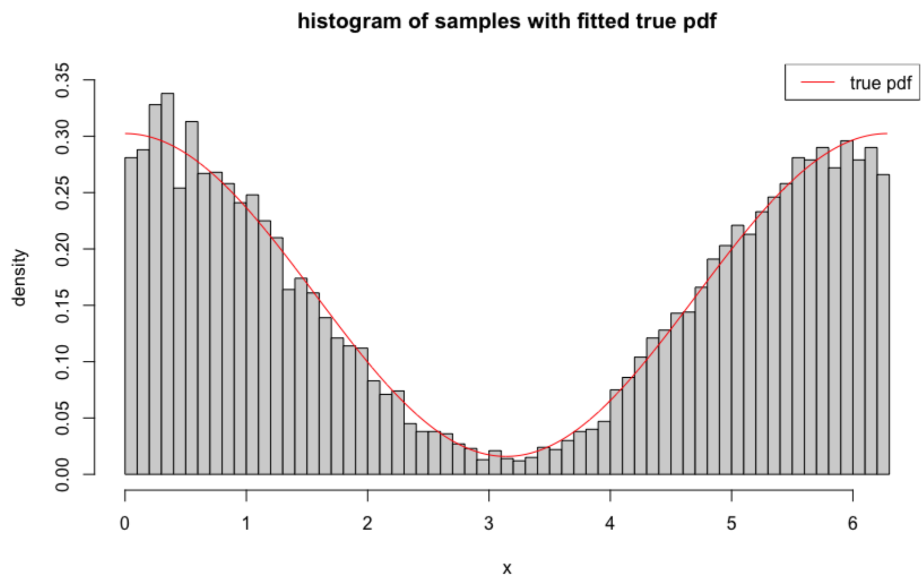
1a) see handwritten work pdf

1b) refer to script file

1c)

refer to script file to see the implementation

Below is the histogram and overlay true pdf plot



1d)

From the lecture notes, we know that Probability of accepting a sample = 1 / M

Where M = Expected number of trials in order to accept a sample

Since from our calculations, we know that

M = sup{f(x) / g(x)} = 1 + r / R

In order to make the algorithm more efficient, we need M smallest possible, i.e. smaller the ratio r / R

For better efficiency algorithm, we can expect

1) Numerically: the mean number of trials should be small and close to 1

2) Graphically: f(x) will be close to the bound Mg(x)

1e) refer to script file as well

Note that we need 0<r<R

Numerical illustrations:

I have tried some combinations of r and R, and compared the sample mean number of trials vs theoretical mean number of trials, which is quite close.

```
> result
         r    R sample_mean_trials theoretical_mean_trials
 [1,] 0.10    1             1.1010                  1.1000
 [2,] 0.10   10             1.0082                  1.0100
 [3,] 0.10  100             1.0011                  1.0010
 [4,] 0.10  500             1.0002                  1.0002
 [5,] 0.10 1000             1.0002                  1.0001
 [6,] 0.01    1             1.0098                  1.0100
 [7,] 0.05    1             1.0501                  1.0500
 [8,] 0.10    1             1.0963                  1.1000
 [9,] 0.50    1             1.5029                  1.5000
[10,] 0.90    1             1.8855                  1.9000
```

Note that higher mean number of trials to accept a sample, means worse efficiency.

So, from this test set of r and R, we can see that

1) fixed r and increase R => decreasing mean number of trials => better efficiency

2) fixed R and increase r => increase mean number of trials => worse efficiency

And we can see why this may happen, by plotting f(x) and the upper bound Mg(x)

Graphical illustrations:

Look at the graph of f and upper bound,

Note that our upper bound is always a straight line, with height = Mg(x) = (1+r/R)/2pi

Here we can see that,

1) first row is fixed r = 0.1, and increase R

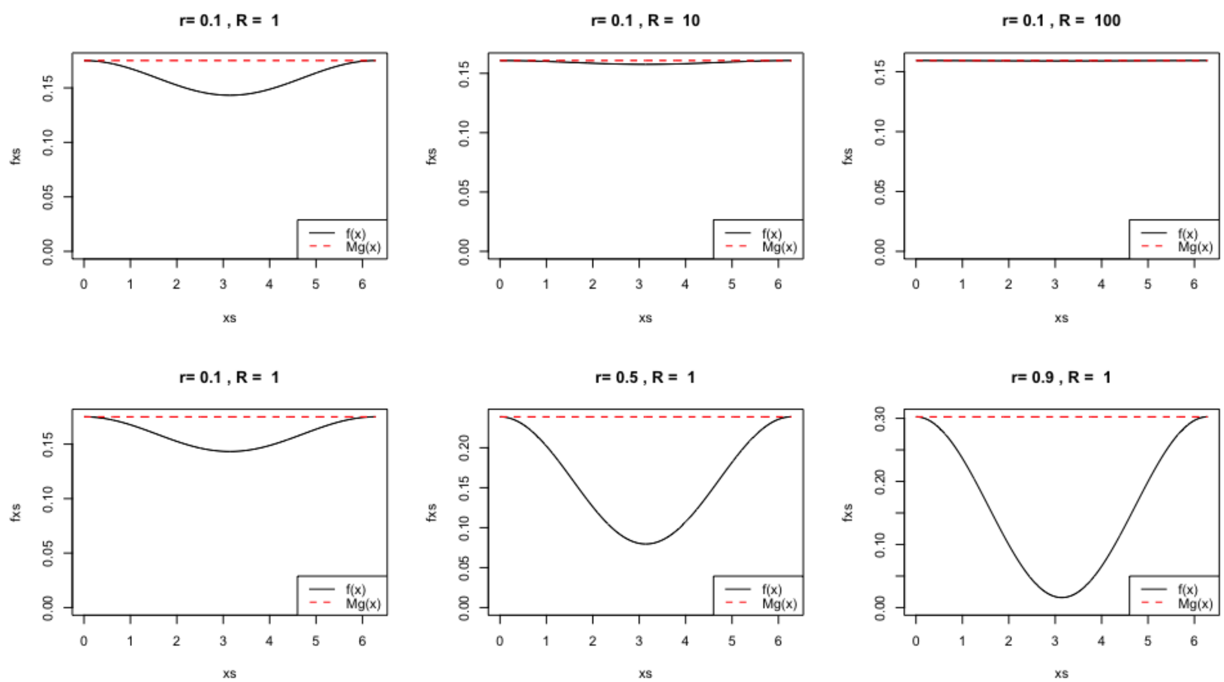
So, M = 1 + r / R, which is decreasing, and we should expect better efficiency

In this case, our f(x) is moving closer to the upper bound, which means better efficiency

2) second row is fixed R = 1, and increase r

So, M = 1 + r / R, which is increasing, and we should expect worse efficiency

In this case, our f(x) is moving away from the upper bound, which means worse efficiency

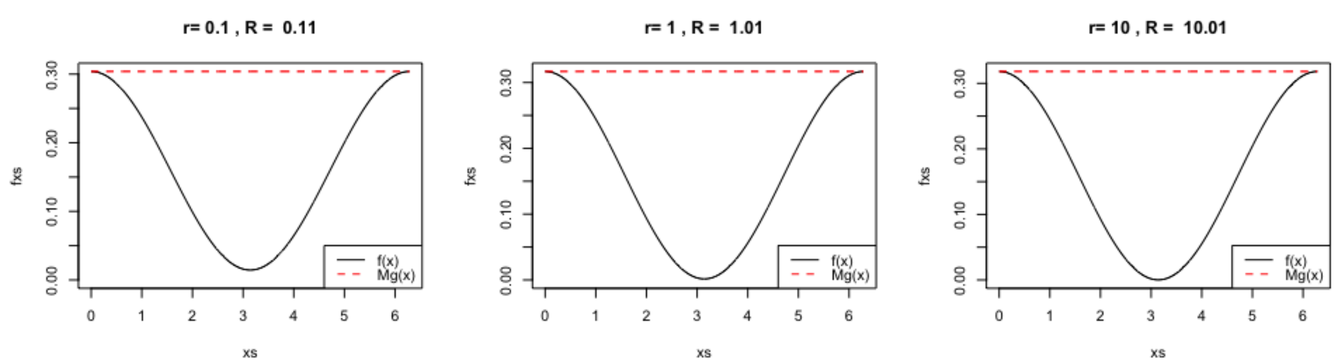


3) for my curious, I have just tried another set of result, change r and R but keep the ratio approximately equal to 1

Because M = 1 + r / R, if r / R is a constant (almost), then M is also fixed

And so, we should expect the shape of graphs should look similar.

* Note that this is obvious because the pdf f(x) is controlled by r and R, and more precisely, is controlled by r / R. And so the shape of the pdf f(x) is almost the same.



2a) see handwritten work pdf

Note that because we can treat yi (given xi, b1, b2) are independent, so we can just multiply the individual pdf in the likelihood function

2b) see handwritten work pdf and also refer to script file

mode of the posterior distribution:

b1 = 0.987296

b2 = 1.819924

2c) see handwritten work pdf

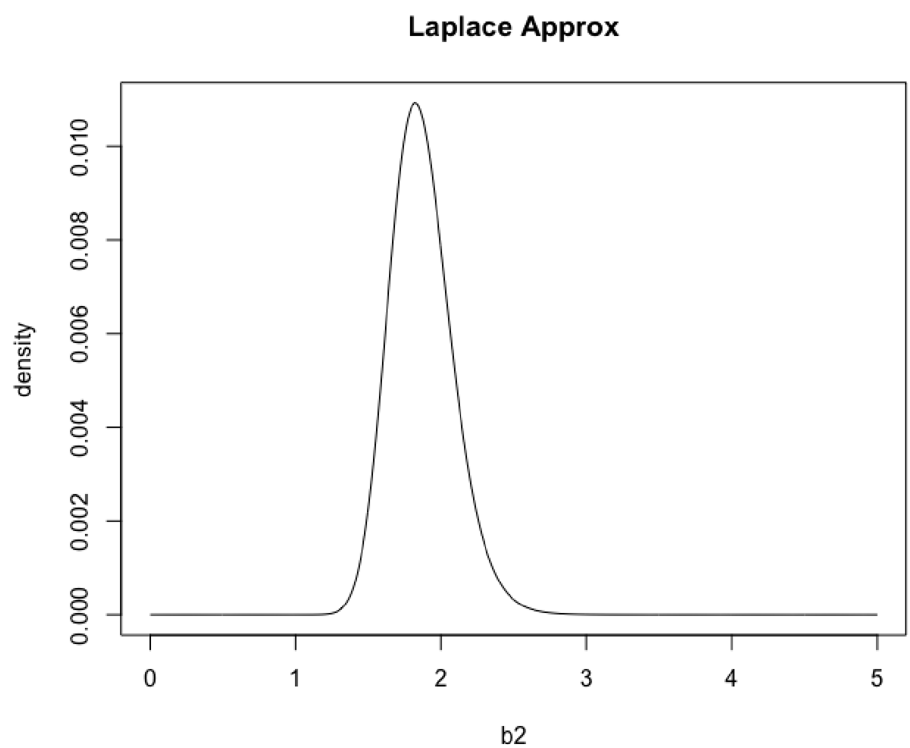
2d) refer to R script file

Note also that because the question (2c to 2g) did not mention I can use posterior up to proportionality, so I need to use FULL posterior distribution, I manually use R commands to compute that, for detail please see the R script file

2e) marginal posterior of b2

p.s.: I tried to plot 0 to 200, and after the function is 0 after 4

So I just plot from 0 to 5



2f) refer to R script file

I guess the question has a typo, should be to optimize $\pi(b2|x,y)$ but not p(…)

it said optimize, but clearly, our target is to maximize the function

2g) the mode of $p(b_2|y)$ to an accuracy of 1 decimal place.

I guess the question has a typo, should be to find the mode of $\pi(b2|x,y)$

Mode of the marginal posterior of b2 (to 1 d.p.)

= 1.8