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MAE 150

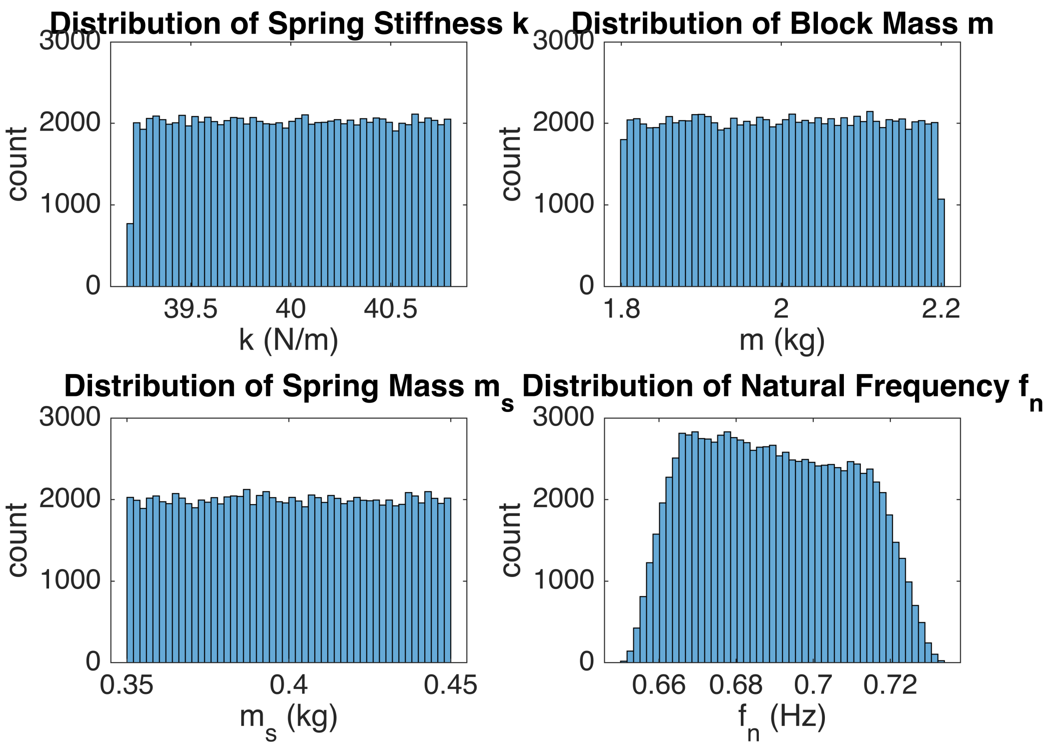
07/14/23

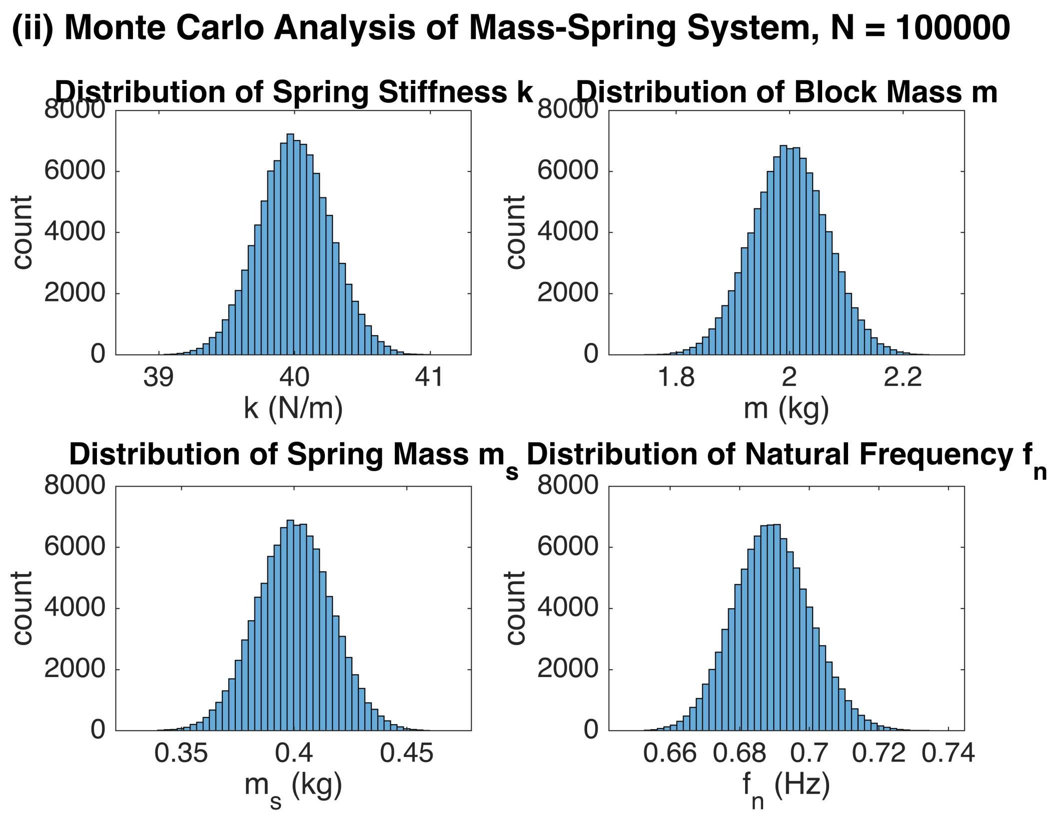
Homework 3

Problem 1

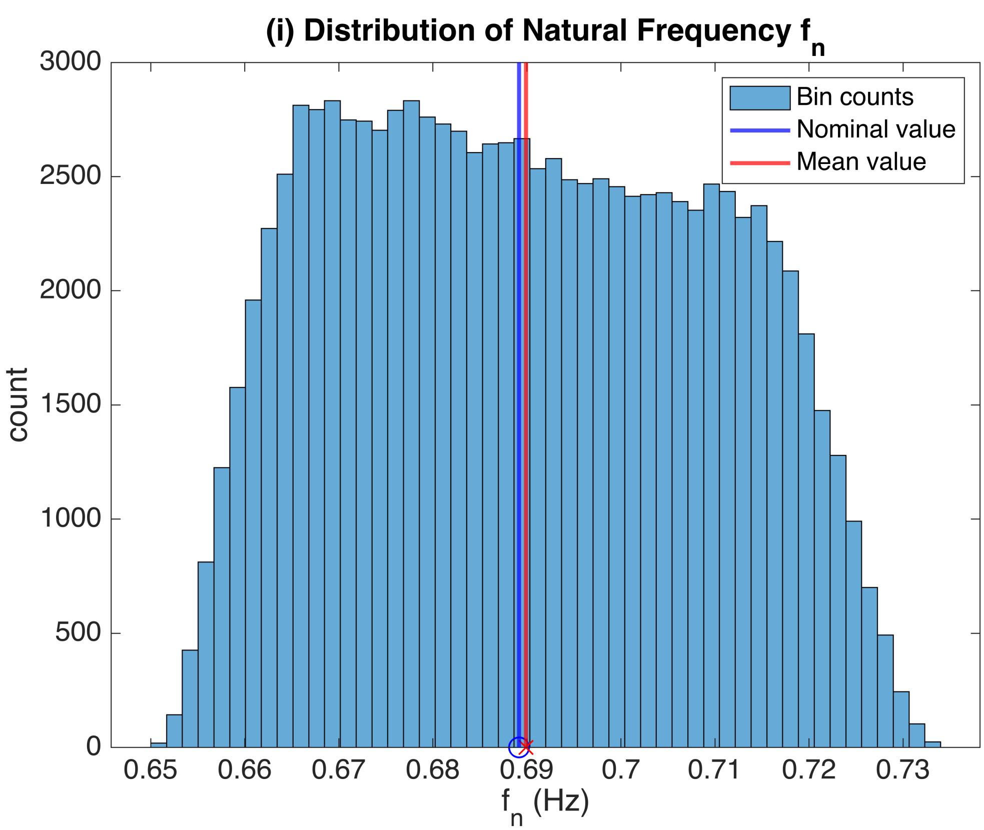
Part (a)

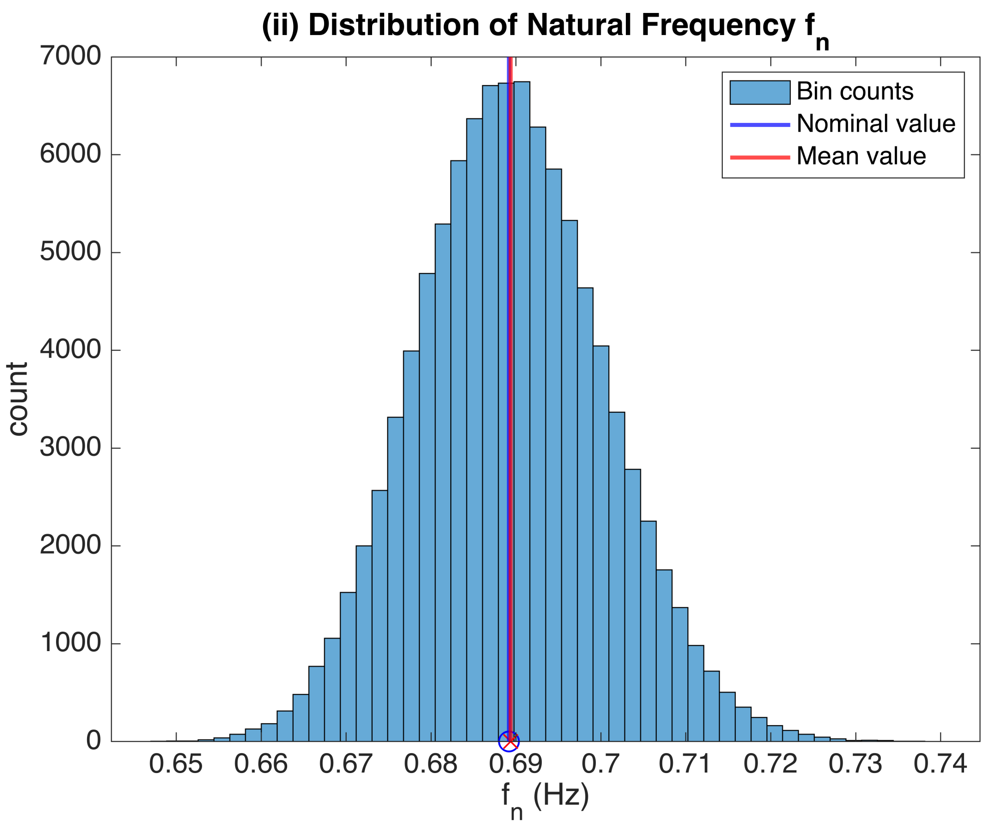
(i) Monte Carlo Analysis of Mass-Spring System, N = 100000




(ii) Monte Carlo Analysis of Mass-Spring System, N = 100000
Distribution of Spring Stiffness k
Distribution of Block Mass m
Distribution of Spring Mass m_s
Distribution of Natural Frequency f_n
count
k (N/m)
m (kg)
m_s (kg)
f_n (Hz)

Part (b)


(i) Distribution of Natural Frequency f_n
Bin counts
Nominal value
Mean value
count
f_n (Hz)



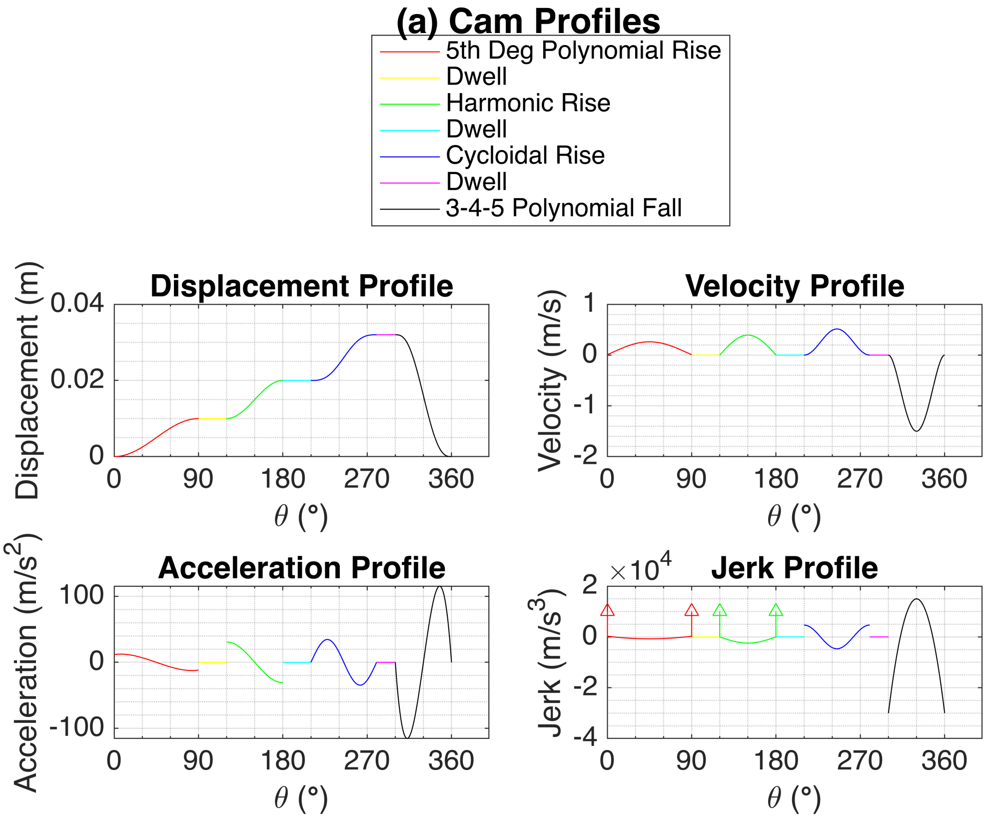
Part (c)

In part (i), each independent parameter appears to be uniformly distributed as intended. The natural frequency appears to be neither uniformly nor normally distributed, but it is closer to being normally distributed. The mean of the natural frequency is close to the nominal value, but there is skew due to the asymmetry of the distribution.

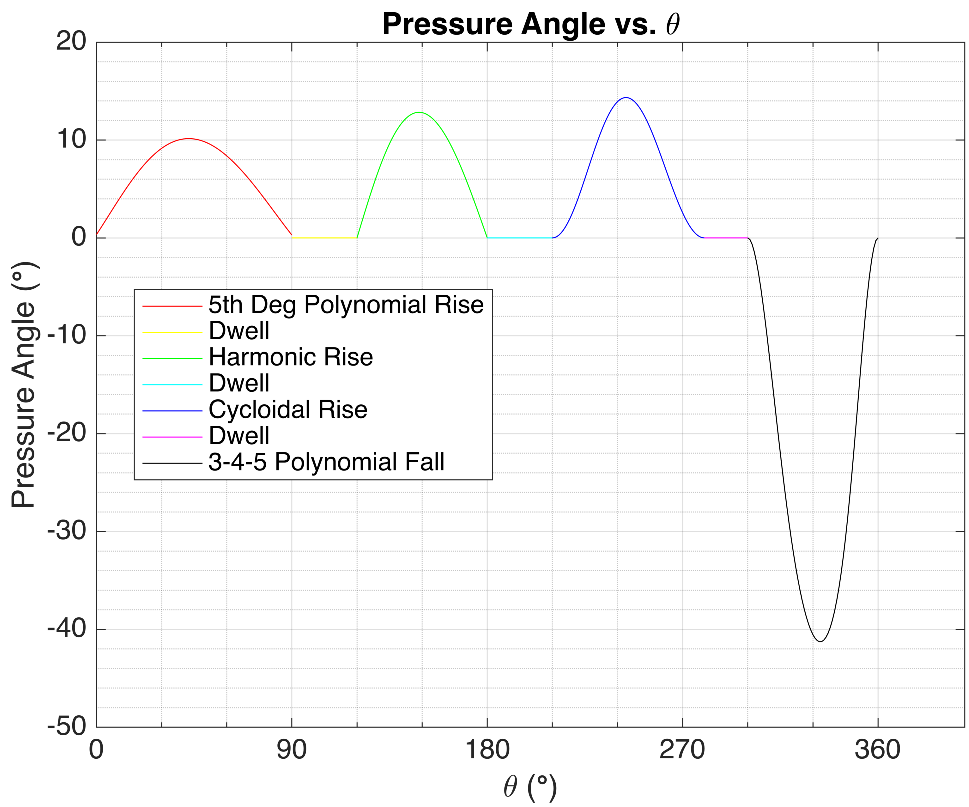
In part (ii), each independent parameter appears to be normally distributed as intended. The natural frequnecy appears to be normally distributed. The mean of the natural frequency is very close to the nominal value. The distribution does not appear to be skewed.

Problem 2

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.263429e-20.
Part (a)
```




```
Part (b)
```




```
ans = 13×2 table
```

	theta (deg)	phi (deg)
1	0	0
2	30	9.1502
3	60	8.3818
4	90	0.2889
5	120	0
6	150	12.8043
7	180	0
8	210	0
9	240	13.9238
10	270	2.5580
11	300	0
12	330	-40.5358
13	360	0

The follower experiences high jerk in the transitions of the first and second rises, but this performance might be acceptable. A possibly more serious issue is the maximum pressure angle during the fall. The pressure angle is greater than 30 deg, which is concerning since high pressure angles can cause jamming. Therefore, this cam is designed poorly. The size of the cam should be increased to reduce the chance of jamming.

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MAE 150 HW 3 Problem 1

```
clear
clc
close all
fontsize = 14;
```

Parameters

```
k_n = 40;
dk = 0.8;
m_n = 2;
dm = 0.2;
m_s_n = 0.4;
dm_s = 0.05;

f_n = 1/(2*pi)*sqrt(k_n./(m_n+m_s_n/3));

N = 100000;
nbins = 50;

fprintf('Part (a)\n')
```

(i) Uniformly Distributed Random Numbers

```
k = 2*dk*rand(N,1) + k_n - dk;
m = 2*dm*rand(N,1) + m_n - dm;
m_s = 2*dm_s*rand(N,1) + m_s_n - dm_s;

f_n_i = 1/(2*pi)*sqrt(k./(m+m_s/3));

% Figure for (i)
figure

ax(1) = subplot(2,2,1);
histogram(k,nbins)
title('Distribution of Spring Stiffness k')
xlabel('k (N/m)')
ylabel('count')
set(gca,'fontsize',fontsize)
```

```
ax(2) = subplot(2,2,2);
histogram(m,nbins)
title('Distribution of Block Mass m')
xlabel('m (kg)')
ylabel('count')
set(gca,'fontsize',fontsize)

ax(3) = subplot(2,2,3);
histogram(m_s,nbins)
title('Distribution of Spring Mass m_s')
xlabel('m_s (kg)')
ylabel('count')
set(gca,'fontsize',fontsize)

ax(4) = subplot(2,2,4);
histogram(f_n_i,nbins)
title('Distribution of Natural Frequency f_n')
xlabel('f_n (Hz)')
ylabel('count')
set(gca,'fontsize',fontsize)

linkaxes(ax,'y')
sgtitle(sprintf('(i) Monte Carlo Analysis of Mass-Spring System, N =
 %d',N),'fontsize',1.25*fontsize,'fontweight','bold')
```

(ii) Normally Distributed Random NumbersN = 1000000;

```
k = k_n + dk/3*randn(N,1);
m = m_n + dm/3*randn(N,1);
m_s = m_s_n + dm_s/3*randn(N,1);

f_n_ii = 1/(2*pi)*sqrt(k./(m+m_s/3));

% Figure for (ii)
figure

ax(1) = subplot(2,2,1);
histogram(k,nbins)
title('Distribution of Spring Stiffness k')
xlabel('k (N/m)')
ylabel('count')
set(gca,'fontsize',fontsize)

ax(2) = subplot(2,2,2);
histogram(m,nbins)
title('Distribution of Block Mass m')
xlabel('m (kg)')
ylabel('count')
set(gca,'fontsize',fontsize)

ax(3) = subplot(2,2,3);
```

```
histogram(m_s,nbins)
title('Distribution of Spring Mass m_s')
xlabel('m_s (kg)')
ylabel('count')
set(gca,'fontsize',fontsize)

ax(4) = subplot(2,2,4);
histogram(f_n_ii,nbins)
title('Distribution of Natural Frequency f_n')
xlabel('f_n (Hz)')
ylabel('count')
set(gca,'fontsize',fontsize)

linkaxes(ax,'y')
sgtitle(sprintf('(ii) Monte Carlo Analysis of Mass-Spring System, N =
 %d',N),'fontsize',1.25*fontsize,'fontweight','bold')
```

(b) Figure 3

```
fprintf('Part (b)\n')
markersize = 10;
linewidth = 2;

f_n_i_mean = mean(f_n_i);

figure
histogram(f_n_i,nbins)
hold on
xline(f_n,'b','LineWidth',linewidth)
xline(f_n_i_mean,'r','LineWidth',linewidth)
plot(f_n,0,'bo','MarkerSize',markersize,'LineWidth',linewidth)
plot(f_n_i_mean,0,'rx','MarkerSize',markersize,'LineWidth',linewidth)
hold off
title('(i) Distribution of Natural Frequency f_n')
xlabel('f_n (Hz)')
ylabel('count')
legend('Bin counts','Nominal value','Mean value','Location','northeast')
set(gca,'fontsize',fontsize)

f_n_ii_mean = mean(f_n_ii);

figure
histogram(f_n_ii,nbins)
hold on
xline(f_n,'b','LineWidth',linewidth)
xline(f_n_ii_mean,'r','LineWidth',linewidth)
plot(f_n,0,'bo','MarkerSize',markersize,'LineWidth',linewidth)
plot(f_n_ii_mean,0,'rx','MarkerSize',markersize,'LineWidth',linewidth)
hold off
title('(ii) Distribution of Natural Frequency f_n')
xlabel('f_n (Hz)')
ylabel('count')
legend('Bin counts','Nominal value','Mean value','Location','northeast')
```

```
set(gca,'fontsize',fontsize)
```

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MAE 150 HW 3 Problem 2

```
clear
close all

% 5th Degree Polynomial Fit
y = [0 0.1 0.3 0.7 1.2 1.8 2.5 3.3 4.1 5 5.9 6.7 7.5 8.2 8.8 9.3 9.7 9.9 10]';
theta = (0:5:90)';

M = length(y);
x = theta;
A = [        M    sum(x) sum(x.^2) sum(x.^3) sum(x.^4) sum(x.^5);
        sum(x) sum(x.^2) sum(x.^3) sum(x.^4) sum(x.^5) sum(x.^6);
     sum(x.^2) sum(x.^3) sum(x.^4) sum(x.^5) sum(x.^6) sum(x.^7);
     sum(x.^3) sum(x.^4) sum(x.^5) sum(x.^6) sum(x.^7) sum(x.^8);
     sum(x.^4) sum(x.^5) sum(x.^6) sum(x.^7) sum(x.^8) sum(x.^9);
     sum(x.^5) sum(x.^6) sum(x.^7) sum(x.^8) sum(x.^9) sum(x.^10)];
b = [      sum(y);
        sum(y.*x);
     sum(y.*x.^2);
     sum(y.*x.^3);
     sum(y.*x.^4);
     sum(y.*x.^5)];

a = A\b;
```

Define functions

```
dtheta = 1;
omega = 250*360/60;

% 5th Degree Polynomial Rise
beta = 90;
theta = (0:dtheta:beta)';
y_1 = (a(1) + a(2)*theta + a(3)*theta.^2 + a(4)*theta.^3 + a(5)*theta.^4 +
 a(6)*theta.^5)/10^3;
v_1 = (a(2)*omega + 2*a(3)*omega*theta + 3*a(4)*omega*theta.^2 +
 4*a(5)*omega*theta.^3 + 5*a(6)*omega*theta.^4)/10^3;
a_1 = (2*a(3)*omega^2 + 6*a(4)*omega^2*theta + 12*a(5)*omega^2*theta.^2 +
 20*a(6)*omega^2*theta.^3)/10^3;
j_1 = (6*a(4)*omega^3 + 24*a(5)*omega^3*theta +
 60*a(6)*omega^3*theta.^2)/10^3;
```

```
% Dwell
beta = 120 - 90;
theta = (0:dtheta:beta)';
L = 10/10^3;
y_2 = L*ones(length(theta),1);
v_2 = zeros(length(theta),1);
a_2 = zeros(length(theta),1);
j_2 = zeros(length(theta),1);

% Harmonic Rise
beta = 180 - 120;
theta = (0:dtheta:beta)';
L = (20 - 10)/10^3;
y_3 = L/2*(1-cos(pi*theta/beta));
v_3 = L/2*pi*omega/beta*sin(pi*theta/beta);
a_3 = L/2*(pi*omega/beta)^2*cos(pi*theta/beta);
j_3 = -L/2*(pi*omega/beta)^3*sin(pi*theta/beta);

% Dwell
beta = 210 - 180;
theta = (0:dtheta:beta)';
L = 20/10^3;
y_4 = L*ones(length(theta),1);
v_4 = zeros(length(theta),1);
a_4 = zeros(length(theta),1);
j_4 = zeros(length(theta),1);

% Cycloidal Rise
beta = 280 - 210;
theta = (0:dtheta:beta)';
L = (32 - 20)/10^3;
y_5 = L*(theta/beta - 1/(2*pi)*sin(2*pi*theta/beta));
v_5 = L*omega/beta*(1 - cos(2*pi*theta/beta));
a_5 = 2*L*pi*(omega/beta)^2*sin(2*pi*theta/beta);
j_5 = 4*L*pi^2*(omega/beta)^3*cos(2*pi*theta/beta);

% Dwell
beta = 300 - 280;
theta = (0:dtheta:beta)';
L = 32/10^3;
y_6 = L*ones(length(theta),1);
v_6 = zeros(length(theta),1);
a_6 = zeros(length(theta),1);
j_6 = zeros(length(theta),1);

% 3-4-5 Polynomial Fall
beta = 360 - 300;
theta = (0:dtheta:beta)';
L = 32/10^3;
y_7 = L - L*(10*theta.^3/beta^3 - 15*theta.^4/beta^4 + 6*theta.^5/beta^5);
v_7 = -L*(30*omega*theta.^2/beta^3 - 60*omega*theta.^3/beta^4 +
 30*omega*theta.^4/beta^5);
a_7 = -L*(60*omega^2*theta/beta^3 - 180*omega^2*theta.^2/beta^4 +
 120*omega^2*theta.^3/beta^5);
```

```
j_7 = -L*(60*omega^3/beta^3 - 360*omega^3*theta/beta^4 + 360*omega^3*theta.^2/
beta^5);
```

(a) Plotting

```
theta = (0:dtheta:360)';

color = ('rygcbmk');
linewidth = 3;
fontsize = 14;

fprintf('Part (a)\n')
figure

subplot(2,2,1)
plot(theta(1:91),y_1,color(1))
hold on
plot(theta(91:121),y_2,color(2))
plot(theta(121:181),y_3+10/10^3,color(3))
plot(theta(181:211),y_4,color(4))
plot(theta(211:281),y_5+20/10^3,color(5))
plot(theta(281:301),y_6,color(6))
plot(theta(301:361),y_7,color(7))
hold off
title('Displacement Profile')
xlabel('\theta (\circ)')
ylabel('Displacement (m)')
xticks(0:90:360)
h = gca;
h.XAxis.MinorTick = 'on';
h.XAxis.MinorTickValues = 0:30:360;
grid on
grid minor
set(h,'DefaultLineLineWidth',linewidth,'FontSize',fontsize)

subplot(2,2,2)
plot(theta(1:91),v_1,color(1))
hold on
plot(theta(91:121),v_2,color(2))
plot(theta(121:181),v_3,color(3))
plot(theta(181:211),v_4,color(4))
plot(theta(211:281),v_5,color(5))
plot(theta(281:301),v_6,color(6))
plot(theta(301:361),v_7,color(7))
hold off
title('Velocity Profile')
xlabel('\theta (\circ)')
ylabel('Velocity (m/s)')
xticks(0:90:360)
h = gca;
h.XAxis.MinorTick = 'on';
h.XAxis.MinorTickValues = 0:30:360;
grid on
```

```
grid minor
set(h,'DefaultLineLineWidth',linewidth,'FontSize',fontsize)

subplot(2,2,3)
plot(theta(1:91),a_1,color(1))
hold on
plot(theta(91:121),a_2,color(2))
plot(theta(121:181),a_3,color(3))
plot(theta(181:211),a_4,color(4))
plot(theta(211:281),a_5,color(5))
plot(theta(281:301),a_6,color(6))
plot(theta(301:361),a_7,color(7))
hold off
title('Acceleration Profile')
xlabel('\theta (\circ)')
ylabel('Acceleration (m/s^2)')
xticks(0:90:360)
h = gca;
h.XAxis.MinorTick = 'on';
h.XAxis.MinorTickValues = 0:30:360;
grid on
grid minor
set(h,'DefaultLineLineWidth',linewidth,'FontSize',fontsize)

subplot(2,2,4)
plot(theta(1:91),j_1,color(1))
hold on
plot(theta(91:121),j_2,color(2))
plot(theta(121:181),j_3,color(3))
plot(theta(181:211),j_4,color(4))
plot(theta(211:281),j_5,color(5))
plot(theta(281:301),j_6,color(6))
plot(theta(301:361),j_7,color(7))
stem(0,10^4,color(1),'marker','^','ShowBaseLine','off')
stem(90,10^4,color(1),'marker','^','ShowBaseLine','off')
stem(120,10^4,color(3),'marker','^','ShowBaseLine','off')
stem(180,10^4,color(3),'marker','^','ShowBaseLine','off')
hold off
title('Jerk Profile')
xlabel('\theta (\circ)')
ylabel('Jerk (m/s^3)')
xticks(0:90:360)
h = gca;
h.XAxis.MinorTick = 'on';
h.XAxis.MinorTickValues = 0:30:360;
grid on
grid minor
set(h,'DefaultLineLineWidth',linewidth,'FontSize',fontsize)

legend('5th Deg Polynomial Rise','Dwell','Harmonic Rise','Dwell','Cycloidal
 Rise','Dwell','3-4-5 Polynomial Fall')
legend("Position", [0.36,0.7,0.32,0.28])
```

```
sgtitle(sprintf('(a) Cam Profiles\n\n\n\n\n
\n'),'fontsize',1.25*fontsize,'fontweight','bold')
```

(b) Pressure Angle

```
R_0 = (45 + 6)/10^3;

dy1dth = v_1/omega*180/pi;
dy2dth = v_2/omega*180/pi;
dy3dth = v_3/omega*180/pi;
dy4dth = v_4/omega*180/pi;
dy5dth = v_5/omega*180/pi;
dy6dth = v_6/omega*180/pi;
dy7dth = v_7/omega*180/pi;

phi_1 = atand(dy1dth./(R_0 + y_1));
phi_2 = atand(dy2dth./(R_0 + y_2));
phi_3 = atand(dy3dth./(R_0 + y_3 + 10/10^3));
phi_4 = atand(dy4dth./(R_0 + y_4));
phi_5 = atand(dy5dth./(R_0 + y_5 + 20/10^3));
phi_6 = atand(dy6dth./(R_0 + y_6));
phi_7 = atand(dy7dth./(R_0 + y_7));

fprintf('Part (b)\n')
figure
plot(theta(1:91),phi_1,color(1))
hold on
plot(theta(91:121),phi_2,color(2))
plot(theta(121:181),phi_3,color(3))
plot(theta(181:211),phi_4,color(4))
plot(theta(211:281),phi_5,color(5))
plot(theta(281:301),phi_6,color(6))
plot(theta(301:361),phi_7,color(7))
hold off
title('Pressure Angle vs. \theta')
xlabel('\theta (\circ)')
ylabel('Pressure Angle (\circ)')
xticks(0:90:360)
h = gca;
h.XAxis.MinorTick = 'on';
h.XAxis.MinorTickValues = 0:30:360;
grid on
grid minor
set(h,'DefaultLineLineWidth',linewidth,'FontSize',fontsize)
legend('5th Deg Polynomial Rise','Dwell','Harmonic Rise','Dwell','Cycloidal
 Rise','Dwell','3-4-5 Polynomial Fall','Location','best')

phi = [phi_1; phi_2(2:end); phi_3(2:end); phi_4(2:end); + phi_5(2:end);
 phi_6(2:end); phi_7(2:end)];
idx = ismember(theta,30:30:360);
table((0:30:360)',[0; round(phi(idx),4)],'VariableNames',{'theta (deg)','phi
 (deg)'})
```

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