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ECE 397Q

Final Project

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Analysis of the Deutsch-Jozsa Algorithm using Transmon Qubits

This project will be centered around the Deutsch-Jozsa Algorithm running on a form of a quantum computer known as superconducting loops, which feature transmon qubits. As explained in Lab 8, Deutsch-Jozsa attempts to figure out how a function 'f' will respond to virtually all of its potential inputs without needing to input them all into the function to find the corresponding output. In other words, the function f(x) will have an input of a binary bit string 'x' and output '0' or '1'. If the function outputs 0 or 1 regardless of the input, the function is constant, whereas if there is an even split (50/50) of outputting 0 and 1, the function is balanced. The Deutsch-Jozsa Algorithm's task is to figure out what type, constant or balanced, the given function is.

In finding the classical solution, the best case scenario only needs two queries to the oracle to determine if the boolean function is balanced, since if any arbitrary binary bit string is used as the input and 0 is the output, and another arbitrary binary bit string is used as another input and 1 is the output, then the function must be balanced since there are two different outputs, meaning it cannot be constant. In the worst case situation, if the same output is seen continuously for different arbitrary inputs that are tried, exactly half of all possible inputs plus one will have to be checked to know that our function is constant (or not). Since the inputs are in binary, the total possible number of inputs is 2^n, where n is the number of bits in the string. So, $2^{n-1} + 1$ inputs are needed to see if our function is constant in this worse case situation. As a result, the probability that the given function is constant as a function of k binary bit string inputs is $P(k) = 1 - (1/2^{k-1})$ for $1 < k \leq 2^{k-1}$. This would work well if it is desired to shorten our classical algorithm to see if there is a certain percentage threshold of confidence that needs to be crossed in the likelihood of the function being constant. Nevertheless, to be 100% sure, $2^{n-1} + 1$ inputs should be checked.

With a quantum computer, however, the Deutsch-Jozsa Algorithm can solve this problem of determining the type of the function with only a single call to the function, assuming there is a function f(x) implemented as a quantum oracle, which maps the state $|x\rangle|y\rangle$ to $|x\rangle|y\oplus f(x)\rangle$, where $\oplus$ represents addition of the binary numbers, then modulo of 2. To carry out this algorithm, two quantum registers are involved, where the first is an n-qubit register initialized to $|0\rangle$, and the second is a one-qubit register initialized to $|0\rangle$. After applying a Hadamard gate to each qubit, the quantum oracle is then applied. Ignoring the second single qubit register, a Hadamard gate is applied to each of the n qubits in the first register before the first register is finally measured.

With superconducting loops, a number of Josephson junctions interpret micrometer sized loops of superconducting metal known as flux qubits or persistent current qubits. This is effectively what represents a qubit with this type of quantum computer. A single qubit gate is executed by a rotation in the Bloch sphere, where rotations between the varying energy levels of a single qubit are achieved by using microwave pulses sent to a nearby antenna or transmission line which is coupled to the qubit. To run two qubit gates, there must be coupling of two qubits, which can be achieved by connecting them to an intermediate electrical coupling circuit. The major limitations of qubits in superconducting loops are a result of the fast decoherence times, or in other words, the short lived memory of the qubits. As a result, qubits that have the sole purpose of error correcting are needed to compensate for potential inaccuracies. A version of transmon qubits, known as fixed-frequency transmons, are being heavily researched in the world of quantum due to having significantly greater coherence times. When dealing with gate fidelity in the context of forming a CNOT gate, it's been observed that that fidelity is generally between 99% and 99.99%; superconducting loops show great promise in delivering accurate results, even if occasional errors come about in terms of gate fidelity. These occasional errors can still dampen what are ostensibly exponential benefits when computing with transoms over classical computers since error checking is needed with transmons to ensure the Deutsch-Jozsa algorithm outputs accurate results. Nevertheless, superconducting loops are being heavily investigated for their use in quantum computing by companies such as Google with their Sycamore quantum computer. Other companies include IBM, Intel, IMEC, and BBN Technologies.

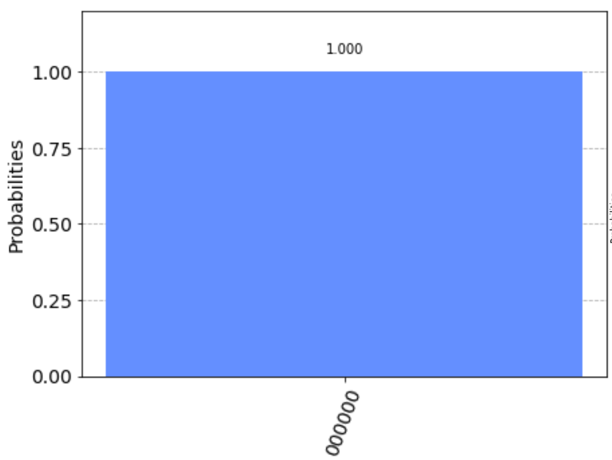
When it comes to the transmon qubit that is used in superconducting loops, error can arise where microscopic sources of flux noise are caused by fluctuating charges of quasiparticles, fluctuating magnetic spins, and fluctuating magnetic vortices. Regardless, superconducting loops present relatively high immunity to noise. The Deutsch-Jozsa Algorithm has effectively 0 error when working with quantum computers. The problem can be solved so quickly with quantum computers to the point where any additional resources for error checking are virtually negligible. A composite pulse is a sequence of individual pulses, which is used to reduce certain types of errors in superconducting quantum computing, but often at the cost of longer total duration of the computation. Composite pulses feature an unparalleled combination of great accuracy, precision, robustness and flexibility, that can help with fixing inaccuracies in using superconducting loops for quantum computing. Transmon qubits have around a 99.4% logic success rate. As a result, there is inherently some level of error in our Deutsch-Jozsa computation, so while exponentially more efficient computation of the algorithm cannot be idealized; the advantages of using a quantum computer over a classical computer in this scenario are mitigated by even the small amount of error that is present in transmon qubits (error checking would have to be performed, slowing down the computation).

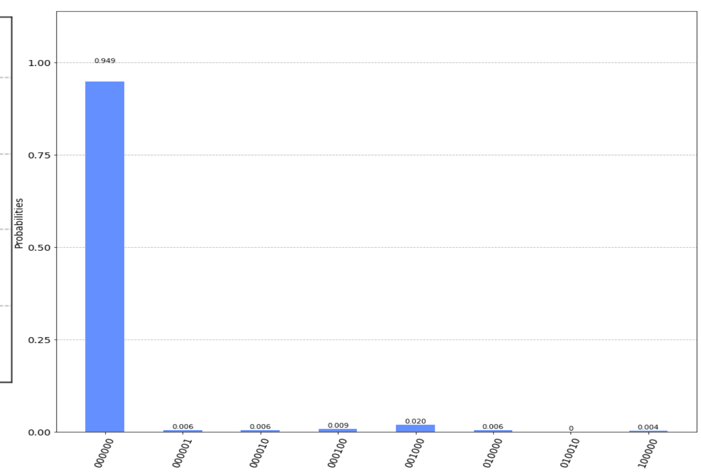
For a bare single qubit, a Rabi Oscillation, or external oscillating electric field, is applied at the qubit frequency corresponding to the energy difference between the ground and excited state to induce rotations on the transmon qubit. To apply it, we use a microwave line that connects to the transmon. With the axis of rotation lying in the x-y plane, the resulting rotation can be impacted by changing the phase and by applying sine or cosine waves. For two qubit operation, there must be coupling of the two qubits. This can be achieved by introducing a flux which changes the energy level of the system. Coupling of the two qubits allows for the realization of the c-phase gate. Having a c-phase gate and a Hadamard gives us the universal set of gates to work with. To break down the problem, an arbitrary binary bit string input will be carried out through the classical solution to provide a basic worked example for how the algorithm functions, before the same input will be explained through the lens of the quantum solution (in a similar way to how a worked example was provided on the Qiskit website). The same input will be used in the code to allow the results to be verified to provide visuals detailing the error rates and other relevant data. A combination of 3D models of the Block sphere to show the position of individual qubits, as well as graphs to show the relevant data.

In writing code to analyze the impact of using transmon qubits to run the Deutsch-Jozsa algorithm, circuits of both the constant and balanced oracles would be generated by creating a base Deutsch-Jozsa circuit before adding the respective oracle's circuit to it. For both oracles, one execution would exclude any noise and another would have a specific noise model added to replicate the sources of error inherent to transmons. This allows a holistic analysis of the impact of using transmon qubits for this algorithm due to obtaining a result from a "control" execution that can be compared to a noisy "experimental" execution. To provide a more complex illustration of the effects of noise on running the algorithm, a randomly-generated arbitrary input bitstring consisting of six bits would be used with six transmon qubits in every scenario. To facilitate this, the ibmq-guadalupe quantum computer was used because of its high number of qubit registers, being 16. Guadalupe's noise model consists of depolarizing error as well as a thermal radiation error in both single qubit and two qubit gates, with an average readout error of 2.506e-2 and an average CNOT error of 1.325e-2.

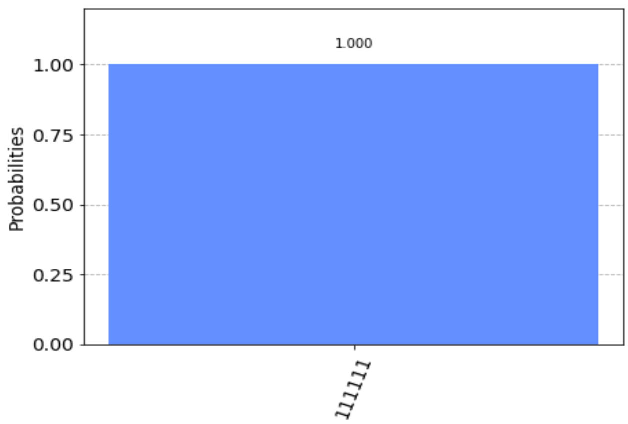
When executing the Deutsch-Jozsa algorithm for the constant oracle without noise, the probability of $|000000\rangle$ was 100%. This meant that there was no phase kickback and the qubits did not get flipped. When we introduced noise into our simulation, the probabilities were distributed across different states in the histogram, but the probability of the state $|000000\rangle$ was much higher (>90%) than any other state. In both cases, there was no phase kickback, and we could correctly predict that the function was a constant oracle.

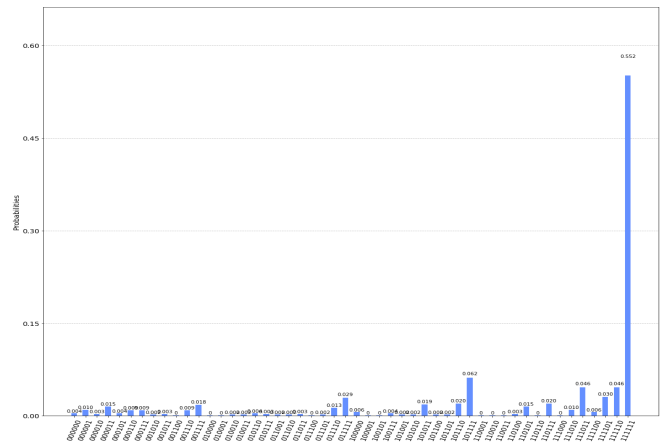
For the balanced oracle, executing the algorithm without the noise model yielded a 100%





chance of the function being balanced: there was a 0% chance of measuring $|000000\rangle$ or any state other than $|111111\rangle$, indicating that the result perfectly computed that the inputted function was indeed balanced. When the input and the output qubits are in superposition, there is a chance that a phase kickback occurs in the input qubit. This reverse operation allows us to query the oracle in one step. In the case of a balanced oracle, the phase kickback sets in and flips all the qubits. After introducing noise, there was about a 60% chance of getting $|111111\rangle$, 0.02% chance of getting $|000000\rangle$, and just over a 39% chance of getting any other final state of the qubit registers. With a 0.02% chance of incorrectly finding that the function is constant, the transmon noise model had a relatively small but noticeable impact on the accuracy of running the algorithm.





While there might be a minor but measurable noisy footprint in each oracle for the Deutsch-Jozsa algorithm, transmons still demonstrate a high logic success rate for the computation of this algorithm. As stated earlier, research oriented around fixed-frequency transmons could unlock the potential for even better accuracy in using quantum computers for tackling even more complex algorithms in the future.

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