1. A. $\vec{a}+\vec{b} = \langle 5+0, 1+2, 14-4, 3+10, -5+4, 12-3, -20-4 \rangle$

$\vec{a}+\vec{b} = \boxed{\langle 5, 3, 10, 13, -1, 9, -24 \rangle}$

B. $3\vec{a}-4\vec{b} = \langle 3(5)-4(0), 3(1)-4(2), 3(14)-4(-4), 3(3)-4(10), 3(-5)-4(4), 3(12)-4(-3), 3(-20)-4(-4) \rangle$

$3\vec{a}-4\vec{b} = \boxed{\langle 15, -5, 58, -31, -31, 48, -44 \rangle}$

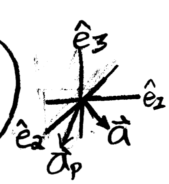
C. $\|\vec{a}\| = \sqrt{(5)^2+(1)^2+(14)^2+(3)^2+(-5)^2+(12)^2+(-20)^2} = \boxed{20\sqrt{2}}$

D. $\vec{a}\cdot\vec{b} = (5\cdot 0)+(1\cdot 2)+(14\cdot -4)+(3\cdot 10)+(-5\cdot 4)+(12\cdot -3)+(-20\cdot -4) = \boxed{0}$

E. Vectors $\vec{a}$ & $\vec{b}$ are orthogonal

2. A. $\vec{a} = \langle 10, 6, -5 \rangle = \boxed{10\hat{e}_1 + 6\hat{e}_2 - 5\hat{e}_3}$

B. $\cos\theta = \dfrac{\vec{a}\cdot\hat{e}_2}{\|\vec{a}\|\,\|\hat{e}_2\|} = \dfrac{(10\cdot 0)+(6\cdot 1)+(-5\cdot 0)}{\left(\sqrt{(10)^2+(6)^2+(-5)^2}\right)(1)} = \boxed{\dfrac{6}{\sqrt{161}}}$

C. 

$\vec{a}_p = \boxed{\langle 0, 6, -5 \rangle}$

3. A. $\underset{1\times 8}{a'}\ \underset{8\times 12}{A} = \underset{1\times 12}{a'A}\ \underset{12\times 3}{B} = \boxed{\underset{1\times 3}{a'AB}}$

C. $\underset{12\times 1}{a} \times \underset{3\times 9}{B} =$ Not Defined

B. $\underset{12\times 37}{A}\ \underset{37\times 9}{B} = \boxed{\underset{12\times 9}{AB}}$

D. $\underset{1\times 12}{a'}\ \underset{12\times 37}{A} = \underset{1\times 37}{a'A}\ \underset{37\times 9}{B} = \underset{1\times 9}{a'AB}\ \underset{9\times 1}{b} = \underset{1\times 1}{a'ABb}\ \underset{1\times 4}{c'} = \underset{1\times 4}{a'ABbc'}\ \underset{4\times 12}{C} = \boxed{\underset{1\times 12}{a'ABbc'C}}$

4\. $\begin{vmatrix} 2 & 6 & 8 \\ 4 & 2 & 6 \\ 3 & -2 & 1 \end{vmatrix} = 2\begin{vmatrix} 2 & 6 \\ -2 & 1 \end{vmatrix} - 6\begin{vmatrix} 4 & 6 \\ 3 & 1 \end{vmatrix} + 8\begin{vmatrix} 4 & 2 \\ 3 & -2 \end{vmatrix} = 2(2+12) - 6(4-18) + 8(-8-6) = 0$

The determinant of the matrix is 0, therefore it's <u>non-invertable</u>

5\. $A - \lambda I = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-\lambda & 3 \\ 2 & 4-\lambda \end{bmatrix}$

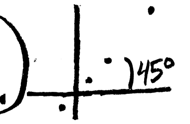
$|A - \lambda I| = (1-\lambda)(4-\lambda) - 6 = \boxed{\lambda^2 - 5\lambda - 2 = 0 \quad \text{Characteristic Equation}}$

$\boxed{\lambda_1 = -0.372 \quad , \quad \lambda_2 = 5.372 \quad \text{Eigenvalues}}$

$A - \lambda I \vec{a} = \vec{0}$:

$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} - (-0.372)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \vec{a}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \vec{a}_1 = \begin{bmatrix} -0.909 \\ 0.416 \end{bmatrix}$

$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} - (5.372)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \vec{a}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \vec{a}_2 = \begin{bmatrix} -0.566 \\ -0.825 \end{bmatrix}$ Eigenvectors

6\. 

Let $\vec{e}_1 = \frac{\sqrt{2}}{2}\vec{e}_1^* - \frac{\sqrt{2}}{2}\vec{e}_2^*$

Let $\vec{e}_2 = \frac{\sqrt{2}}{2}\vec{e}_1^* + \frac{\sqrt{2}}{2}\vec{e}_2^*$

#1: $a_1^* = \frac{\sqrt{2}}{2}(-1) + \frac{\sqrt{2}}{2}(-1) = -\sqrt{2}$
$a_2^* = -\frac{\sqrt{2}}{2}(-1) + \frac{\sqrt{2}}{2}(-1) = 0$, $\quad -1\hat{i} - 1\hat{j} \mapsto -\sqrt{2}\vec{e}_1^*$

#2: $a_1^* = \frac{\sqrt{2}}{2}(1) + \frac{\sqrt{2}}{2}(1) = \sqrt{2}$
$a_2^* = -\frac{\sqrt{2}}{2}(1) + \frac{\sqrt{2}}{2}(1) = 0$, $\quad 1\hat{i} + 1\hat{j} \mapsto \sqrt{2}\vec{e}_1^*$

#3: $a_1 = \frac{\sqrt{2}}{2}(2) + \frac{\sqrt{2}}{2}(2) = 2\sqrt{2}$
$a_2^* = -\frac{\sqrt{2}}{2}(2) + \frac{\sqrt{2}}{2}(2) = 0$, $\quad 2\hat{i} + 2\hat{j} \mapsto 2\sqrt{2}\vec{e}_1^*$

#4: $a_1^* = \frac{\sqrt{2}}{2}(5) + \frac{\sqrt{2}}{2}(5) = 5\sqrt{2}$
$a_2^* = -\frac{\sqrt{2}}{2}(5) + \frac{\sqrt{2}}{2}(5) = 0$, $\quad 5\hat{i} + 5\hat{j} \mapsto 5\sqrt{2}\vec{e}_1^*$

7\. A. $\begin{bmatrix} 132 & 255 & 130 & 130 \\ 130 & 255 & 132 & 130 \\ 156 & 255 & 156 & 1 \\ 210 & 0 & 1 & 9 \end{bmatrix}$

C. $\begin{bmatrix} -0.0664 & 0.107 & -0.002 & 0.031 \\ -0.114 & 0.240 & -0.142 & 0.069 \\ 0.188 & -0.137 & 0.089 & -0.066 \\ -0.007 & -0.210 & 0.055 & 0.018 \end{bmatrix}$

D. -10298

B. $\begin{bmatrix} 4771 & 4789 & 4639 & 678 \\ 662 & 654 & 1245 & 1235 \\ 1489 & 1461 & 1846 & 3582 \\ 12293 & 12293 & 10792 & 4180 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

F. 552

G. 552