\lambda_i \mid \mu \sim Exp(\mu))$

$= \mu^4 e^{-\mu \sum_{i=1}^{4} \lambda_i}$

and $\pi(\mu) \propto \frac{1}{\mu}$

so $(*)$: $\pi(\lambda, \mu \mid y) \propto \left[e^{-5 \sum_{i=1}^{4} \lambda_i} \prod_{i=1}^{4} \lambda_i^{\sum_{j=1}^{5} y_{ij}} \right] \left[\mu^4 e^{-\mu \sum_{i=1}^{4} \lambda_i} \right] \left[\frac{1}{\mu} \right]$

$\propto \mu^3 \cdot e^{-(\mu+5) \sum_{i=1}^{4} \lambda_i} \cdot \prod_{i=1}^{4} \lambda_i^{\sum_{j=1}^{5} y_{ij}}$

2b denote $\lambda_{(i)}$ be the vector λ with removed the i^{th} element to find the full conditional

$$\pi(\lambda_i \mid \lambda_{(i)}, \mu, y) \propto e^{-(\mu+5)\lambda_i} \lambda_i^{\sum_{j=1}^{5} y_{ij}}$$

$$\propto \lambda_i^{(\sum_{j=1}^{5} y_{ij}+1)-1} e^{-(\mu+5)\lambda_i}$$

so, $\lambda_i \mid \lambda_{(i)}, \mu, y \sim \Gamma(\sum_{j=1}^{5} y_{ij}+1, \mu+5)$

and $\pi(\mu \mid \lambda, y) \propto \mu^3 e^{-\mu \sum_{i=1}^{4} \lambda_i}$

$$\propto \mu^{4-1} e^{-\mu \sum_{i=1}^{4} \lambda_i}$$

so, $\mu \mid \lambda, y \sim \Gamma(4, \sum_{i=1}^{4} \lambda_i)$

Algorithm: step 0 : initialize $\mu^{(0)}$

step 1 : Sample $\lambda_1^{(n+1)}$ from $\Gamma(\sum_{j=1}^{5} y_{1j}+1, \mu^{(n)}+5)$
Sample $\lambda_2^{(n+1)}$ from $\Gamma(\sum_{j=1}^{5} y_{2j}+1, \mu^{(n)}+5)$
sample $\lambda_3^{(n+1)}$ from $\Gamma(\sum_{j=1}^{5} y_{3j}+1, \mu^{(n)}+5)$
Sample $\lambda_4^{(n+1)}$ from $\Gamma(\sum_{j=1}^{5} y_{4j}+1, \mu^{(n)}+5)$

Sample $\mu^{(n+1)}$ from $\Gamma(4, \sum_{i=1}^{4} \lambda_i^{(n+1)})$

step 2 : update $\mu^{(n)} = \mu^{(n+1)}$, and repeat step 1 & 2 until desired length of iteration