```
%% Exercise 1.4

clear
clc

% Define A matrix
A = sym([ ...
   754     0  -377; ...
     0   754   377; ...
   377  -377     0; ...
]);

% Define B matrix
B = sym([ ...
   47.1250         0; ...
         0   47.1250; ...
         0         0; ...
]);

% Define K matrix
K = sym([ ...
   -32     0     0; ...
     0   -32     0; ...
]);

% Define A_c matrix
A_c = A + B*K
```

A_c =

$$\begin{pmatrix} -754 & 0 & -377 \\ 0 & -754 & 377 \\ 377 & -377 & 0 \end{pmatrix}$$

```
% (a) Compute the eigenvalues of A
lambda = eig(A_c)
```

lambda =

$$\begin{pmatrix} -754 \\ -377 - 377\,\mathrm{i} \\ -377 + 377\,\mathrm{i} \end{pmatrix}$$

```
% (b) Compute the diagonalization of A
[Q,D] = eig(A_c)
```

Q =

$$\begin{pmatrix} 1 & -\frac{1}{2}-\frac{1}{2}\,\mathrm{i} & -\frac{1}{2}+\frac{1}{2}\,\mathrm{i} \\ 1 & \frac{1}{2}+\frac{1}{2}\,\mathrm{i} & \frac{1}{2}-\frac{1}{2}\,\mathrm{i} \\ 0 & 1 & 1 \end{pmatrix}$$

D =

$$\begin{pmatrix} -754 & 0 & 0 \\ 0 & -377-377\,\mathrm{i} & 0 \\ 0 & 0 & -377+377\,\mathrm{i} \end{pmatrix}$$

```
Qinv = Q^-1
```

Qinv =

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2}\,\mathrm{i} & -\frac{1}{2}\,\mathrm{i} & \frac{1}{2}+\frac{1}{2}\,\mathrm{i} \\ -\frac{1}{2}\,\mathrm{i} & \frac{1}{2}\,\mathrm{i} & \frac{1}{2}-\frac{1}{2}\,\mathrm{i} \end{pmatrix}$$

```
% Define t
syms t

% Define x_0
x_0 = [0; 0; 1];

% (d) Compute x(t) with equilibrium initial condition
x = Q*[exp(D(1,1)*t) 0 0; 0 exp(D(2,2)*t) 0; 0 0 exp(D(3,3)*t)]*Qinv*x_0
```

x =

$$\begin{pmatrix} -\sigma_1+\sigma_2 \\ \sigma_1-\sigma_2 \\ \mathrm{e}^{t\,(-377-377\,\mathrm{i})}\left(\frac{1}{2}+\frac{1}{2}\,\mathrm{i}\right)+\mathrm{e}^{t\,(-377+377\,\mathrm{i})}\left(\frac{1}{2}-\frac{1}{2}\,\mathrm{i}\right) \end{pmatrix}$$

where

$$\sigma_1=\frac{\mathrm{e}^{t\,(-377-377\,\mathrm{i})}\,\mathrm{i}}{2}$$

$$\sigma_2=\frac{\mathrm{e}^{t\,(-377+377\,\mathrm{i})}\,\mathrm{i}}{2}$$

```
% (f) Plot output y
y1 = -exp(-377*t)*sin(377*t);
```

```
y2 = exp(-377*t)*sin(377*t);

myplot = tiledlayout(2,1);
title(myplot,"Exercise 1.4(f)","FontSize",16)
nexttile
fplot(y1,[0,0.02],"LineWidth",1)
xlabel("t")
ylabel("y_1(t)")
set(gca,"FontSize",14)
nexttile
fplot(y2,[0,0.02],"LineWidth",1)
xlabel("t")
ylabel("y_2(t)")
set(gca,"FontSize",14)
```