Riddler 210813

You are given 4 dice, and roll them. At each stage you must freeze the value of at least one die, and can roll the remaining. What score can you achieve with optimal play?

Suppose you have N dice. Assume that the value with optimal play of N dice is E(N). You roll the dice and get some set of values. Now, try each possible value i of a number of dice to freeze, 1 <= i <= N. The score for freezing i will be the sum of the values of the frozen die plus E(N – i). Choose the value of i with the highest average score. It makes sense to freeze the dice with the highest values since E(N – i) is independent of the values of the dice you choose to roll.

It is straightforward to write a program to find E(N) for increasing values of N since E(N) only depends on E(n<N). I can't find a better way than brute forcing all 6^N possible assignments of N dice, although it would be possible to take advantage of the partitions of the set of values rolled for more computational efficiency. But it is easier to let my computer run for longer.

The optimal algorithm is to precompute the table of optimal scores, shown below, then at each stage with N dice, roll the dice and perform the calculation described above to choose the number of dice to freeze. This is O(N) since the table is precomputed.

The solutions are:

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level 1 7/2
level 2 593/72
level 3 13049/972
level 4 989065/52488
level 5 1108166095/45349632
level 6 332273594663/11019960576
level 7 4621176159903031/128536820158464
level 8 902639921215721019595 1/21589249972727866982 4
level 9 5189777334358211193220362301 7/1087849490465798670885322752
level 10
11759263296226066501435361866248389 45/2192603291733843480477266711307 8
784
level 11
29619303722436447922271833379787089100980057/4971687620105922189262161
48618312620703744
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