Quantum noise and error correction

Argonne Quantum Computing Tutorial
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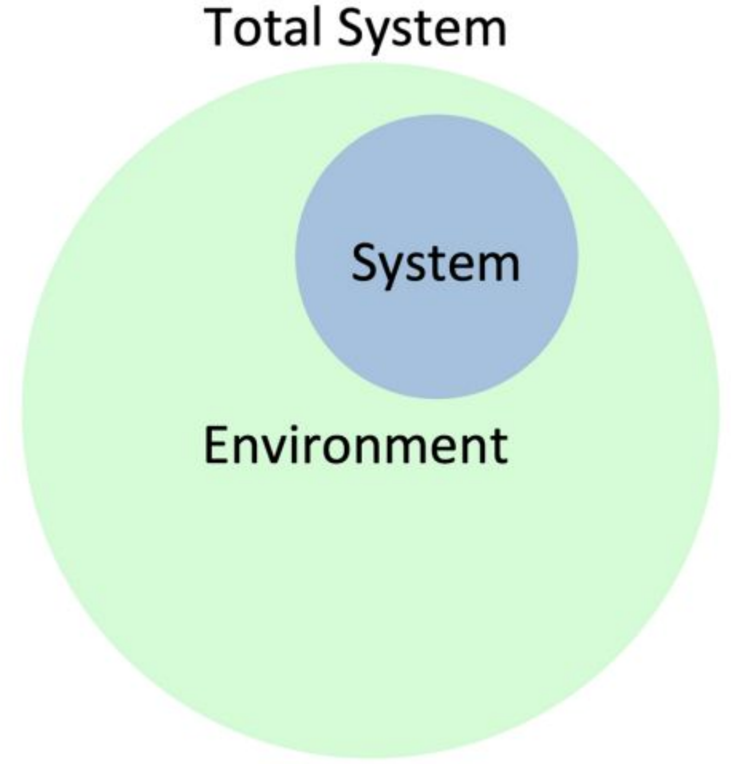
Quantum noise

- Quantum systems often interact with environment, over which we have little control
- Use a new formalism: density matrix

Three “faces” of density matrix:

1. Represents an ensemble of quantum states
2. Represents a system of which we don’t know certain state information
3. Represents a part of quantum system.



Types of errors in quantum computing

- State preparation error
- Gate error
- Crosstalk between gates
- Measurement error

Density matrix formalism

State: $|\psi\rangle$ $\rho \equiv \sum_i p_i |\psi_i\rangle\langle\psi_i|.$

Outcome: $P(a_i) = |\langle\psi|a_i\rangle|^2.$ $P(a_i) = \mathrm{Tr}[|a_i\rangle\langle a_i|\rho],$

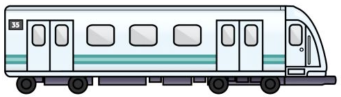
$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} = \rho_{00}|0\rangle\langle 0| + \rho_{01}|0\rangle\langle 1| + \rho_{10}|1\rangle\langle 0| + \rho_{11}|1\rangle\langle 1|,$$

and it should fulfill $\rho_{00} + \rho_{11} = 1$ and $\rho_{01} = \rho_{10}^*$.

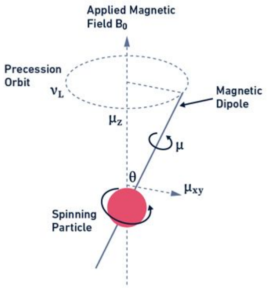
1. A density matrix ρ has unit trace (Tr[ρ] = 1)
2. A density matrix is a positive matrix ρ > 0.

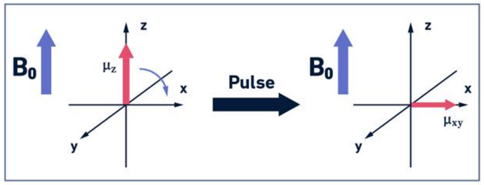
What creates noise in quantum systems?

1. Quantum gates are result of evolution of quantum system
2. The evolution is directed by system Hamiltonian
3. The Hamiltonian may have small interactions with the outside world



Magnetic field





Kraus operators

- Noisy operation described by "Error channels"
- Each type of process has its error channel

$$\mathcal{N}(\rho) = \sum_j A_j \rho A_j^\dagger.$$

Dephasing:

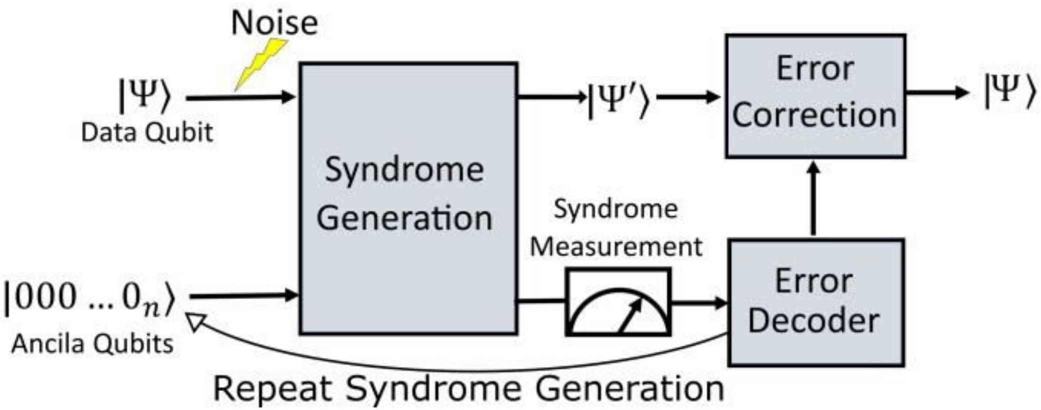
$$\rho \to (1-p)\rho + pZ\rho Z.$$

Depolarizing:

$$\rho \to (1-p)\rho + p\pi,$$

Error correction theory

Motivation: for correct operation quantum computers must correct errors due to *decoherence* and *limited control accuracy*.



Key points:

- “Physical” and “Logical” qubits
- “Data” and “Ancilla” qubits
- Error syndromes
- Error correction

Error correction theory

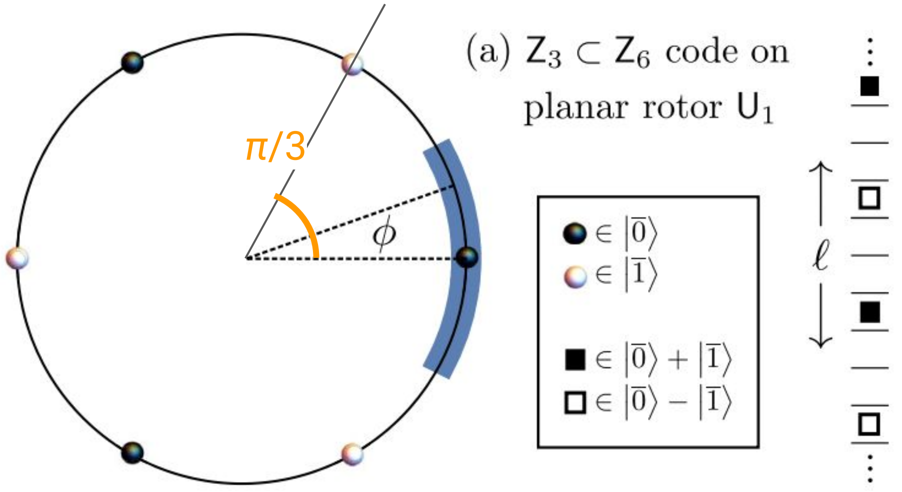
A. A protected qubit

For example, the two orthonormal basis states of a protected qubit can be chosen to be [see Fig. 2(a)]

$$|\bar{0}\rangle = \frac{1}{\sqrt{3}}\left(|\phi = 0\rangle + \left|\phi = \frac{2\pi}{3}\right\rangle + \left|\phi = \frac{4\pi}{3}\right\rangle\right), \quad (10a)$$

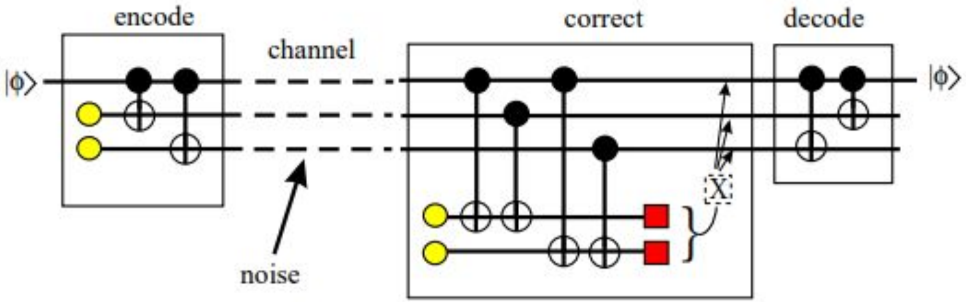
$$|\bar{1}\rangle = \frac{1}{\sqrt{3}}\left(\left|\phi = \frac{\pi}{3}\right\rangle + |\phi = \pi\rangle + \left|\phi = \frac{5\pi}{3}\right\rangle\right). \quad (10b)$$

Both basis states are eigenstates with eigenvalue 0 of $\hat{\phi}$ modulo $\pi/3$. Suppose that $|\bar{\psi}\rangle$ is an arbitrary state in the code space spanned by $|\bar{0}\rangle$ and $|\bar{1}\rangle$. If an error occurs which causes ϕ to shift by $\delta\phi \in [-\pi/6, \pi/6]$, we can unambiguously diagnose the error by measuring $\hat{\phi}$ modulo $\pi/3$. Once $\delta\phi$ is known, we can correct the error by applying a unitary transformation that shifts ϕ by $-\delta\phi$, restoring the state of the rotor to the initial undamaged state $|\bar{\psi}\rangle$.



General error correction procedure

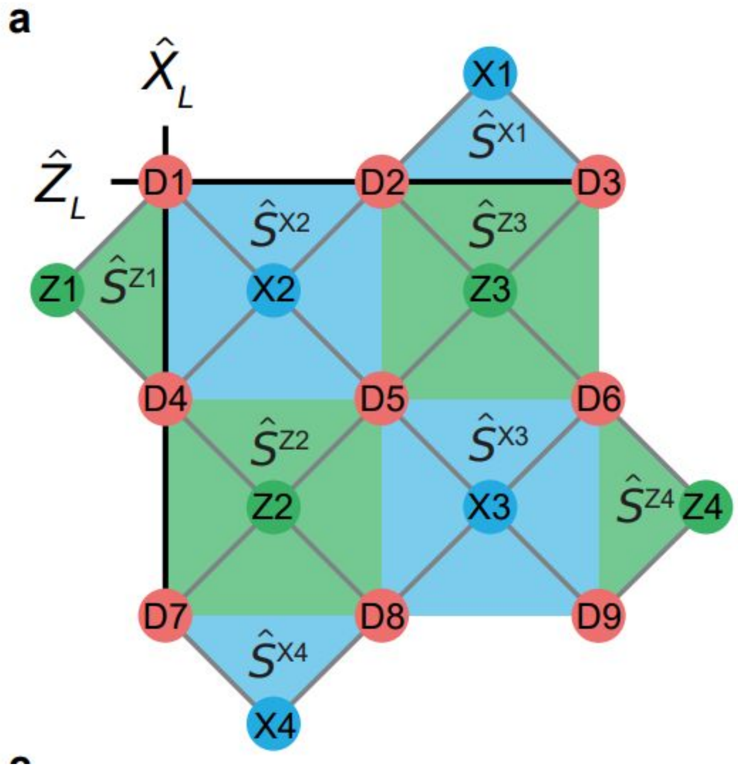
Example of a simple 3-bit code:

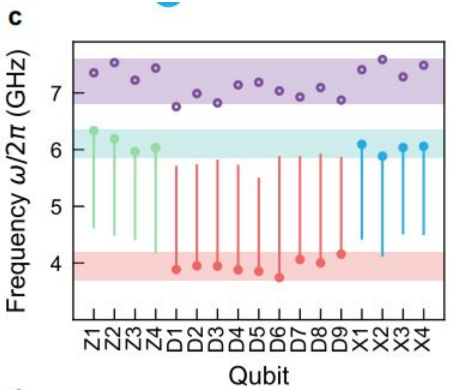


Device description

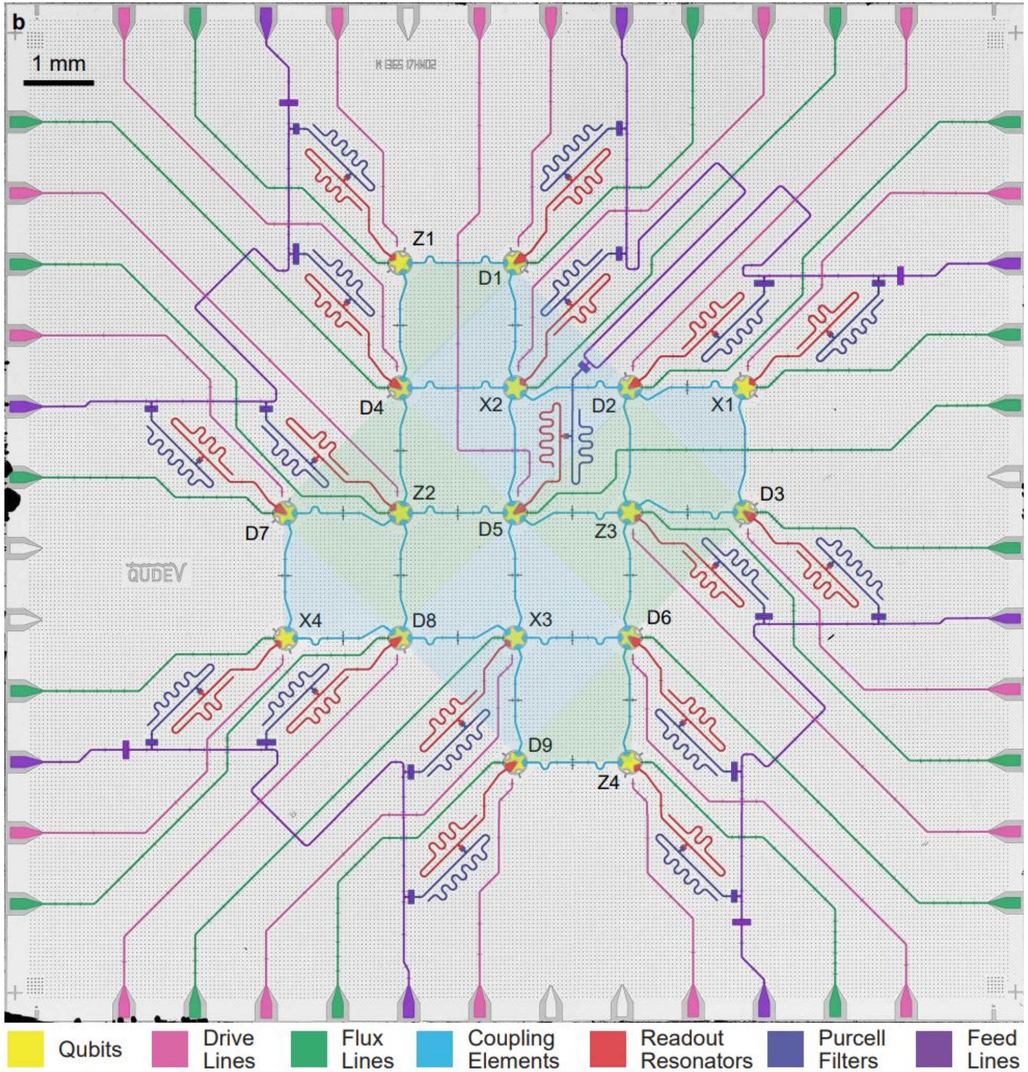
Superconducting circuit with 17 qubits

- Red, Dn- Data qubits - 9x
- Green, Zn - Ancilla in Z-basis - 4x
- Blue, Xn - Ancilla in X-basis - 4x





c
Frequency ω/2π (GHz)
7
6
5
4
Z1 Z2 Z3 Z4 D1 D2 D3 D4 D5 D6 D7 D8 D9 X1 X2 X3 X4
Qubit

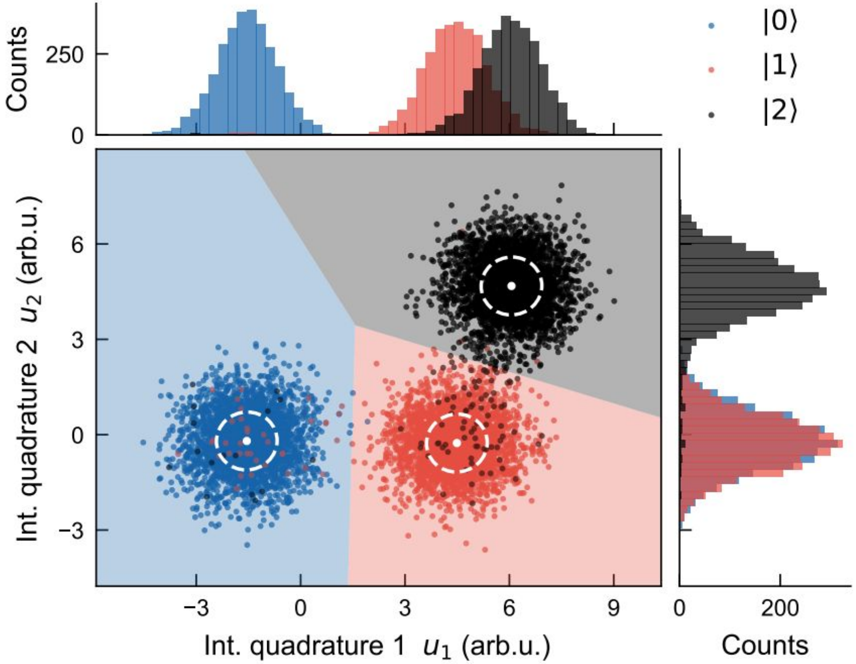


b
1 mm
QUDEV
Z1
D1
D4
X2
D2
X1
Z2
D5
Z3
D3
D7
X4
D8
X3
D6
D9
Z4
Qubits
Drive Lines
Flux Lines
Coupling Elements
Readout Resonators
Purcell Filters
Feed Lines

Single-qubit errors

Three states: 0, 1 and “Leak”

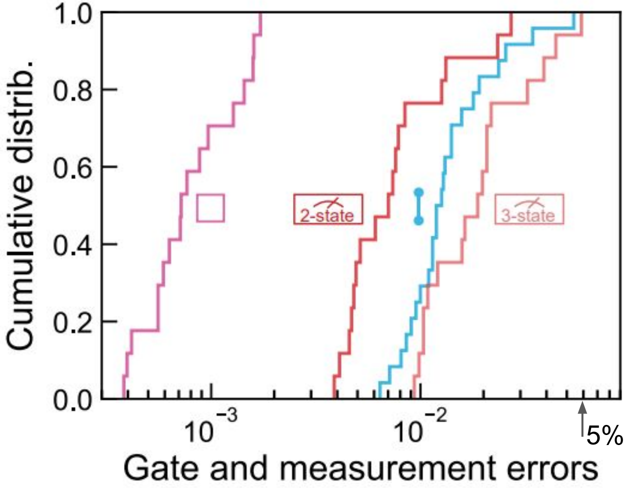
Whenever a “Leak” is detected - the sequence had failed.

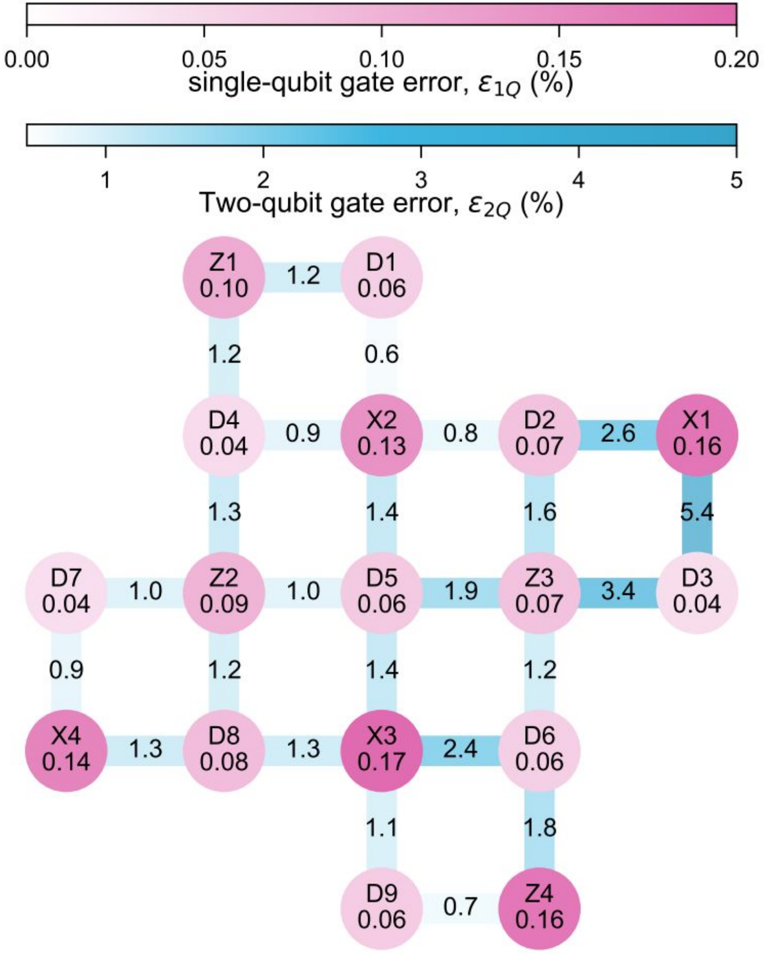


Single-qubit errors

Single- and two-qubit gate errors as characterized by randomized benchmarking and interleaved randomized benchmarking, respectively.

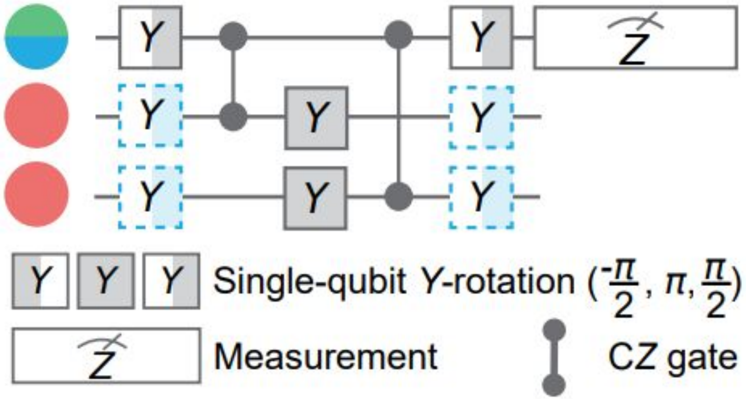
The average measured single-qubit (two-qubit) gate error across the device is 0.09(4) % [1.5(1.0) %]

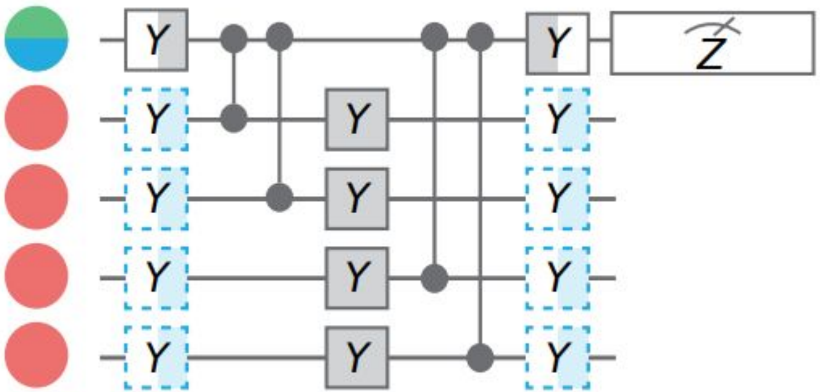




Protocol description

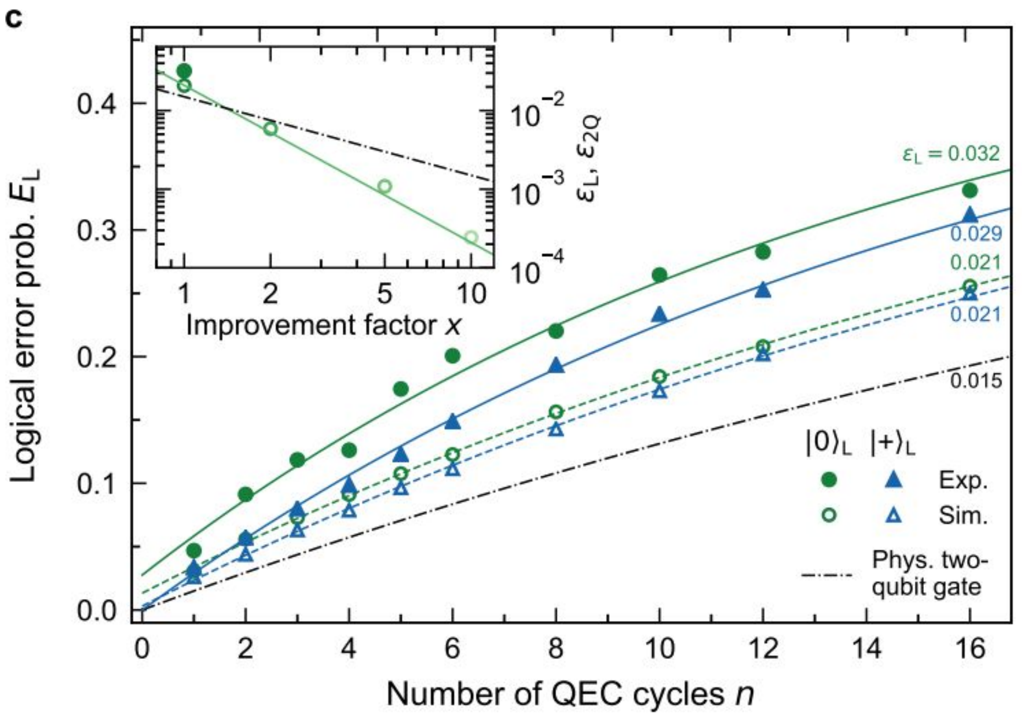
Stabilizer circuits: determine error syndrome





Circuit performance

- Repeated application of error-correcting circuit to preserve the state

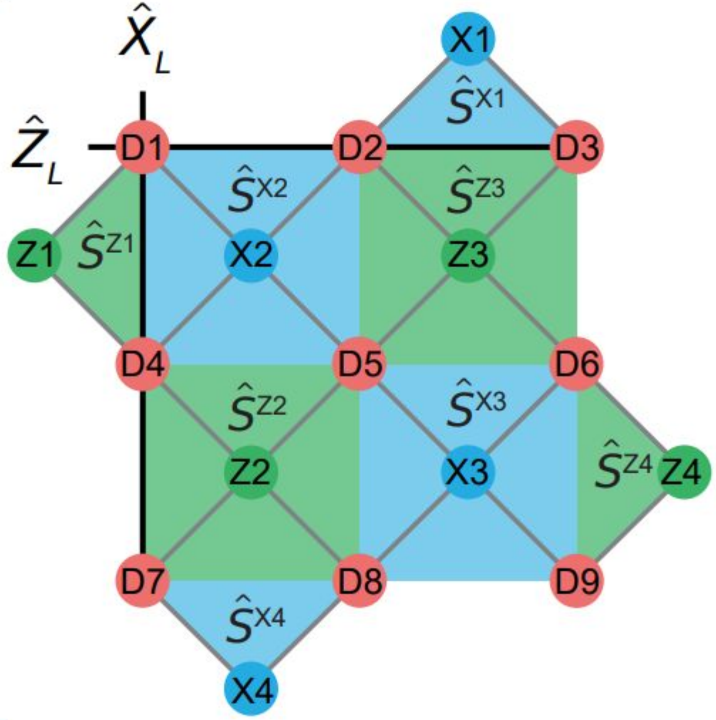


Protocol description

Error syndromes

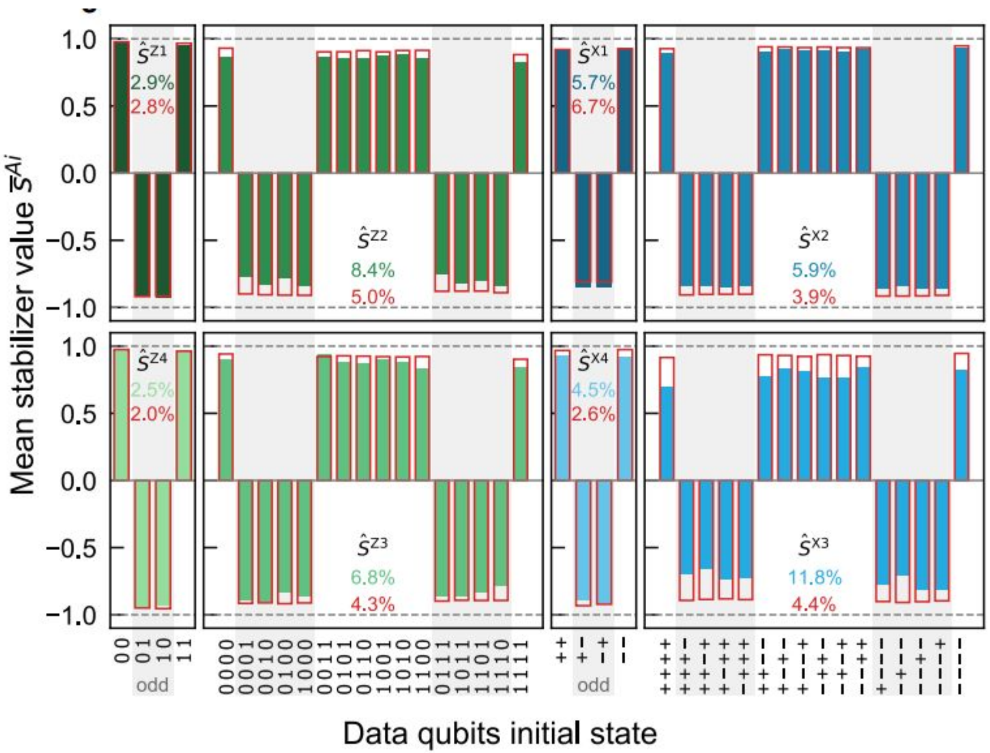
Symbol	Stabilizer	ϵ (%)	ϵ_{sim} (%)
$\hat{S}^{Z1}$	$\hat{Z}_1\hat{Z}_4$	2.9	2.8
$\hat{S}^{Z2}$	$\hat{Z}_4\hat{Z}_5\hat{Z}_7\hat{Z}_8$	8.4	5.0
$\hat{S}^{Z3}$	$\hat{Z}_2\hat{Z}_3\hat{Z}_5\hat{Z}_6$	6.8	4.3
$\hat{S}^{Z4}$	$\hat{Z}_6\hat{Z}_9$	2.5	2.0
$\hat{S}^{X1}$	$\hat{X}_2\hat{X}_3$	5.7	6.7
$\hat{S}^{X2}$	$\hat{X}_1\hat{X}_2\hat{X}_4\hat{X}_5$	5.9	3.9
$\hat{S}^{X3}$	$\hat{X}_5\hat{X}_6\hat{X}_8\hat{X}_9$	11.8	4.4
$\hat{S}^{X4}$	$\hat{X}_7\hat{X}_8$	4.5	2.6
	Weight-two average	3.9	3.5
	Weight-four average	8.2	4.4
	Average	6.1	3.9

a



Protocol description

Ancilla values depending on data qubit values



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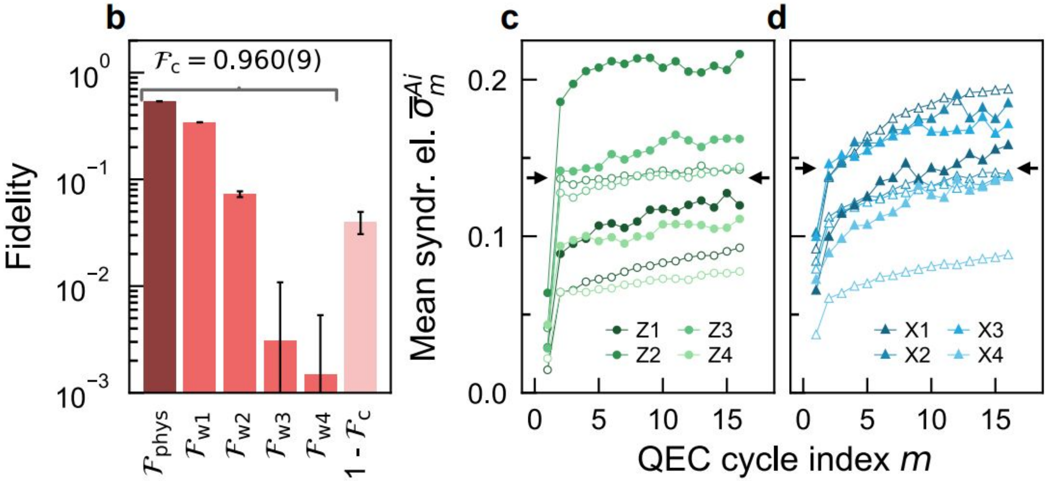

References

1. Krinner, S., Lacroix, N., Remm, A. et al. Realizing repeated quantum error correction in a distance-three surface code. Nature 605, 669–674 (2022). https://doi.org/10.1038/s41586-022-04566-8

2. A Tutorial on Quantum Error Correction Andrew M. Steane https://www2.physics.ox.ac.uk/sites/default/files/ErrorCorrectionSteane06.pdf