Riddler 220311

The Riddler 220311 asks you to pick 4 numbers, such that when 4 dice are rolled, the probability that one of the 4 numbers that you have picked is contained in the set of numbers that are the sum of any two of the 4 dice values, is maximized.

This shows results for 4,5 and 6 dice, with 2 or 3 dice per sum, and varying the number of guesses.

Let's start with the base case of 4 dice, 2 dice per sum, and 4 guesses.

We can see that there are 6^4 = 1296 possible ways to roll the dice, and a counting exercise shows that there are 330 different sets of numbers we could choose. The latter is seen by observing that each number must be in the range [2,12], and we need 4 unique numbers, so there are $\binom{11}{4} = 330$ possibilities.

Since there are very few possibilities, it is easy to solve it in a program. We break it into two phases for simplicity, to avoid cascading loops absurdly deep.

Phase 1: for every possible roll of the dice, determine all 6 possible sums (since there are exactly 2 out of 4 dice in each sum) and set the Boolean array entry combo_covers [i] [j] to true iff dice combination i has a sum that evaluates to j.

Phase 2: for every possible set s of distinct integers in [2,12], look at every possible dice roll i, and for each integer j in s, if combo_covers [i] [j], then increment the total success count for s.

The answer is that the set {4,6,8,10} covers 1264 out of the 1296 possible dice rolls, or 79/81 chance.

So why is the number 7, which is the most likely sum of two dice, not included in the best set? And why isn't it the numbers closest to 7, which have the most likely sums? (i.e. 5 and 9 are not included, but 4 and 10 are.)

The worst choice is {2,3,11,12} which seems obvious since the least likely sums of a pair of dice are 2,12 and 3,11.

If we look at other numbers of picked numbers, we get the following:

Number of guesses	Best Guesses	Score / 1296
1	7	834
2	6,7	1083
3	6,7,8	1192
4	4,6,8,10	1264
5	2,4,6,8,10	1283
6	2,4,6,8,10,12	1296

What if we look at sums of 3 of the 4 dice? It takes a lot more guesses to do perfectly. Here is a table of 4, 5, 6 dice, with 2 or 3 dice per sum. For 4 dice the score is out of 1296, for 5 out of 7776 and 6 dice of of 46656.

guesses	dice	dice per sum	best guesses	score
1	4	2	7	834
2	4	2	6,7	1083
3	4	2	6,7,8	1192
4	4	2	4,6,8,10	1264
5	4	2	2,4,6,8,10	1283
6	4	2	2,4,6,8,10,12	1296
1	4	3	10	498
2	4	3	9,11	787
3	4	3	8,10,13	966
4	4	3	7,10,11,13	1086
5	4	3	7,8,10,13,14	1164
6	4	3	7,8,9,12,13,14	1210
7	4	3	6,7,9,10,12,13,15	1252
8	4	3	5,6,8,9,11,12,14,15	1268
9	4	3	3,6,7,9,10,12,13,15,16	1281
10	4	3	3,5,6,8,9,11,12,14,15,18	1290
11	4	3	3,4,6,7,9,10,12,13,15,16,18	1296
1	5	2	7	6210
2	5	2	6,7	7211
3	5	2	6,7,8	7542
4	5	2	4,6,8,10	7724
5	5	2	2,4,6,8,10	7760
6	5	2	2,4,6,8,10,12	7776
7	5	2	2,3,4,6,8,10,12	7776
1	5	3	10	4965
2	5	3	8,13	6490
3	5	3	8,10,13	7105
4	5	3	6,9,12,15	7574
5	5	3	3,6,9,12,15	7675
6	5	3	3,6,9,12,15,18	7776
7	5	3	2,3,6,9,12,15,18	7776
1	6	2	7	41586

2	6	2	6,7	45175
3	6	2	5,7,8	46143
4	6	2	4,6,8,10	46568
5	6	2	2,4,6,8,10	46637
6	6	2	2,4,6,8,10,12	46656
7	6	2	2,3,4,6,8,10,12	46656
1	6	3	10	37428
2	6	3	8,13	43846
3	6	3	7,12,13	45343
4	6	3	6,9,12,15	46346
5	6	3	3,6,9,12,15	46511
6	6	3	3,6,9,12,15,18	46656
7	6	3	2,3,6,9,12,15,18	46656