Riddler 210813

You are given 4 dice, and roll them. At each stage you must freeze the value of at least one die, and can roll the remaining. What score can you achieve with optimal play?

Suppose you have N dice. Assume that the value with optimal play of N dice is E(N). You roll the dice and get some set of values. Now, try each possible value i of a number of dice to freeze, 1 <= i <= N. The score for freezing i will be the sum of the values of the frozen die plus E(N – i). Choose the value of i with the highest average score. It makes sense to freeze the dice with the highest values since E(N – i) is independent of the values of the dice you choose to roll.

It is straightforward to write a program to find E(N) for increasing values of N since E(N) only depends on E(n<N). I can't find a better way than brute forcing all 6^N possible assignments of N dice, although it would be possible to take advantage of the partitions of the set of values rolled for more computational efficiency. But it is easier to let my computer run for longer.

The optimal algorithm is to precompute the table of optimal scores, shown below, then at each stage with N dice, roll the dice and perform the calculation described above to choose the number of dice to freeze. This is O(N) since the table is precomputed.

The solutions are:

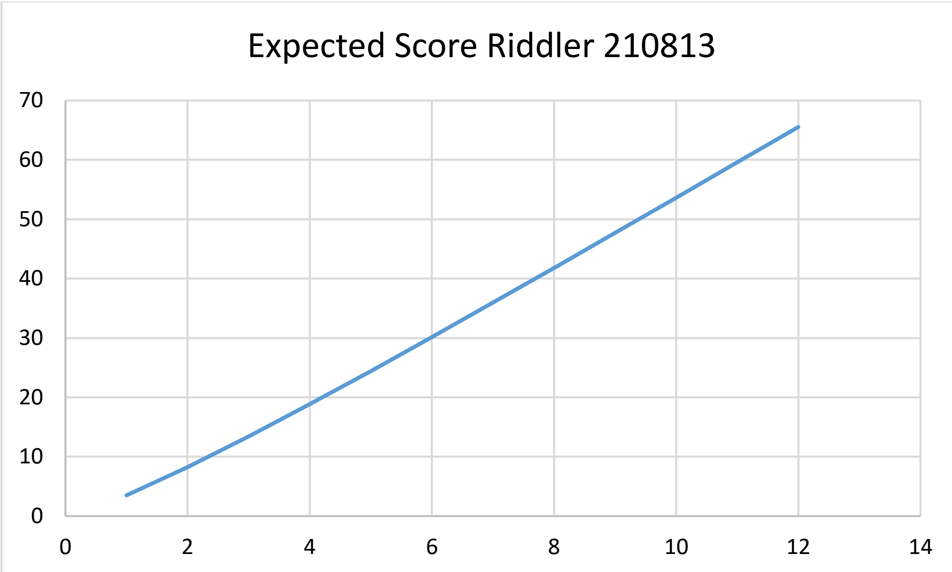
```
level 1 7/2
level 2 593/72
level 3 13049/972
level 4 989065/52488
level 5 1108166095/45349632
level 6 332273594663/11019960576
level 7 4621176159903031/128536820158464
level 8 9026399212157210195951/215892499727278669824
level 9 51897773343582111932203623017/1087849490465798670885322752
level 10
1175926329622606650143536186624838945/21926032917338434804772667113078
784
level 11
29619303722436447922271833379787089100980057/4971687620105922189262161
48618312620703744
```

Improvement Using Partitions

Using partitions is actually much faster. We can partition N throws into 6 bins, and the number of possible partitions grows much slower than 6^N. We can easily generate all partitions with a simple recursive procedure. Given some partition, say of 11 rolls into [6,3,0,2,0,0], this can represent 6 1's, 3 2's, etc. The number of ways of rolling that partition is 11! / 6! / 3! / 0! / 2! / 0! / 0! = 4620. By using partitions we effectively have generated 4620 different rolls in one partition.

Using partitions we can generate all answers up to fairly large values of N using double precision; well beyond the point where it clearly reaches some asymptote. It only takes a few minutes to generate all values up to 100 dice.

1	3.5
2	8.23611
3	13.4249
4	18.8436
5	24.4361
6	30.152
7	35.9522
8	41.8097
9	47.7068
10	53.6315
11	59.576
12	65.5346
13	71.5035
14	77.4801
15	83.4623
16	89.4487
17	95.4383
18	101.43
19	107.424
20	113.419
...	
94	557.402
95	563.402
96	569.402
97	575.402
98	581.402
99	587.402
100	593.402



It seems to be heading to 100 * N – 6.598

The exact rational values are more time consuming to compute and kind of boring. Here are the first 25

```
fast level 1 7/2
fast level 2 593/72
fast level 3 13049/972
fast level 4 989065/52488
fast level 5 1108166095/45349632
fast level 6 332273594663/11019960576
fast level 7 4621176159903031/128536820158464
fast level 8 902639921215721019595l/215892499727278669824
fast level 9 518977733435821119322036230l7/108784949046579867088532275 2
fast level 10
117592632962260665014353618662483894 5/219260329173384348047726671130787 84
fast level 11
296193037224364479222718333797870891009800 57/497168762010592218926216148 61
8312620703744
fast level 12
70923365813321046283314123145046748978307097489755685/108222817915564498 70
5763219963029371887377782826598 4
fast level 13
561487469041525142464941448001497627740799254695737442968278 29/78525839463
58171341713341287133496992453964633859451602862 08
fast level 14
476782352396323731279571500104935878521950207856807479658940994404558940 3/
6153611769500270006125087664173812117749849582935426232072542603758796 8
fast level 15
60371100219595114957771106122669519552035627720536156315883224406079757502
2053298015/72333396373229413001443994222655292959183373049470335393275356 0
16535818825807953 92
fast level 16
6084313660634852865944790629903809557834695372790659484742642386372442206 0
```

8693206705824689677381/680201537161531317827869565786140240595567913096417
2746371344032551160555112808648922663747584
fast level 17
54941620155808943219429425430772688823965437287147257879039390904721594323
23130052456485559965957357551952183/57567698866595896828818805010177309941
020005089175903641701779170442227215659079585607356957143113150758912
fast level 18
59301946574309939636498390982594138582051037140715188812599829698883028090
1758493868495243652049387595257714108566280154741/584657300239190015732994
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fast level 19
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fast level 20
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95754185702799558156670387134246005601398968310729964852100918593937665400
6033/542736338058506521602598688149482386707291597181306749338123421904389
60971783922706486020322260169639182130872939571359119983067896219284360760
4461568
fast level 21
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367733857028705598470587593997475119104
fast level 22
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fast level 23
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0100448148846011036089175769088
fast level 24
37304558478982315517990756943358779273487581164479581288136327117544247826
12234147597862830870085525793933202173809785308731809535124485770278610202
584612263782435348906016135560322928503728432348005391609397246314944631/2
71486152932971848036322978008130780864714876746012805920219179460101431074
13096347792817284219159647198002688393118861390505185711234278540846939619
294421858790966927330768124237854282110987795019457121169796495310848
fast level 25
18447988578447758784125146889336634320209289489368215456738358387676283966
21242811508514960763864488087411168169006605835247793068277626157459649302
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2580528408419373039958411519465128259977821492910069341293121287257880919197021050155445250992817748888780 8