Exploring Air Resistance and its Effects on Acceleration

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Abstract

In this experiment we studied the effects of increasing the cross-sectional area of a cart on its acceleration and velocity. Through this, we proved that air resistance exists and is directly correlated to the cross-sectional area of an object, as well as its shape. Furthermore, we found that air resistance can be decreased by keeping cross-sectional area constant and changing the shape of an object.

Introduction

Throughout high school physics, we have been calculating the range of projectiles, the velocity of carts, and even the acceleration of planes - all by assuming that air resistance is negligible. In this experiment, we set out to prove that air resistance does exist, and that it has a significant impact on acceleration and velocity of an object relative to its cross-sectional area

Background

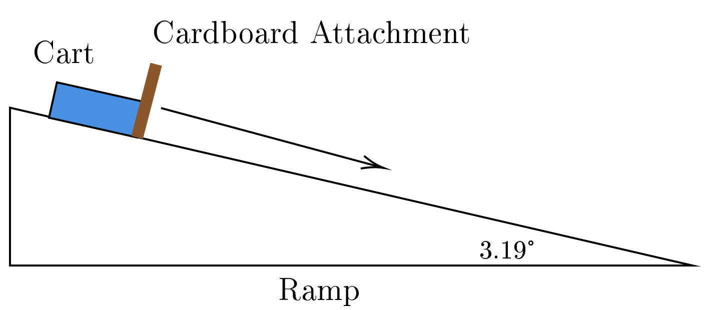


Figure 1. General experimental setup with a Smart Cart rolling down a ramp. Cross-sectional area of the cart is changed using a cardboard attachment.

Figure 1 shows the basic experimental configuration of this experiment. The cart was rolled down a ramp with an attachment on its front. Said attachment modified the cart's cross-sectional area as well as its aerodynamic properties.

We can break down the forces experienced by the cart into components 1 as shown in the free body diagram below.

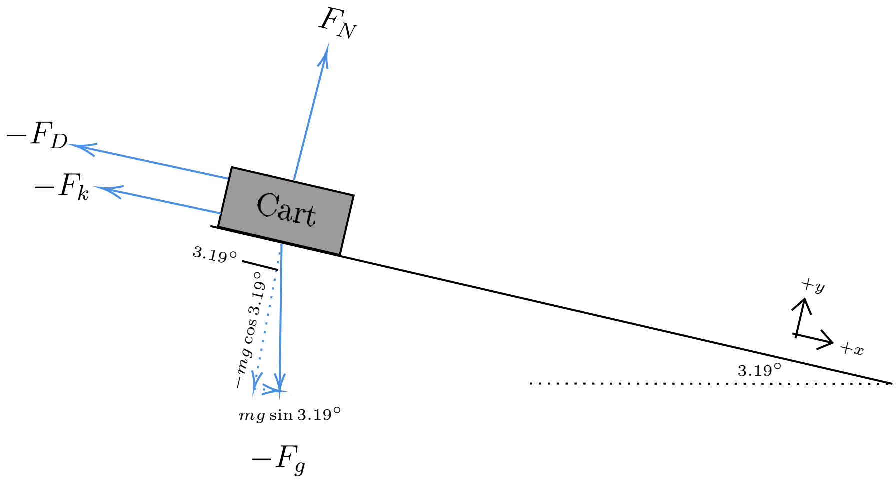


Figure 2. Free body diagram illustrating various forces acting upon the Smart Cart

In the free body diagram in figure 2, the cart is on the ramp, with the forces of air resistance (F_D) and kinetic friction (F_k) acting against the cart in the horizontal direction. The cart is also accelerated down the ramp by the horizontal component of gravity ($mg\sin(3.19°)$). Given that the force of air resistance upon an object with a drag coefficient of K is Kv^2, the following equation can be derived to solve for the drag coefficient where acceleration and velocity are a function of time.

$$F = mg\sin(\theta) - F_k \tag{1}$$

$$F_{\text{net}} = F - Kv(t)^2$$

$$ma(t) = F - Kv(t)^2$$

$$F - ma(t) = Kv(t)^2$$

$$K = \frac{F - a(t)m}{v(t)^2} \tag{2}$$

Methods

1. A 250g weight was taped onto the Smart Cart to make the effect of additional weight negligible
2. A 1.22m ramp was placed on top of a textbook so that it made a 3.19 degree angle with the lab bench counter
3. Depending on the trial being run, a cardboard attachment was measured and cut to proper dimensions and taped to the front of the Smart Car
4. The Smart Car's position, velocity, force, and acceleration were all recorded relative to time using Pasco Capstone software
5. Steps 3 and 4 were repeated seven more times until all trial data were successfully collected

Results

Table 1

K values relative to cross-sectional area and the shape of the attachment

Attachment	**Length (cm)**	**Width (cm)**	**Area (cm^2)**	**K**
None	3.4	8.5	33.85	0.002
Orange	29	33	957	0.184
Cardboard	12	20.75	249	0.105
Cardboard	15.78	15.78	249	0.109
Orange Pocket	29	33	957	0.198
Equilateral Wedge	9.4	26.5	249	0.087
Sharp Wedge	9.4	26.5	249	0.050
Sharp Wedge on Orange	29	33	957	0.127

Discussion

Methodology

One unique part of our methodology that proved crucial to the statistical significance and accuracy of our data was the addition of a 250g weight on top of the Smart Cart. Adding the various cardboard attachments used to modify the aerodynamic properties of the Smart Cart would increase the total weight of our modified Smart Cart system. It would be possible to account for the variance in weight by measuring each cardboard attachment along with the tape used to affix it; however, a lack of time meant that it would be more efficient to make the additional weight negligible by increasing the overall weight of the Smart Cart itself. Although this may introduce some variance into our results, the relative effect of the additional weight is acceptable given how little the attachments weighed relative to the weighted Smart Cart.

Air Resistance

A major goal of this experiment was to determine whether or not air resistance actually existed, or if it had enough of an effect to affect our data in a manner that was statistically significant. If air resistance were to exist and be statistically significant with respect to the data collected, then there should be a statistically significant decrease in acceleration due to the increasing drag forces. After collecting multiple sets of data and removing outliers based on inter-quartile ranges, the data for each trial were individually plotted on a graph. Specifically, when acceleration relative to time was plotted for each trial, as well as a linear regression for acceleration relative to time, it becomes clear that when cross-sectional area is increased, there is a time-dependent, increasing drag force that decreases acceleration. Referring to A3, A4, A5, A2, A6, A7, and A8, each of their respective linear regressions of acceleration all have negative slopes and decrease with respect to time - signifying an overall deceleration. Furthermore, when referring to the base case, A1, it can be seen that the slope of the linear regression of acceleration is practically zero. The base case did not have a cardboard attachment nor an increased cross-sectional area. This shows that increasing cross-sectional area will affect the aerodynamic properties of the cart, which will create drag forces that decelerate the cart.

Calculating the Drag Coefficient ($\boldsymbol{K}$)

To calculate the drag coefficient for a given time, we can use equation 2. Since all the variables needed to solve for K at a given time are known, then these values can be substituted to solve for K at a given time - with the exception of F_k from equation 1.

Calculating the Force of Kinetic Friction

The friction forces on the cart are a system composed of static friction from the wheels on the ramp, and kinetic friction on the axles. It becomes difficult to find kinetic friction since it presents two unknowns: F_k and K. To determine F_k without needing to use two unknown variables, we let $K = 0$ in the base case where the cart had no additional attachments affecting its cross-sectional area. We were able to set K to zero in this case because in A1, the linear regression of acceleration for the base case was equal to zero, thus showing that the deceleration due to air resistance was also zero. To find for F_k given K, we can solve for F_k as a function of time by dividing acceleration by the mass of the cart and then average over the time interval of the respective trial. This will give us a numeric value for F_k if it is needed in other calculations. For the purposes of this experiment, F_k is included in the variable F[1] from 1, and not directly substituted into any calculations.

The Correlation Between Shape, Cross-Sectional Area, Angle of the Wedge, and *K*

Cross-Sectional Area. From the results in 1, we are able to see that for most cases, an increase in K correlates to an increase in the overall cross-sectional area of that attachment. We also see that changing the dimensions of an attachment, but keeping surface area constant (as shown in both cardboard trials) does not affect K in a significant manner. In fact, the change in K for this case is marginal, and one could argue that it is not a statistically significant change.

Angle of the Wedge. Another interesting observation was the effects of adding an equilateral wedge onto the Smart Cart system. The equilateral wedge was of the same surface area as the cardboard. However, it had a significantly lower drag coefficient. This is due to the shape of the attachment itself. A wedge is able to "cut" through the air, which creates less turbulence and frictional drag around the surfaces of the attachment compared to a fully flat piece of cardboard. The shape of the wedge also redirects the air around the attachment instead of directly impacting the air, causing less deceleration upon the Smart Cart system. When the angle of the sharp part of the wedge facing forward was changed so

[1] F is the average force measured by the Smart Cart for the base case, as the average force experienced by the cart when air resistance is negligible would only be the component of gravity down the ramp, and $-F_k$

that it was sharper, we see that K decreases relative to a normal equilateral wedge. Making the leading edges of the wedge have a smaller angle relative to the ramp results in less force being used to push the air out of the way. Normally, for an attachment that is flat, the air directly creates a normal force against that attachment, pushing it up the ramp. With a sharp wedge, the sharper angles redirect that normal force so that it points outwards from the edge, resulting in less push-back against the cart in the horizontal direction.

Linear regressions of the acceleration of both wedges are of the same order of magnitude as that of the cart with no attachments. However, these figures do carry a certain statistical insignificance due to the almost harmonic fluctuation of acceleration in these cases. Therefore, it can be said that adding these wedges make the cart have no air friction, or that air friction cannot be measured properly due to the inaccuracy of the cart's measurements.

Shape. Both wedge, and flat cardboard shapes and their relation to K have been previously discussed. When we flipped the orange cardboard attachment (top cover of a shoebox) over so that the raised edges face forward, it was observed that the K value slightly increased relative to the flat side of the orange cardboard attachment. These attachments both have the same cross-sectional area, however, the overall shape of the "pocketed" orange attachment with its raised edges was more conducive to creating turbulence and frictional drag against the air. It can be theorized that the raised edges effectively trap some of the air impacting the attachment, thereby creating a pocket of air that creates more air resistance and decelerating the Smart Cart system.

General Conclusions. Through these trials, it can be shown that increasing the cross-sectional area of the Smart Cart increases the drag coefficient K and decelerates the cart. Changing the dimensions while retaining the same cross-sectional area does not appear to affect air resistance and the K value. It can also be shown that changing the angle at which the attachment impacts the air can decrease air resistance, and making said angle sharper, or smaller relative to the ramp, can further decrease air resistance. Finally, changing the shape of the attachment by adding raised edges also causes more air resistance, thereby increasing K and causing deceleration.

Experimental Errors

Instrumental Errors

Smart Cart Measurements. The acceleration measured by the Smart Cart appears semi-harmonic. This was problematic in our calculations since the linear regression of acceleration was relied upon to get a clearer picture of the impact of air resistance on a Smart Cart trial. Had the acceleration been closer to a exponential or polynomial fit, the regression which introduced statistical error to the analysis would not have been necessary. Acceleration with respect to time was also used in the calculations of K at a given time; the large fluctuations also seemed to affect the average K values for each trial. In the end, since these values are averaged out through regression and outlier trimming, the effect on the results should be marginal, however, it did present increased complexity to the interpretation of experimental data.

Appendix A

Trial Charts

Figure A1. Raw Data Collected from Smart Cart and Calculated K value - No Attachment

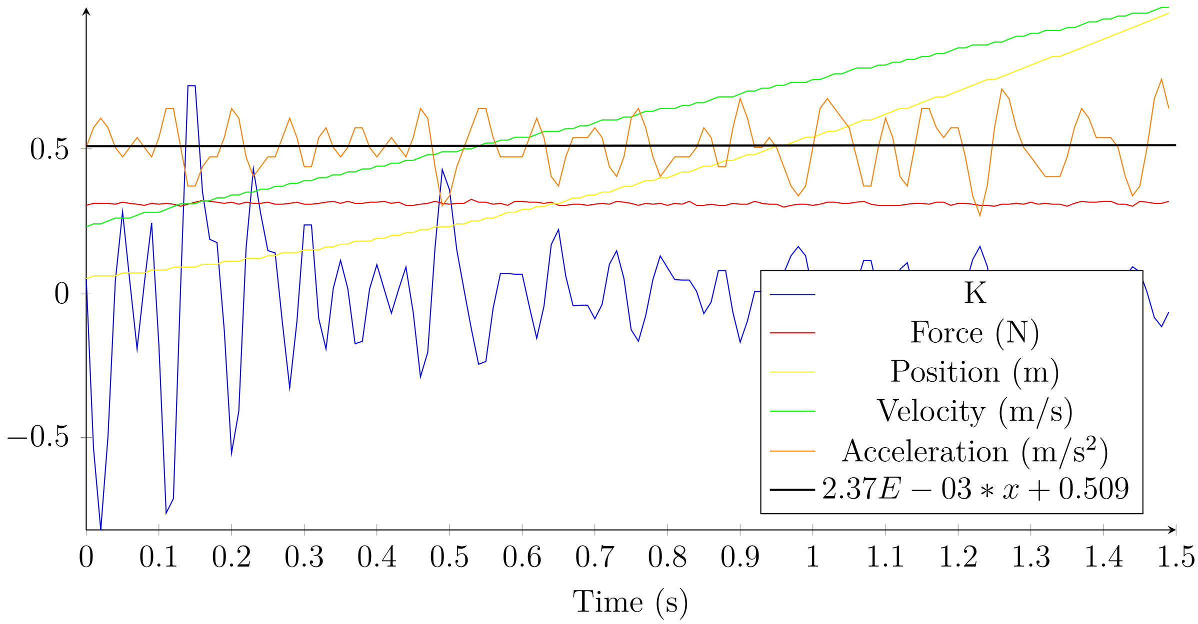


Figure A2. Raw Data Collected from Smart Cart and Calculated K value - Orange

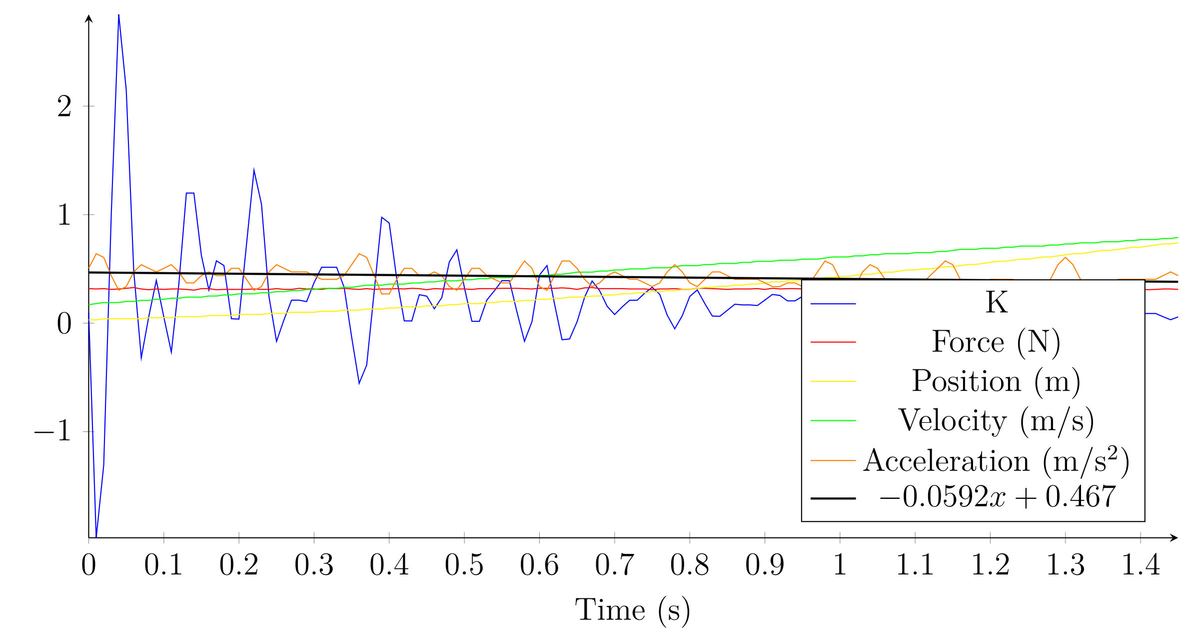


Figure A3. Raw Data Collected from Smart Cart and Calculated K value - Cardboard (12 x 20.75)

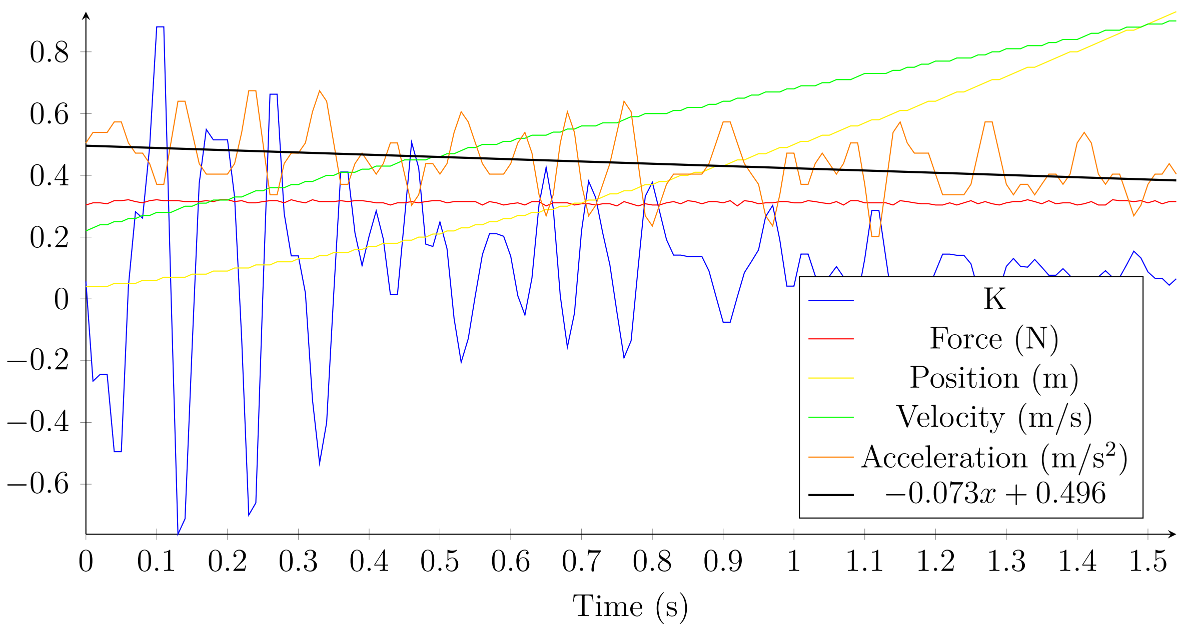


Figure A4. Raw Data Collected from Smart Cart and Calculated *K* value - Cardboard (15.78 x 15.78)

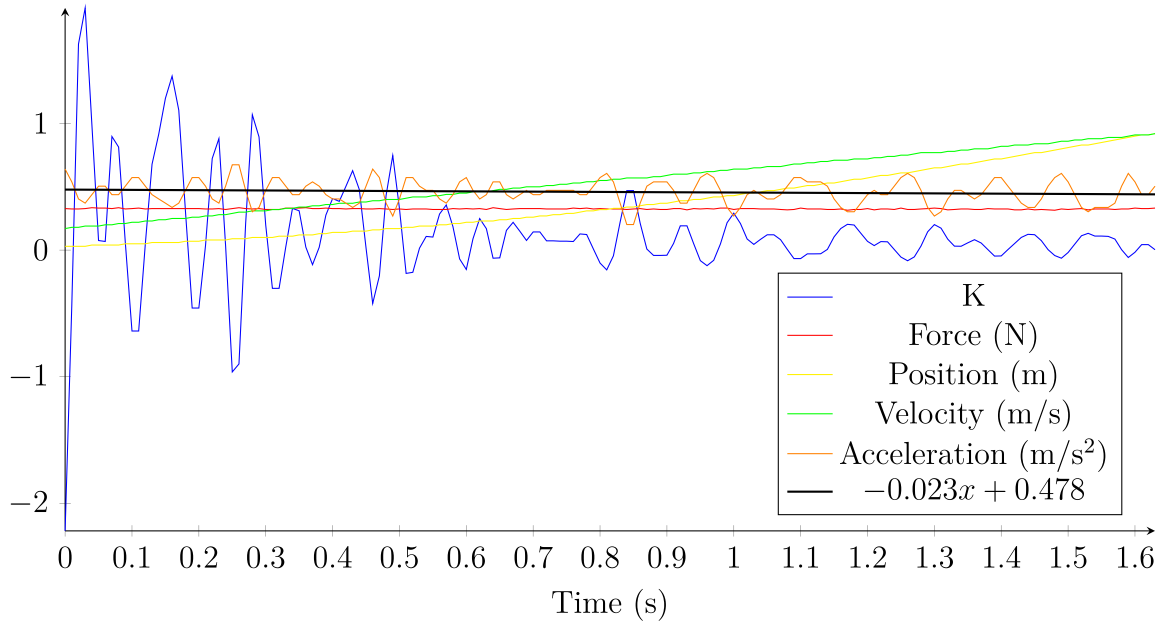


Figure A5. Raw Data Collected from Smart Cart and Calculated K value - Equilateral Wedge

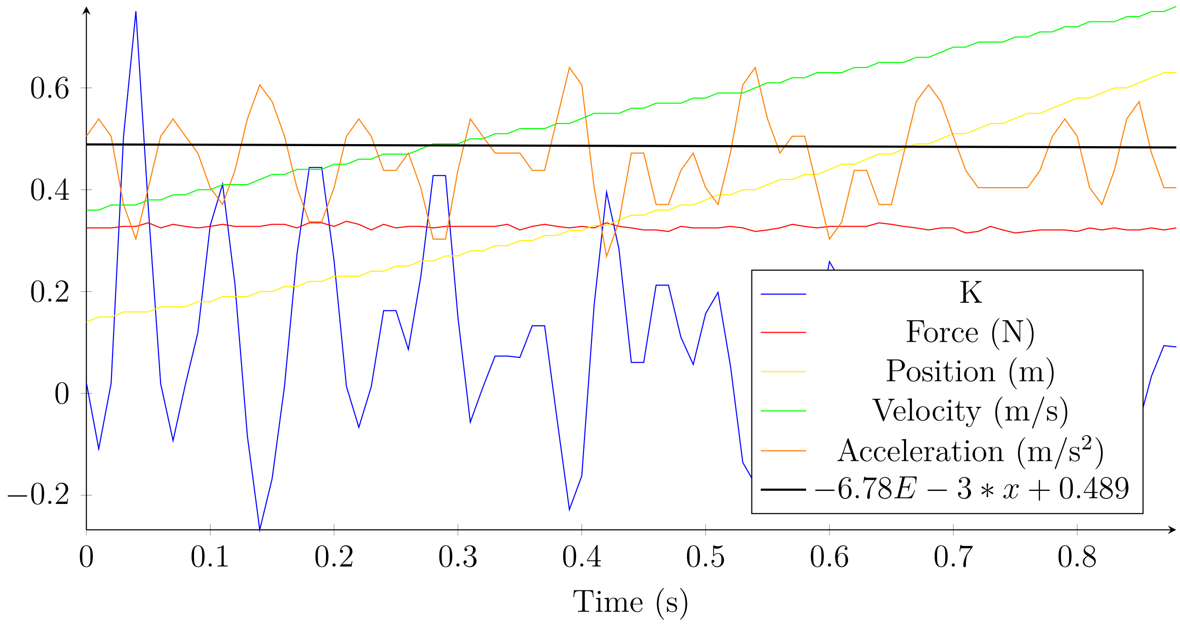


Figure A6. Raw Data Collected from Smart Cart and Calculated K value - Orange with Pocket

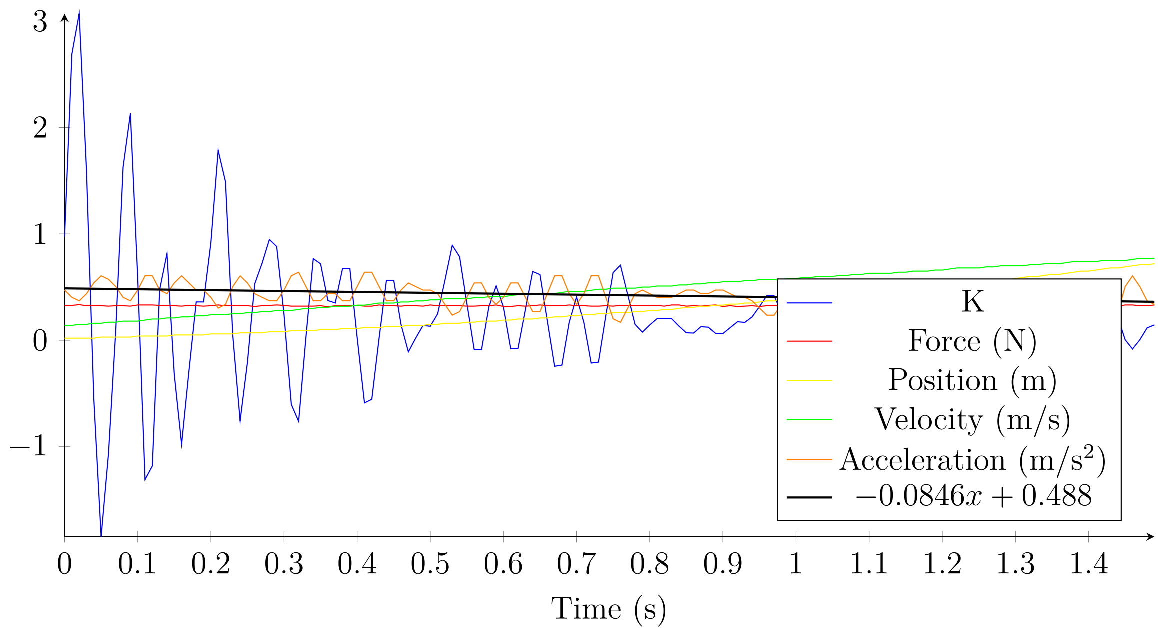


Figure A7. Raw Data Collected from Smart Cart and Calculated K value - Sharp Wedge

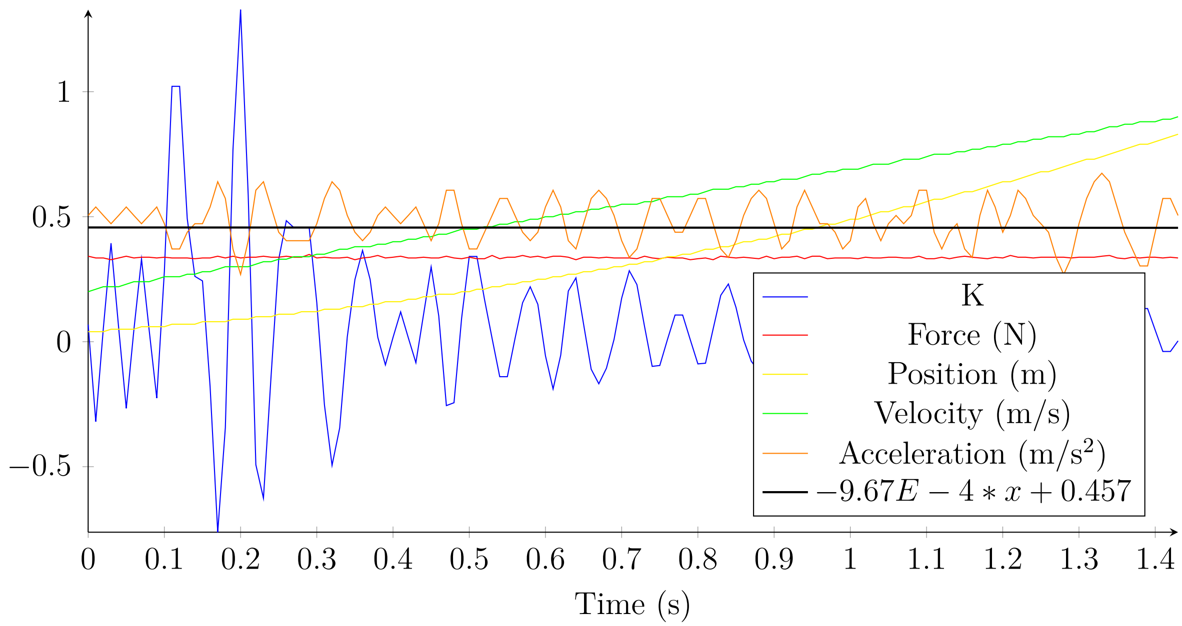
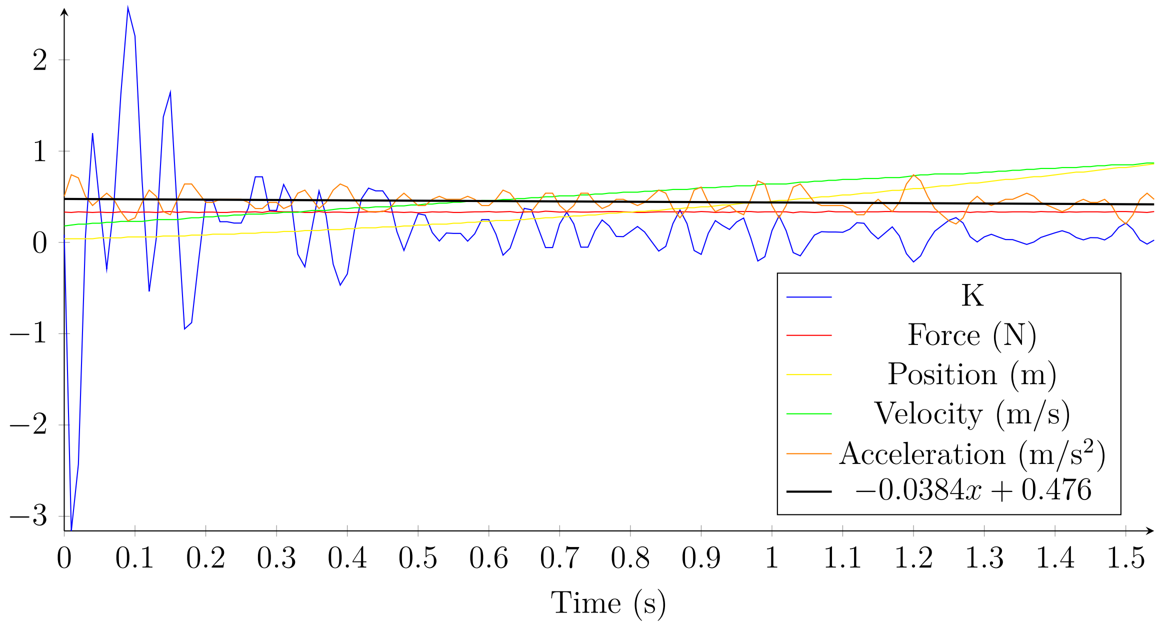


Figure A8. Raw Data Collected from Smart Cart and Calculated K value - Sharp Wedge on Orange



Appendix B

Sample Calculations

Average Range

$$\text{Average Range} = \frac{R_1 + R_2 + R3 + ... + R_n}{n} \tag{3}$$

$$\text{Average Range}_{12.5\text{cm}} = \frac{338 + 333 + 332 + 336 + 332}{5} \tag{4}$$

$$\text{Average Range}_{12.5\text{cm}} = 334.2 \text{ cm} \tag{5}$$