Riddler Marble Puzzle June 25

There are 100 marbles in a bag of 3 different colours, and the probability of drawing 3, and one is of each colour is 20%. How many of each are there?

Call the number of marbles a, b, and c. Then first, a + b + c = 100. There are 6 different orders to draw the marbles, for example a, b, then c, each with a probability of $\frac{1}{100} \times \frac{1}{99} \times \frac{1}{98}$ so $\frac{6 \times a \times b \times c}{100 \times 99 \times 98} = \frac{2}{10}$ and $a \times b \times c = 32340$. We will brute force test solutions, but to reduce the number of trials, factor 32340 into $2^2 \times 3 \times 5 \times 7^2 \times 11$. To help solve, first find the cube root of 32340 ~= 31, and since $3 \times 31$ is reasonably close to 100, we can start by assuming that each of a, b, and c is reasonably close to 31.

So, for the first attempt, start with the largest prime factor and try and find a product slose to 31. Try $a = 3 \times 11 = 33$ which leaves $2^2 \times 5 \times 7^2$ for b and c, with the obvious first guesses of $b = 2^2 \times 7 = 28$ and $c = 5 \times 7 = 35$. The sum of these is only 96, so the values are a bit more divergent from the average.

Therefore, for the next try, use $a = 2^2 \times 11 = 44$ leaving $3 \times 5 \times 7^2$ for b and c, which has only a single partitioning into $b = 3 \times 7 = 21$ and $c = 5 \times 7 = 35$  44 + 21 + 35 = 100, so we are done.

So the solution is 44, 35, 21.