Riddler Game Puzzle June 25

There are 11 cards with values 1 to 10, and a joker. You choose the cards randomly and the score accumulates as long as you don't pick the joker, in which case you score 0. What strategy maximizes your score?

At any point in time you have n cards in your hand, which do not include the joker, and your score is s. The total possible score is 55, so there are 11 – n remaining cards, which include the joker, and have a score of 55 – s.

If you proceed, there is a probability of $\frac{1}{11-n}$ that you will score the joker, and $\frac{10-n}{11-n}$ that you will get a good card. If you get a non-joker card, of which there are 10 - n, your expected score will be $s+\frac{55-s}{10-n}$, but this will only happen with a probability of $\frac{10-n}{11-n}$, otherwise your score is 0. Therefore, if you choose a card, your expected score is $\left(s+\frac{55-s}{10-n}\right)\times\frac{10-n}{11-n}$, so you should proceed if $\left(s+\frac{55-s}{10-n}\right)\times\frac{10-n}{11-n}>s$. This simplifies to $s<\frac{55}{2}$, so the algorithm is simply to proceed if your score is 27 or less.

To determine the expected score with this algorithm I needed to use a program. Build a table of the expected score [n] [m] where n is the number, and m is a set of cards that the player owns, and s [10] [{1,2,...,10}] = 55. Work backwards exhaustively computing the expected outcome for each possible set of n cards, and verify against the algorithm stated above.

The answer is the expected outcome is $\frac{10709}{693}$.