MAE 144 Homework 1

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Problem 1

Link to GitHub Page:

https://github.com/davidlim4318

Problem 2

2a

```
hw1_prob2a
```

```
eps = 1.0000e-10
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eps = 1.0000e-10
x =
  RR_poly with properties:
poly:    -0.6710   -2.2869   10.1755   24.1641
roots:   -5.0009   -2.0030    3.5955
   n: 3
y =
  RR_poly with properties:
poly:     0.6710    2.2869  -21.5818  -62.0409   42.6886   81.5318
roots:   -6.0000   -3.0000   -1.0000    1.2681    5.3235
   n: 5
eps = 1.0000e-10
residual = 1.1369e-13
```

2b

The controller D(s) is improper. The order of the denominator x(s) is less than the order of the numerator y(s).

```
hw1_prob2b
```

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eps = 1.0000e-10
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eps = 1.0000e-10
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eps = 1.0000e-10
eps = 1.0000e-10
k = 5
x =
  RR_poly with properties:
poly:    1.0e+08 *
    0.0000    0.0000   -0.0491   -0.1903    0.5827    1.5346
roots:   1.0e+03 *
   -2.2745   -0.0050   -0.0020    0.0031    2.1584
   n: 5
y =
  RR_poly with properties:
poly:    1.0e+08 *
    0.0492    0.1921   -1.3847   -4.3786    2.2250    5.0759
roots:   -6.0000   -3.0000   -1.0000    1.1646    4.9263
   n: 5
f =
  RR_poly with properties:
poly:    1.0e+09 *
    0.0000    0.0000    0.0000    0.0002    0.0031    0.0339    0.2377    1.0446    2.7935    4.2883    3.
roots: -20.0302 - 0.0219i -20.0302 + 0.0219i -19.9885 - 0.0354i -19.9885 + 0.0354i -19.9628 + 0.0000i  -6.
   n: 11
eps = 1.0000e-10
residual = 1.0832e-05
```

If my code is correct, k needs to be = 5 in order for the controller D(s) to be proper (i.e. semi-causal, relying only on present and past inputs). This might be sufficient for a controller operating in continuous time (i.e. an analog controller). Increasing k further should eventually make D(s) strictly proper (i.e. strictly causal, only relying on past inputs).

Problem 3

Symbolic:

```
syms s z1 p1
as = [1 p1 0];
bs = [1 z1];
Ds = RR_tf(bs,as);
h = 1;
Gz = DHL_C2D_matched(Ds,h)
```

```
Gz =
  RR_tf with properties:
num:[(abs(z1)*abs(exp(-p1) - 1))/(abs(p1)*abs(exp(-z1) - 1)), -(exp(-z1)*abs(z1)*abs(exp(-p1) - 1))/(abs(p
den:[1, - exp(-p1) - 1, exp(-p1)]
Discrete-time transfer function with h=     1
  m=1, n=2, n_r=n-m=1, strictly proper, K=(abs(z1)*abs(exp(-p1) - 1))/(abs(p1)*abs(exp(-z1) - 1))
  z:exp(-z1)
  p:[1, exp(-p1)]
```

By hand:

```
I = imread('hw1_prob3_byhand.jpeg');
imshow(I);
```

Problem 3: solving by hand, symbolically

$$D(s) = \frac{s+z_1}{s(s+p_1)} \quad , h = 1 \quad , \bar{\omega} = 0$$

Step 1: $z = e^{sh}$

poles: $s = 0 \Rightarrow z = e^{0 \cdot 1} = 1$

$s = -p_1 \Rightarrow z = e^{-p_1}$

zeros: $s = -z_1 \Rightarrow z = e^{-z_1}$

Step 2:

Add zero at $s = \infty \Rightarrow z = -1$

include

$$G(z) = K \cdot \frac{(z - e^{-z_1})\ \cancel{(z+1)}}{(z-1)(z-e^{-p_1})}$$

immediately remove it ($\rightarrow z = \infty$)

make it strictly causal

Step 3:

$$\text{DCgain}\left(\frac{s+z_1}{s(s+p_1)}\right)\Bigg|_{s=i\bar{\omega}=0} = \text{DCgain}\left(K \cdot \frac{(z-e^{-z_1})}{(z-1)(z-e^{-p_1})}\right)\Bigg|_{z=e^{i\bar{\omega}h}=1}$$

$$\lim_{s\to 0} \frac{s+z_1}{\cancel{s}(s+p_1)} = \lim_{z\to 1} K \cdot \frac{(z-e^{-z_1})}{(\cancel{z-1})(z-e^{-p_1})}$$

in the limit, 0's should cancel, right?

$$\frac{z_1}{p_1} = K \frac{1-e^{-z_1}}{1-e^{-p_1}}$$

$$K = \frac{z_1 \cdot (1-e^{-p_1})}{p_1 \cdot (1-e^{-z_1})}$$

$$G(z) = \frac{z_1 \cdot (1-e^{-p_1})}{p_1 \cdot (1-e^{-z_1})} \frac{(z-e^{-z_1})}{(z-1)(z-e^{-p_1})}$$

Numerical:

```
z1 = 1;
p1 = 10;
as = [1 p1 0];
bs = [1 z1];
Ds = RR_tf(bs,as);
```

```
eps = 1.0000e-10
eps = 1.0000e-10
```

```
h = 1;
Gz = DHL_C2D_matched(Ds,h)
```

```
eps = 1.0000e-10
eps = 1.0000e-10
eps = 1.0000e-10
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eps = 1.0000e-10
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eps = 1.0000e-10
eps = 1.0000e-10
eps = 1.0000e-10
eps = 1.0000e-10
eps = 1.0000e-10
Gz =
  RR_tf with properties:
num:    0.1582   -0.0582
den:    1.0000   -1.0000    0.0000
Discrete-time transfer function with h=     1
  m=1, n=2, n_r=n-m=1, strictly proper, K=    0.1582
  z:    0.3679
  p:    0.0000    1.0000
```

Built-in function:

```
Ds = tf(bs,as);
Gz = zpk(c2d(Ds,h,'matched'))
```

```
Gz =

  0.15819 (z-0.3679)
  ------------------
  (z-1) (z-4.54e-05)

Sample time: 1 seconds
Discrete-time zero/pole/gain model.
Model Properties
```

The result D(z) generated using my function has nearly the same poles, zeros, and gain as the result generated using MATLAB's built-in function. My code is better because it works with symbolic variables and can give a symbolic result, which is pretty cool.

(My code can also handle omega_bar = 0 by effectively taking the limit and canceling out the probelmatic poles in the calculation of the gain, arriving to the result as MATLAB.)