Riddler Express 220429

What is the sum of all numbers 1/n, such that the decimal representation of 1/n has a finite number of digits?

We see that in base 10, n must have no prime factors other than 2 or 5, which are the prime factors of 10. Values of n that have any other factors won't have a finite decimal representation. So we can summarize this as:

$$x = \sum_{\forall n = 2^i \times 5^j} \frac{1}{n}$$

Then it is easy to write an explicit summation:

$$x = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{1}{2^i \times 2^j}$$

We can move the 2^i term outside the latter summation to get:

$$x = \sum_{i=0}^{\infty} \frac{1}{2^i} \sum_{j=0}^{\infty} \frac{1}{5^j}$$

Now summing an exponential series $\sum_{i=0}^{\infty} \frac{1}{c^n} = \frac{c}{c-1}$ so the previous simplifies to $x = \frac{2}{1} \times \frac{5}{4} = \frac{5}{2}$

Therefore the sum is $\frac{5}{2}$.