P. Set #2

David Lieberman MATH 797 02/10/20

(1) The Centering Matrix Operator is given by $C_n = I_n - 1_N$
The Kernel Matrix K is square, so taking the matrix product $C_n K C_n$ will center both K's rows and colums to mean 0.

$$K_c = C_n K C_n = (I_n - 1_N) K (I_n - 1_N)$$
$$= I_n K I_n - I_n K 1_N - 1_N K I_n + 1_N K 1_N$$
$$= K - K 1_N - 1_N K + 1_N K 1_N \quad \text{as desired} \quad \blacksquare$$

(2) Under the low-rank assumption, we may express:

$$K = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \Lambda \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}^T$$

$$\Rightarrow K_{11} = U_1 \Lambda U_1^T \; ; \; K_{12}^T = K_{21} = U_2 \Lambda U_1^T \; ; \; K_{22} = U_2 \Lambda U_2^T$$

$$\Rightarrow K_{21} K_{11}^{-1} K_{12} = (U_2 \Lambda U_1^T)(U_1 \Lambda U_1^T)^{-1} (U_2 \Lambda U_1^T)^T$$
$$= U_2 \cancel{\Lambda U_1^T (U_1^T \Lambda)^{-1} U_1^{-1}} U_1 \Lambda^T U_2^T$$
$$= U_2 \Lambda^T U_2^T = U_2 \Lambda U_2^T = K_{22} \quad \blacksquare$$