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MAE 150 HW 5 Problem 2

```
clear
close all

% Parameters
L = 1; % m
r = 0.02; % m
I = pi*r^4/4; % 2nd moment of area of circular cross section
E = 200*10^9; % Pa
P = 100; % N
```

Part (a): Vertical deflection at free end

```
delta_L = P*L^2*(3*L-L)/(6*E*I);

fprintf('Part(a)\n\n')
fprintf('delta(x=L) =\n\n')
disp(delta_L)
fprintf('(Units: m)\n')
```

Part (b): FEM of beam

```
m = 5; % number of elements
n = m + 1; % number of nodes

l = L/m; % length of each element

% Initialize arrays in loop
Ke = cell(m,1);
KG = zeros(2*n);

% Loop through elements
for el = 1:m
    i = el;
    j = el + 1;

    Ke{el} = E*I/l^3*[ 12   6*l  -12   6*l;
                      6*l 4*l^2 -6*l 2*l^2;
                      -12  -6*l   12  -6*l;
```

```
                      6*l 2*l^2 -6*l 4*l^2];

    Kt = zeros(2*n);
    Kt([2*i-1,2*i,2*j-1,2*j],[2*i-1,2*i,2*j-1,2*j]) = Ke{el};
    KG = KG + Kt;
end

% Define load vector
F = zeros(n,1); % N
F(1) = nan;
F(2) = nan;
F(3) = 0;
F(4) = 0;
F(5) = 0;
F(6) = 0;
F(7) = 0;
F(8) = 0;
F(9) = 0;
F(10) = 0;
F(11) = -100;
F(12) = 0;

% Indexing array for reduction (displacement = 0 at reaction forces)
redux = ~isnan(F);

% Reduce KG and F
Kr = KG(redux,redux);
Fr = F(redux);

% Solve for unknown displacements
dr = Kr\Fr;

% Define complete displacement vector
d = zeros(2*n,1);
d(redux) = dr;
d(2:2:end) = d(2:2:end)*180/pi; % convert angles to degrees

% Reduce KG (reduce opposite rows)
Kr = KG(~redux,redux);

% Solve for unknown loads
Fr = Kr*dr;

% Define complete load vector
F(~redux) = Fr;
```

Print results for Part (b)

```
fprintf('\n\n\n')
fprintf('Part(b)\n\n')
fprintf('K_e1 =\n\n')
disp(Ke{1})
fprintf('K_e2 =\n\n')
```

```
disp(Ke{2})
fprintf('K_e3 =\n\n')
disp(Ke{3})
fprintf('K_e4 =\n\n')
disp(Ke{4})
fprintf('K_e5 =\n\n')
disp(Ke{5})
fprintf('(Units: N/m)\n')

fprintf('\n\n\n')
fprintf('K_G =\n\n')
disp(KG(:,1:10))
disp(KG(:,11:12))
fprintf('(Units: N/m)\n')

fprintf('\n\n\n')
fprintf('d =\n\n')
disp(d)
fprintf('(Units: m & deg)\n')

fprintf('\n\n\n')
fprintf('F =\n\n')
disp(F)
fprintf('(Units: N & N*m)\n')
```

Part (c): Plotting

```
x = 0:0.01:L;
y = -P*x.^2.*(3*L-x)/(6*E*I);

d(redux) = dr; % back to angles in radians
y_FEM = zeros(1,length(x));
for el = 1:m

    w1 = d(2*el - 1);
    th1 = d(2*el);
    w2 = d(2*el + 1);
    th2 = d(2*el + 2);

    c1 = (2*w1 + l*th1 - 2*w2 + l*th2)/l^3;
    c2 = (-3*w1 - 2*l*th1 + 3*w2 - l*th2)/l^2;
    c3 = th1;
    c4 = w1;

    idx = x > (el-1)*l & x <= el*l;
    y_FEM(idx) = c1*(x(idx)-(el-1)*l).^3 + c2*(x(idx)-(el-1)*l).^2 +
 c3*(x(idx)-(el-1)*l) + c4;
end

fprintf('\n\n\n')
fprintf('Part(c)\n\n')

linewidth = 2;
```

```
fontsize = 14;
color1 = 'b';
color2 = 'r';

close all
figure
clf
plot(x,y,'-','Color',color1,'Linewidth',linewidth);
hold on
plot(x,y_FEM,'--','Color',color2,'Linewidth',linewidth);
plot(0:l:L,d(1:2:end),'o','Color',color2,'Linewidth',linewidth);
hold off
grid on
title('Beam Under External Load')
xlabel('X axis (m)')
ylabel('Y axis (m)')
set(gca,'FontSize',fontsize)
lgd = legend('Analytical','FEM');
title(lgd,'Solution Methods')
```

Part (f)

```
A = pi*r^2; % m^2
rho = 7850; % kg/m^3

f1 = 1.875^2/(2*pi*L^2)*sqrt(E*I/(rho*A));
```

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