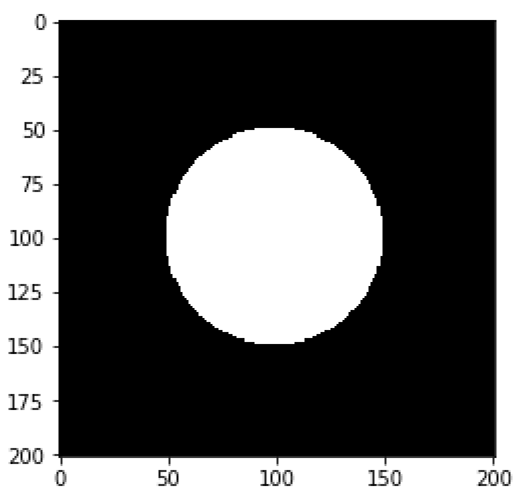
AMATH797 PSet1 David Lieberman

In [1]:
```
import numpy as np
import os
from numpy import linalg
from numpy.linalg import norm
import math
from PIL import Image
from sklearn.feature_extraction.image import extract_patches_2d
from sklearn.decomposition import PCA

import scipy
import scipy.io as sio
from scipy.io import loadmat

import matplotlib.pyplot as plt
from matplotlib import offsetbox
from mpl_toolkits.mplot3d import Axes3D
from mpl_toolkits.mplot3d import proj3d
%matplotlib inline

os.chdir(os.path.expanduser(os.sep.join(["~","Desktop","Homework Scans","2020S_AMATH797"
])))
np.random.seed(0)
```

In [2]:
```
circle = np.array(Image.open("circle.png"))
#circle = circle.astype('float')/255
plt.imshow(circle, cmap = 'gray', vmin=0, vmax=255)
```

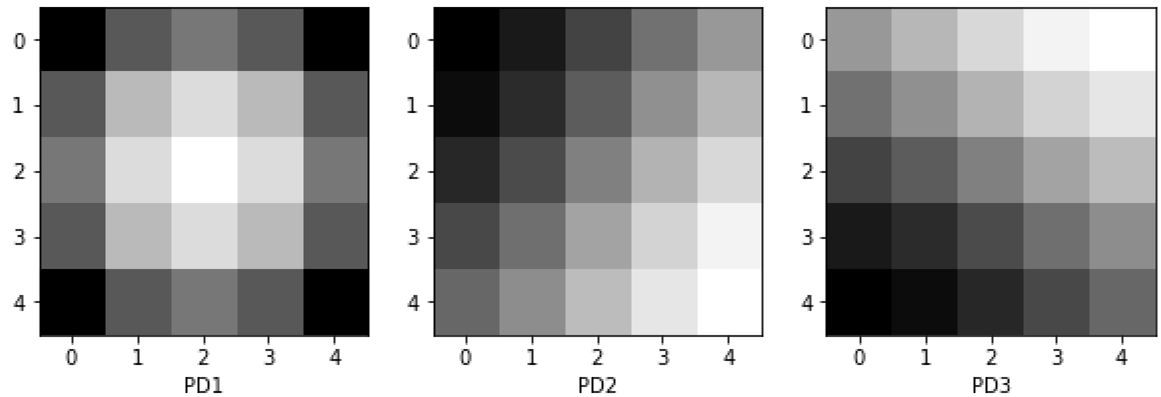
Out[2]: `<matplotlib.image.AxesImage at 0x15cda3ec9c8>`



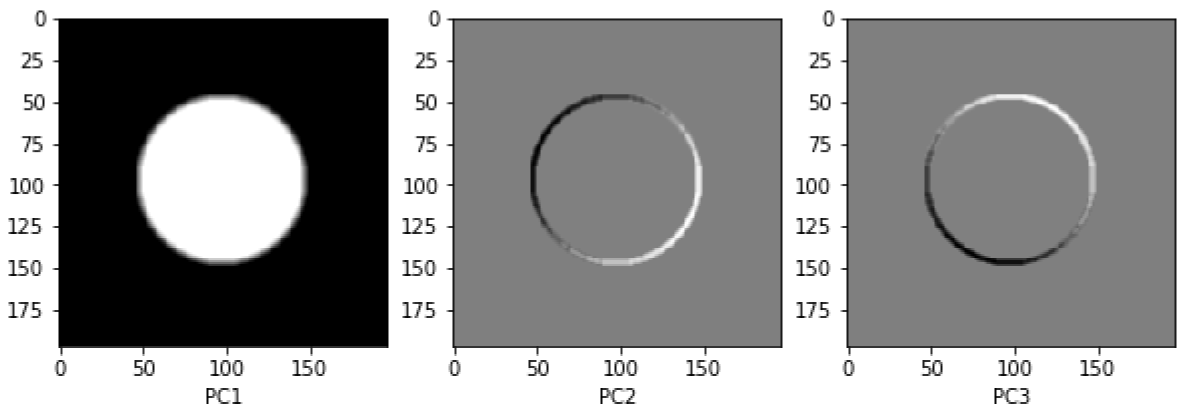
In [3]:
```
patches = extract_patches_2d(circle, (5, 5))
patches = patches.reshape((38809, 25))
#patches_centered = patches - np.mean(patches[:], axis=0)

circle_pca = PCA(n_components = 3)
circle_pca.fit(patches)
print(circle_pca.explained_variance_ratio_)
```

```
[0.95940322 0.01202822 0.01202822]
```

In [4]:
```python
PD = [np.reshape(circle_pca.components_[i], (5,5)) for i in range(3)]

fig = plt.figure(figsize=(10, 10))
for i in range(3):
    fig.add_subplot(1, 3, i + 1)
    plt.xlabel('PD' + str(i + 1))
    plt.imshow(PD[i], cmap = 'gray')
```

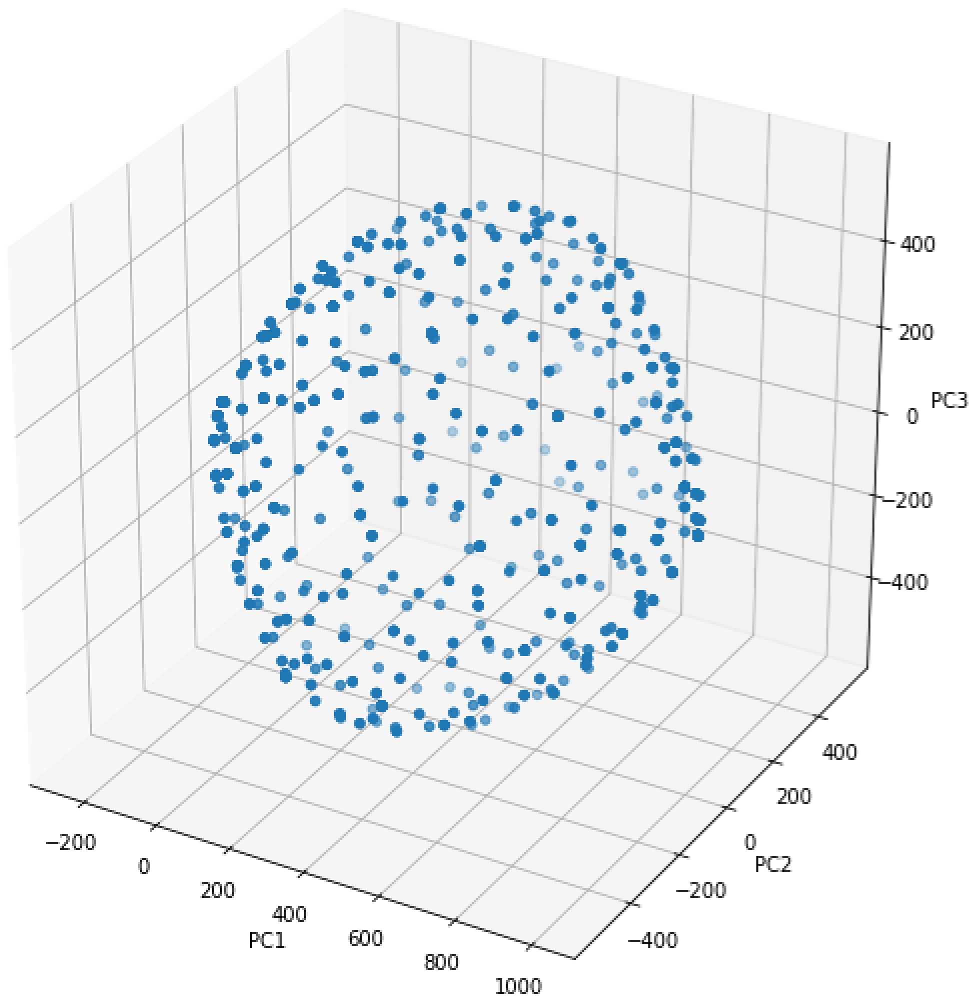


In [5]:
```python
PC = circle_pca.fit_transform(patches)

fig = plt.figure(figsize=(10, 10))
for i in range(3):
    fig.add_subplot(1, 3, i + 1)
    plt.xlabel('PC' + str(i + 1))
    plt.imshow(np.reshape(PC[:, i], ((197, 197))), cmap = 'gray')
```



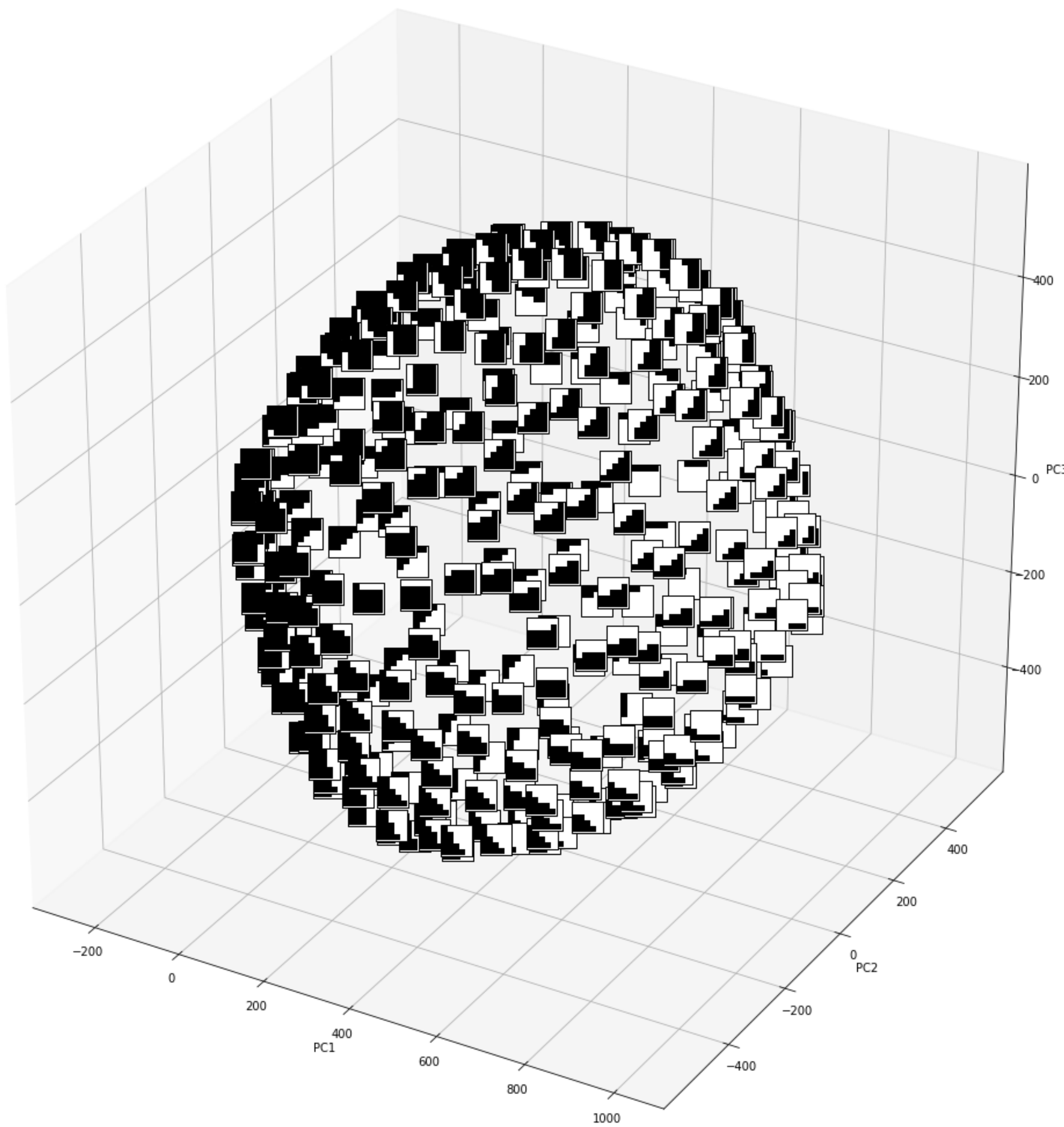
In [6]:

```
fig = plt.figure(figsize=(10, 10))
ax = plt.axes(projection = '3d')
ax.scatter3D(PC[:, 0], PC[:, 1], PC[:, 2])
ax.set_xlabel('PC1')
ax.set_ylabel('PC2')
ax.set_zlabel('PC3')
plt.show()
```



In [7]:

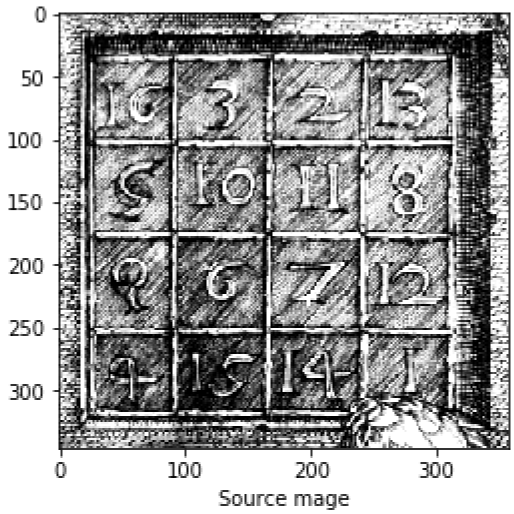
```
xs = PC[:, 0].flatten()
ys = PC[:, 1].flatten()
zs = PC[:, 2].flatten()

fig = plt.figure(figsize=(20, 20))
ax = fig.add_subplot(111, projection=Axes3D.name)
ax.scatter(xs, ys, zs)

ax2 = fig.add_subplot(111, frame_on=False)
ax2.axis("off")
ax2.axis([0,1,0,1])


def proj(X, ax1, ax2):
    """ From a 3D point in axes ax1,
        calculate position in 2D in ax2 """
    x,y,z = X
    x2, y2, _ = proj3d.proj_transform(x,y,z, ax1.get_proj())
    return ax2.transData.inverted().transform(ax1.transData.transform((x2, y2)))

def image(ax,arr,xy):
    """ Place an image (arr) as annotation at position xy """
    im = offsetbox.OffsetImage(arr, zoom = 5, cmap='gray', norm=plt.Normalize(0,255))
    im.image.axes = ax
    ab = offsetbox.AnnotationBbox(im, xy, xycoords = 'data', frameon = True, pad = 0.1)
    ax.add_artist(ab)


i = 0
for s in zip(xs,ys,zs):
    x,y = proj(s, ax, ax2)
    image(ax2, np.reshape(patches[i], ((5, 5))), [x,y])
    i += 1

ax.set_xlabel('PC1')
ax.set_ylabel('PC2')
ax.set_zlabel('PC3')
plt.show()
```



We observe that principal component 1 which captures roughly ~96% of the variance of the data seems to differentiate patches based on their color. Principal components 2 and 3, seem to capture directions for moving vertically and horiztonally (respectively), which are axises of symmetry for the circle. These results are exactly those which we would expect.

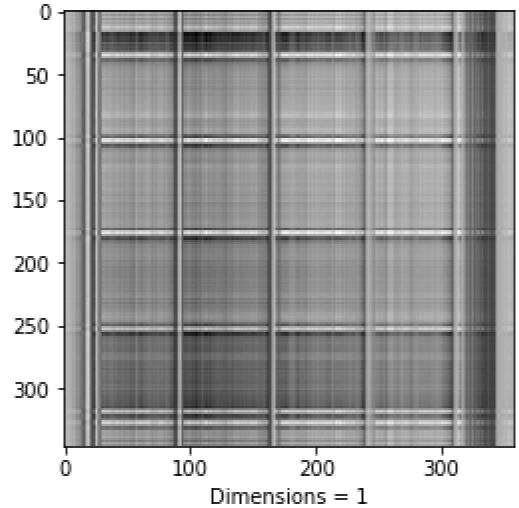
In [8]:
```
numbers = scipy.io.loadmat('numbers.mat')['mat']
numbers_mean = np.mean(numbers[:], axis=0)
plt.imshow(numbers, cmap = 'gray')
plt.xlabel('Source mage')
numbers.shape
```

Out[8]:
```
(346, 358)
```

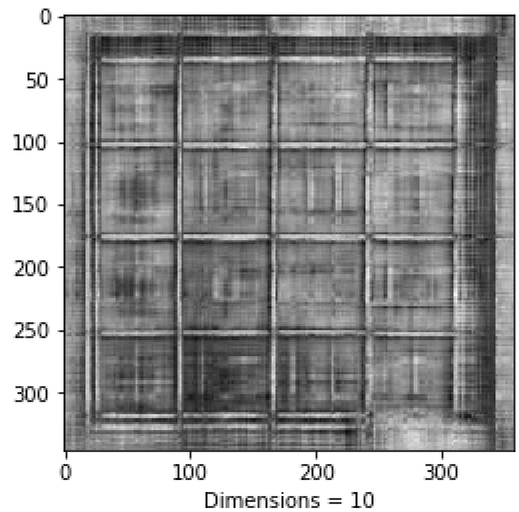


In [9]:

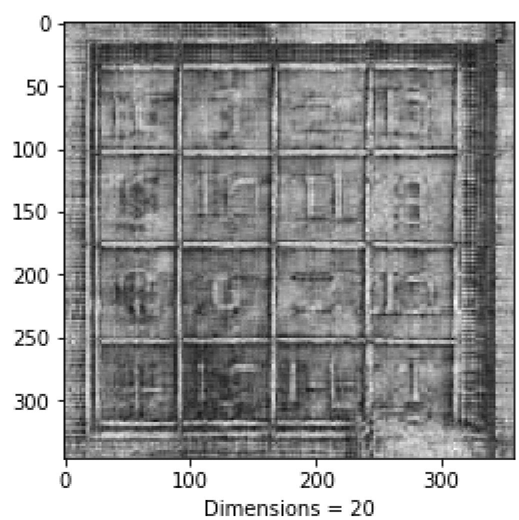
```
for i in np.array([1, 10, 20, 100]):
    numbers_pca = PCA(n_components = i)
    numbers_PC = numbers_pca.fit_transform(numbers)
    numbers_reconstruct = numbers_pca.inverse_transform(numbers_PC)
    plt.imshow(numbers_reconstruct, cmap = 'gray')
    plt.xlabel('Dimensions = ' + str(i))
    plt.show()
    print("Component-wise Percentage Total Variance:", numbers_pca.explained_variance_ra
tio_[:10])
    print("Reconstruction Error:", ((numbers - numbers_reconstruct) ** 2).sum())
    print("Compression Rate:", (346 * 358) / (346 * i + i + i * 358))
```



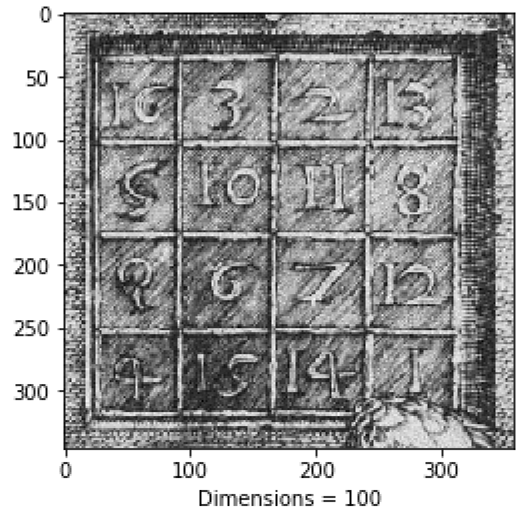
```
Component-wise Percentage Total Variance: [0.22591042]
Reconstruction Error: 8690.909202650431
Compression Rate: 175.69929078014184
```



```
Component-wise Percentage Total Variance: [0.22591042 0.0599177  0.04909906 0.03056869
0.02494228 0.02450543
 0.01909028 0.01847125 0.01619839 0.01573313]
Reconstruction Error: 5788.366782829698
Compression Rate: 17.569929078014184
```



```
Component-wise Percentage Total Variance: [0.22591042 0.0599177  0.04909906 0.03056869
0.02494228 0.02450543
 0.01909028 0.01847125 0.01619839 0.01573313]
Reconstruction Error: 4501.927216110859
Compression Rate: 8.784964539007092
```




```
Component-wise Percentage Total Variance: [0.22591042 0.0599177  0.04909906 0.03056869
0.02494228 0.02450543
 0.01909028 0.01847125 0.01619839 0.01573313]
Reconstruction Error: 962.002224382447
Compression Rate: 1.7569929078014184
```