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MAE 150

07/27/23

Homework 4

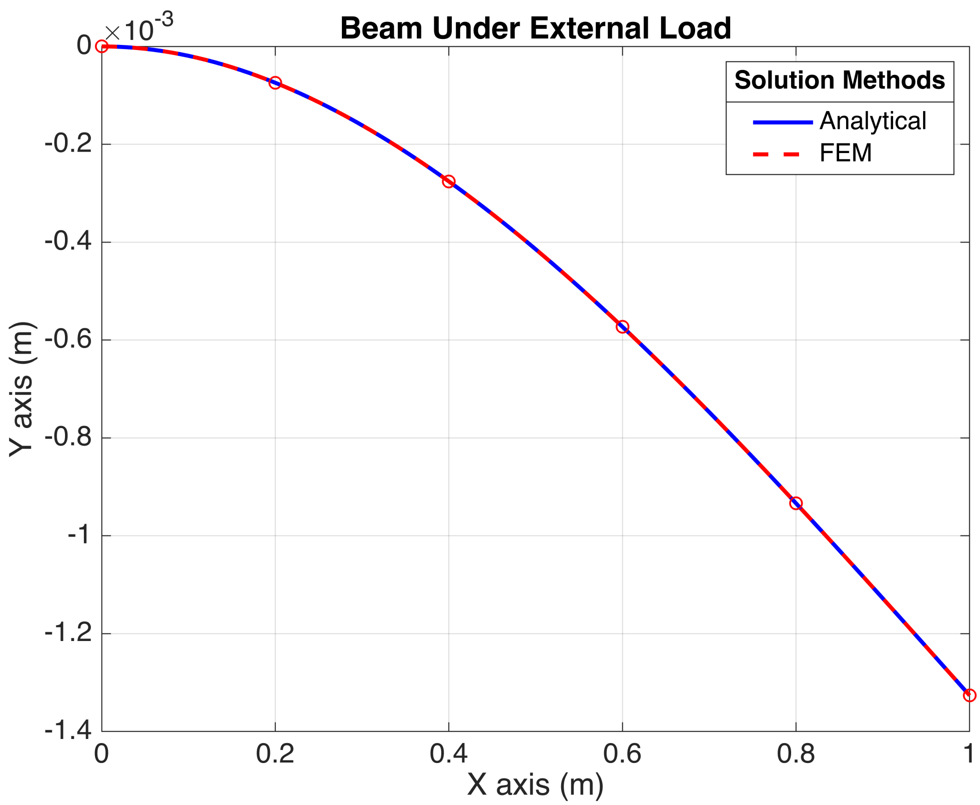
Problem 1

```
Part(a)

delta(x=L) =

    0.0013

(Units: m)



Part(b)

K_e1 =

   1.0e+07 *

    3.7699    0.3770   -3.7699    0.3770
    0.3770    0.0503   -0.3770    0.0251
   -3.7699   -0.3770    3.7699   -0.3770
    0.3770    0.0251   -0.3770    0.0503

K_e2 =

   1.0e+07 *

    3.7699    0.3770   -3.7699    0.3770
    0.3770    0.0503   -0.3770    0.0251
   -3.7699   -0.3770    3.7699   -0.3770
    0.3770    0.0251   -0.3770    0.0503

K_e3 =

   1.0e+07 *

    3.7699    0.3770   -3.7699    0.3770
    0.3770    0.0503   -0.3770    0.0251
   -3.7699   -0.3770    3.7699   -0.3770
    0.3770    0.0251   -0.3770    0.0503

K_e4 =

   1.0e+07 *

    3.7699    0.3770   -3.7699    0.3770
    0.3770    0.0503   -0.3770    0.0251
   -3.7699   -0.3770    3.7699   -0.3770
    0.3770    0.0251   -0.3770    0.0503
```

```
K_e5 =

   1.0e+07 *

    3.7699    0.3770   -3.7699    0.3770
    0.3770    0.0503   -0.3770    0.0251
   -3.7699   -0.3770    3.7699   -0.3770
    0.3770    0.0251   -0.3770    0.0503

(Units: N/m)



K_G =

   1.0e+07 *

    3.7699    0.3770   -3.7699    0.3770         0         0         0         0         0         0
    0.3770    0.0503   -0.3770    0.0251         0         0         0         0         0         0
   -3.7699   -0.3770    7.5398         0   -3.7699    0.3770         0         0         0         0
    0.3770    0.0251         0    0.1005   -0.3770    0.0251         0         0         0         0
         0         0   -3.7699   -0.3770    7.5398         0   -3.7699    0.3770         0         0
         0         0    0.3770    0.0251         0    0.1005   -0.3770    0.0251         0         0
         0         0         0         0   -3.7699   -0.3770    7.5398         0   -3.7699    0.3770
         0         0         0         0    0.3770    0.0251         0    0.1005   -0.3770    0.0251
         0         0         0         0         0         0   -3.7699   -0.3770    7.5398         0
         0         0         0         0         0         0    0.3770    0.0251         0    0.1005
         0         0         0         0         0         0         0         0   -3.7699   -0.3770
         0         0         0         0         0         0         0         0    0.3770    0.0251

   1.0e+07 *

         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
   -3.7699    0.3770
   -0.3770    0.0251
    3.7699   -0.3770
   -0.3770    0.0503

(Units: N/m)



d =

         0
         0
   -0.0001
   -0.0410
   -0.0003
   -0.0730
   -0.0006
   -0.0957
   -0.0009
   -0.1094
   -0.0013
   -0.1140
```

```
(Units: m & deg)


F =

  100.0000
  100.0000
         0
         0
         0
         0
         0
         0
         0
         0
 -100.0000
         0

(Units: N & N*m)
```

```
Part(c)
```



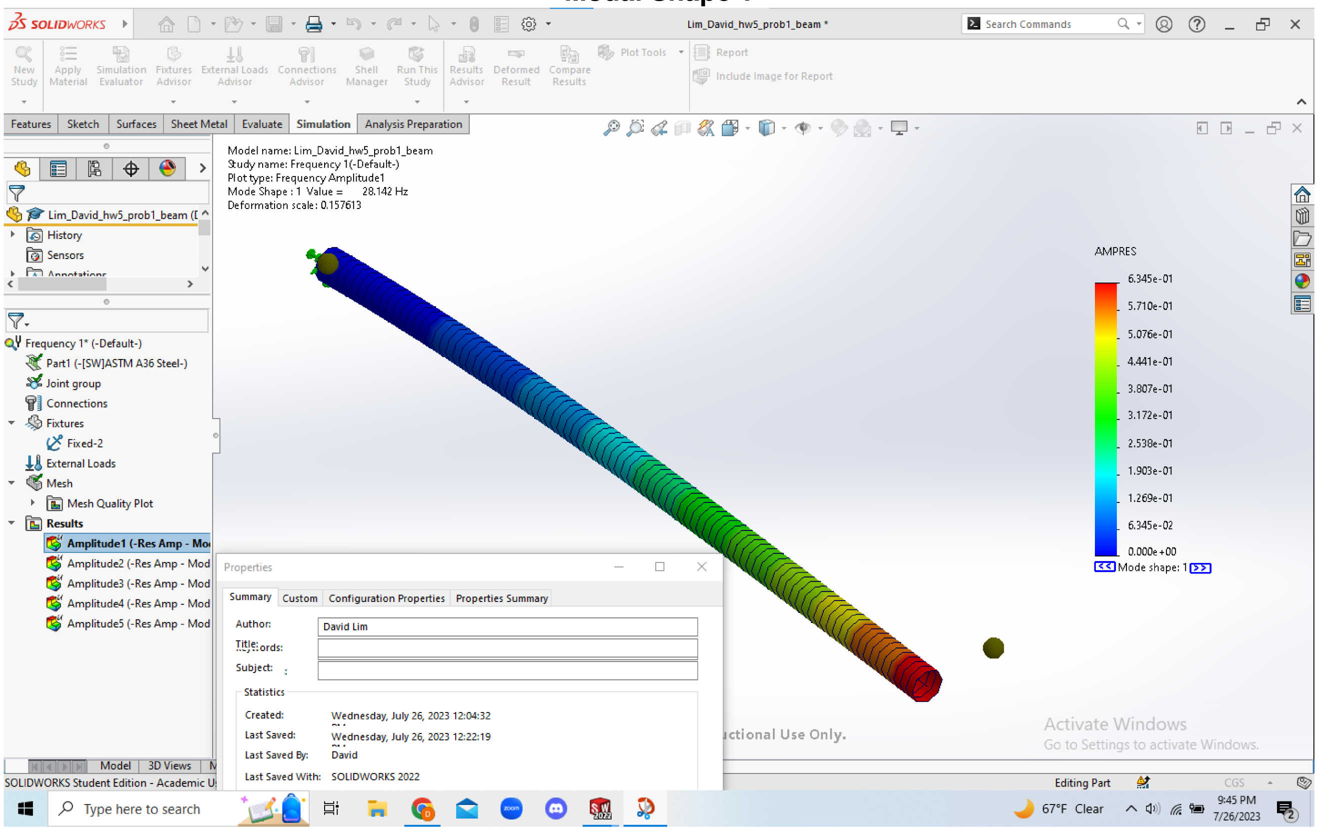
The two solution methods provide nearly equal displacement values at the nodes. The analytical method provides a solution for displacement at every point along the beam while the FEM solution only provides exact value for the displacement at the nodes and approximates the displacement along the beam using a 3rd order polynomial interpolation.

Part (d)

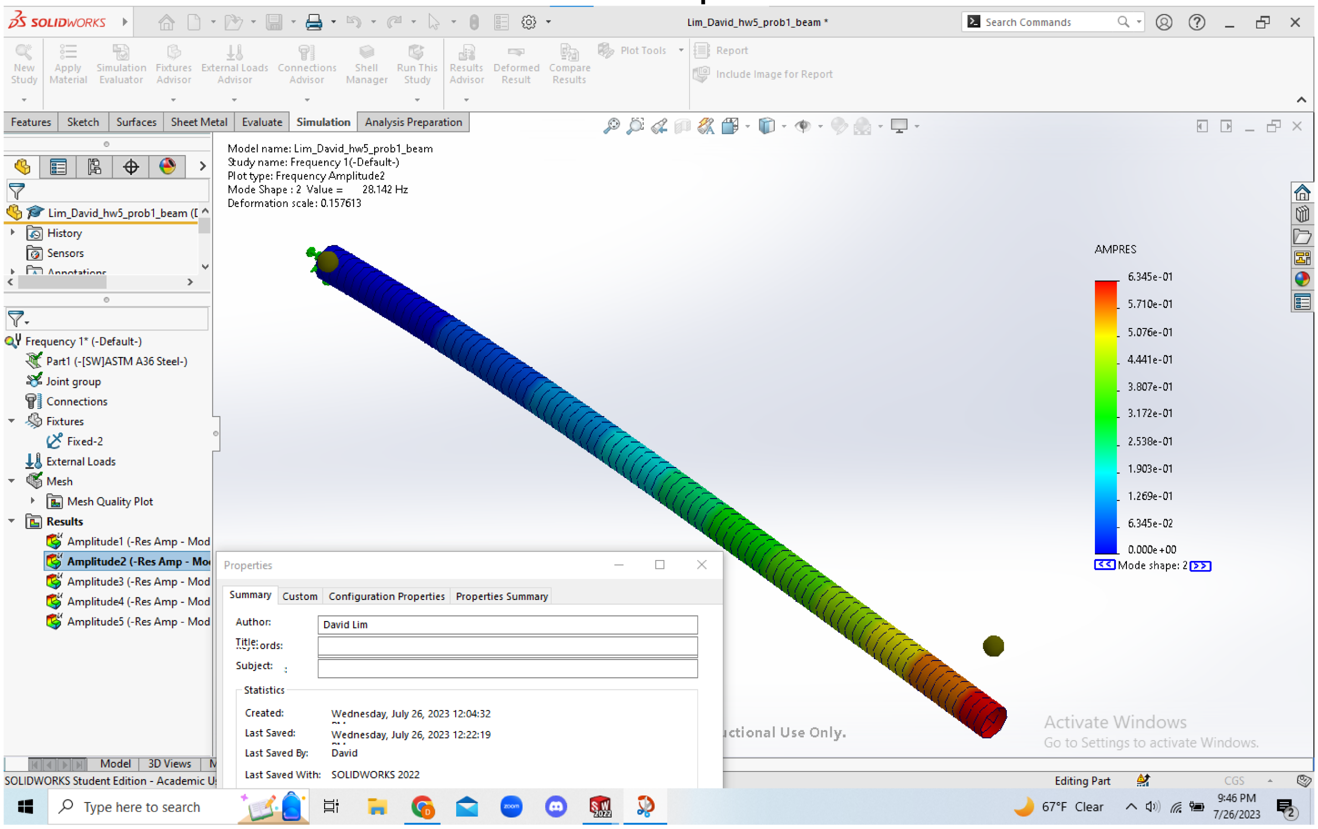
Bending stiffness depends on the elastic modulus of the material and the area moment of inertia. If the material and cross-sectional area are to remain the same, the area moment of inertia could still be changed. Using a cross-sectional shape with a greater area moment of inertia than a circle would increase the bending stiffness while other parameters can be held constant.

Part (e)

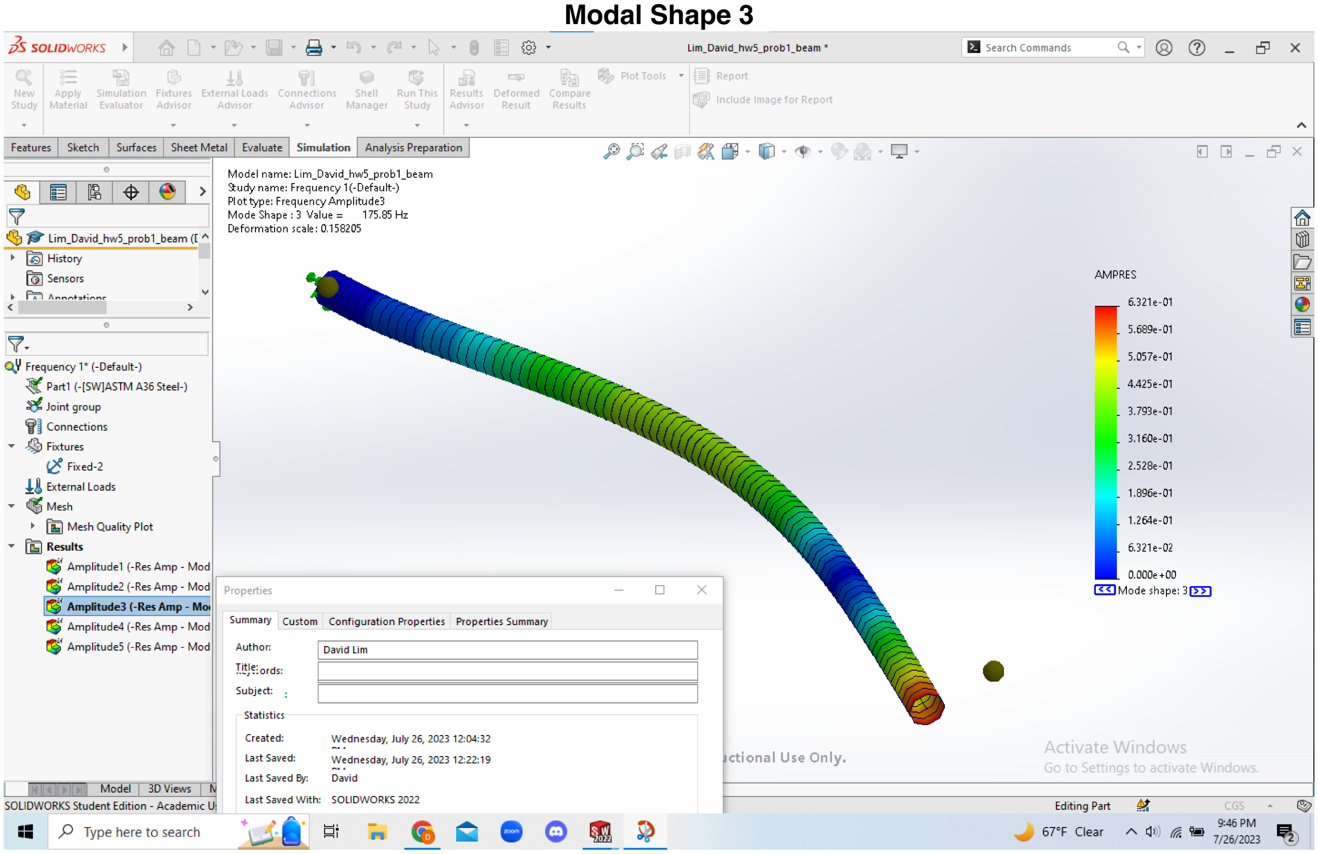
Modal Shape 1



Modal Shape 2



Modal Shape 3



Part (f)

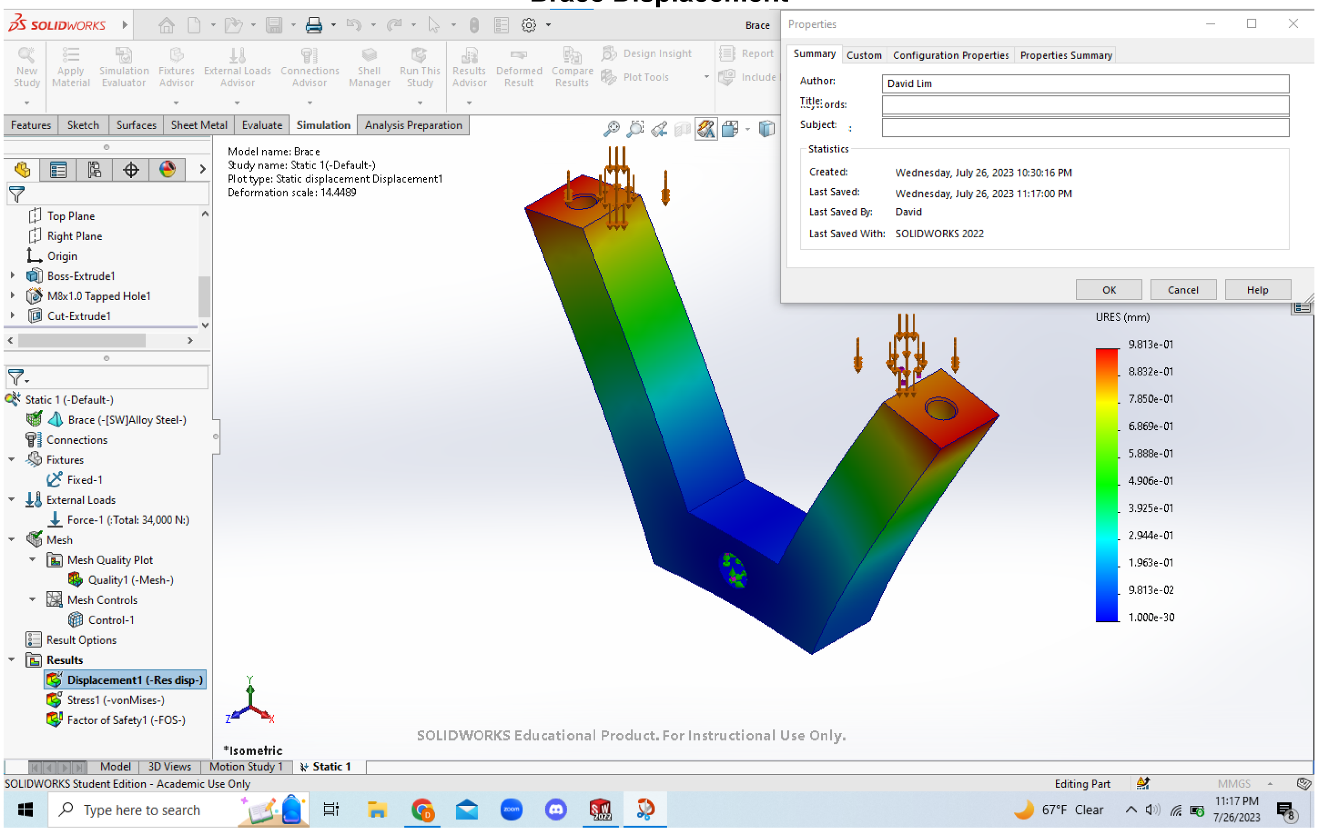
```
f_1 =

    28.2425

(Units: Hz)
```

The first natural frequency computed in Solidworks is close to the analytical solution.

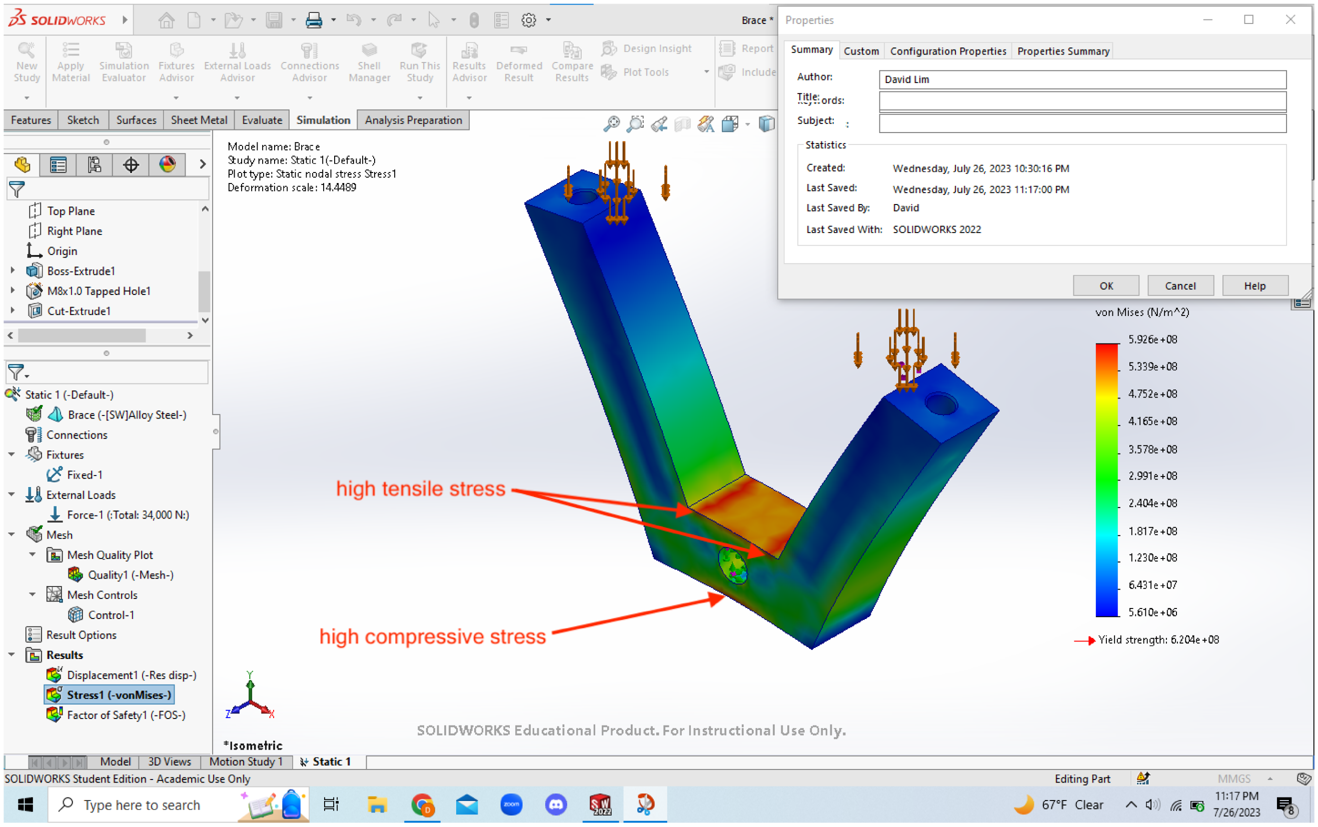
Problem 2

Part (a)

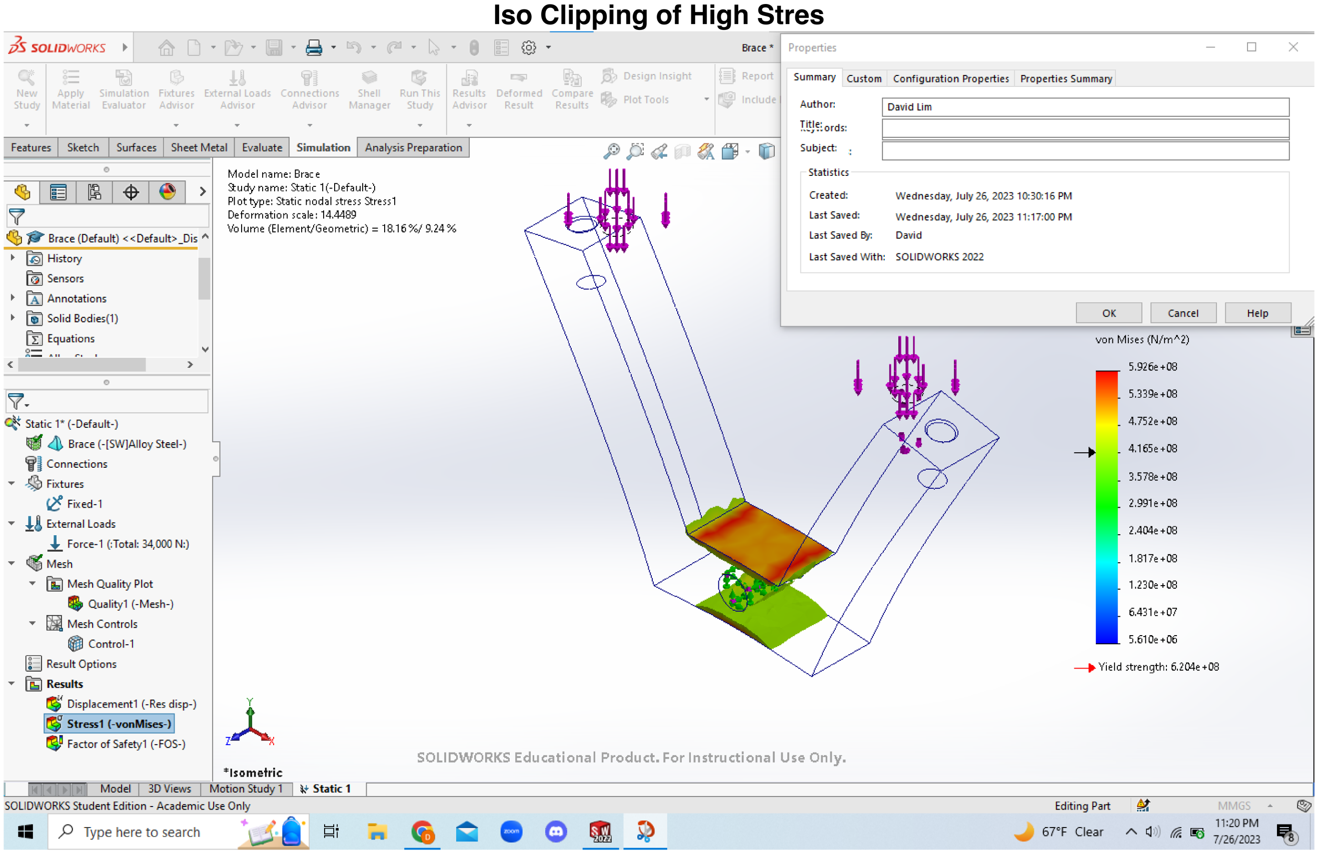
Brace Displacement



Brace Stress



Iso Clipping of High Stres



Part (b)

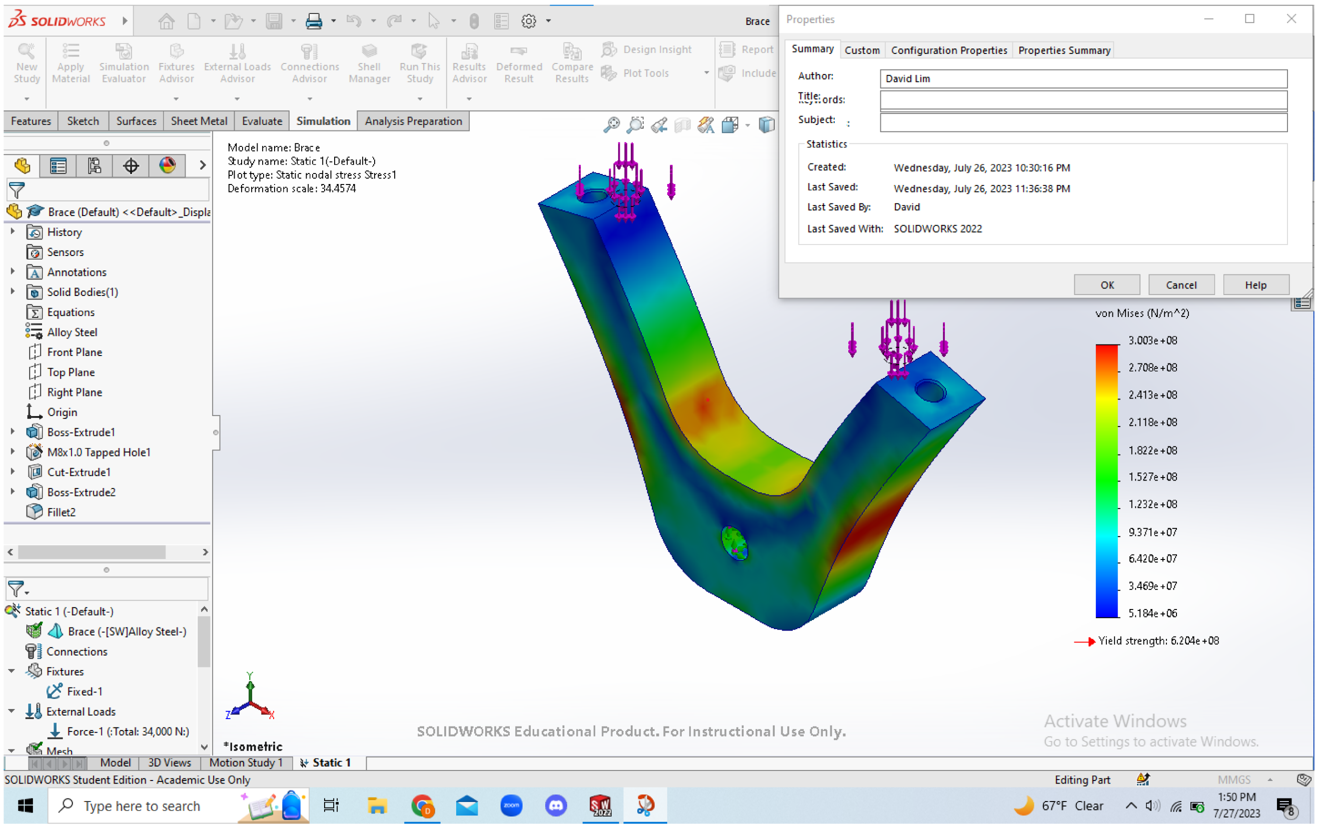
The yield stress of the material is 6.204 * 10^8. The maximum stress is 5.926 * 10^8. Dividing the yield stress by the maximum stress gives the factor of safety, which is about 1.05. The factor of safety is less than the requirement of 2, so design is not safe.

Part (c)

I increased the thickness of the material above and below the bottom hole and added fillets to the inner and outer corners.

Part (d)

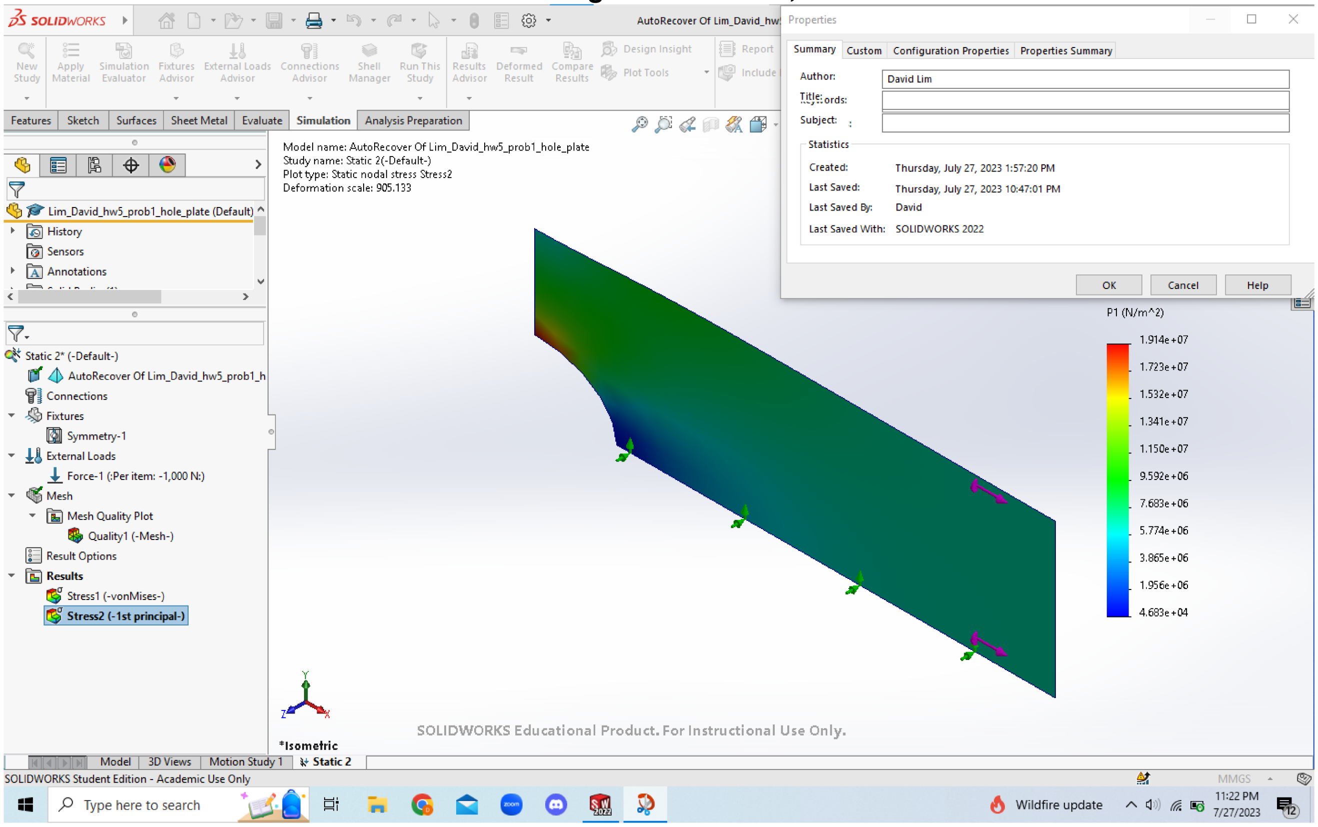
Stress of Modified Brace



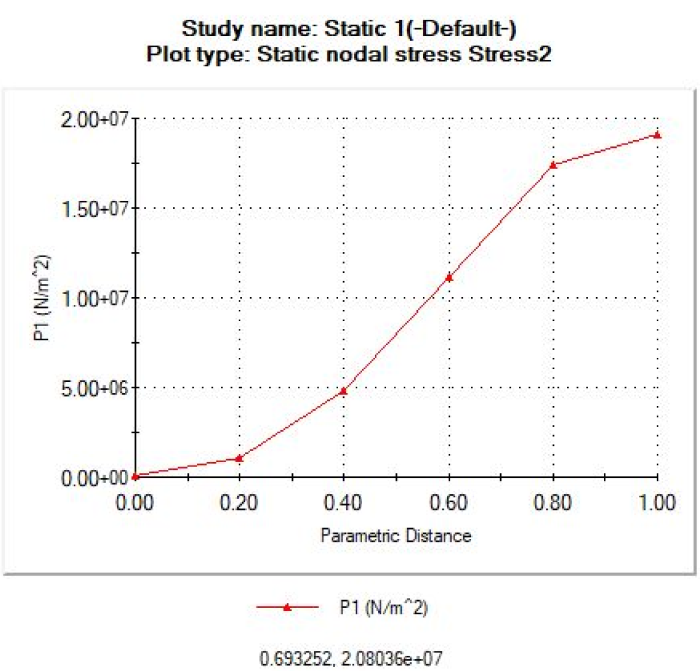
The maximum stress in this design is 3.003 * 10^8 with a factor of safety of 2.065, which satisfies the factor of safety requirement.

Problem 3

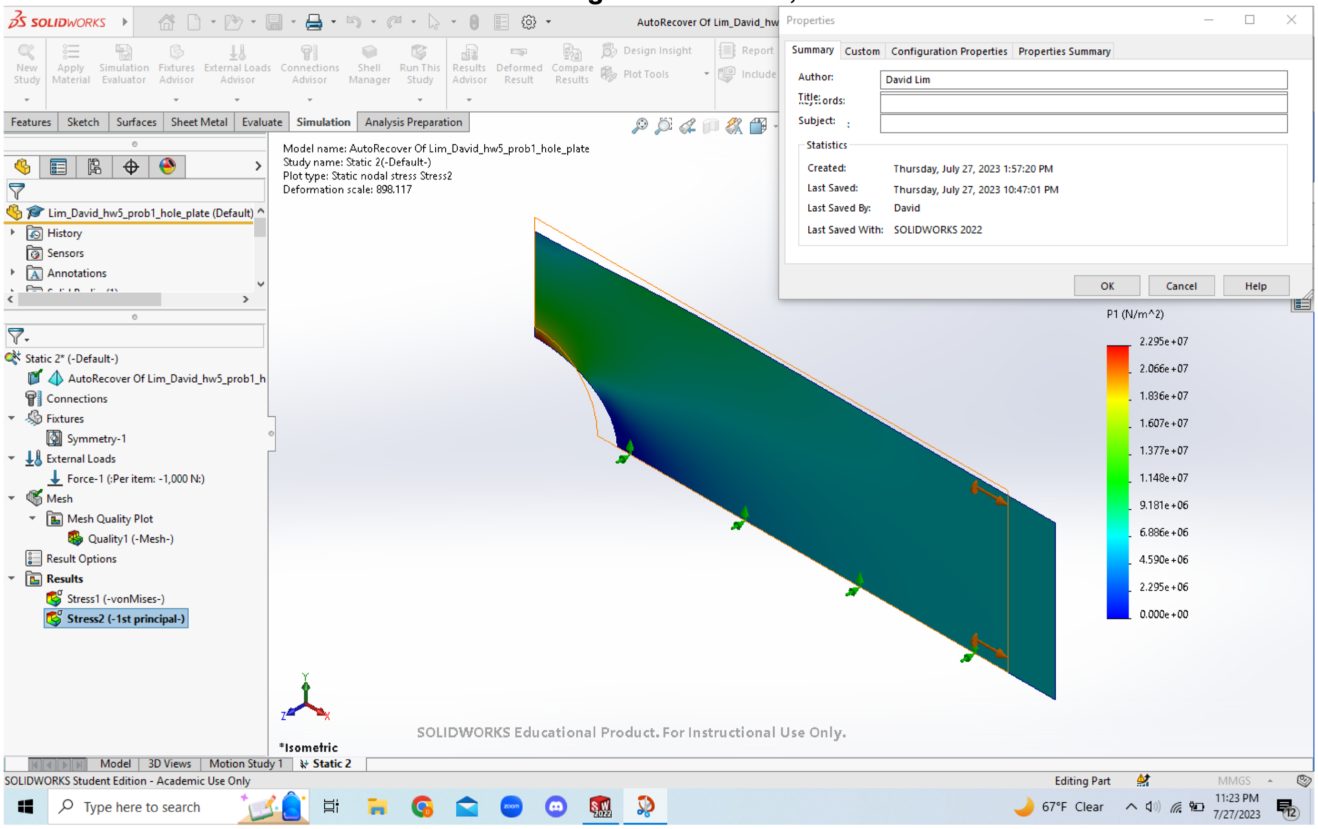
Stress of Large Hole in Plate, Mesh 1



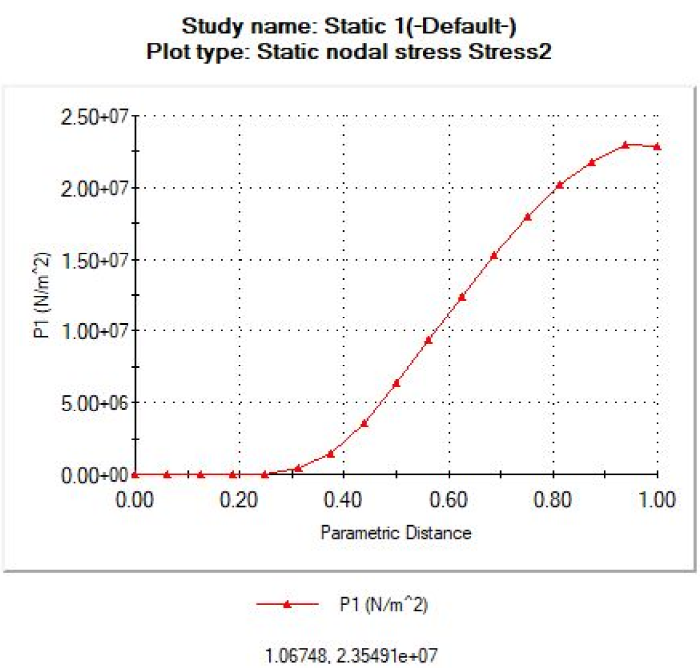
Stress at Edge Nodes of Large Hole, Mesh 1



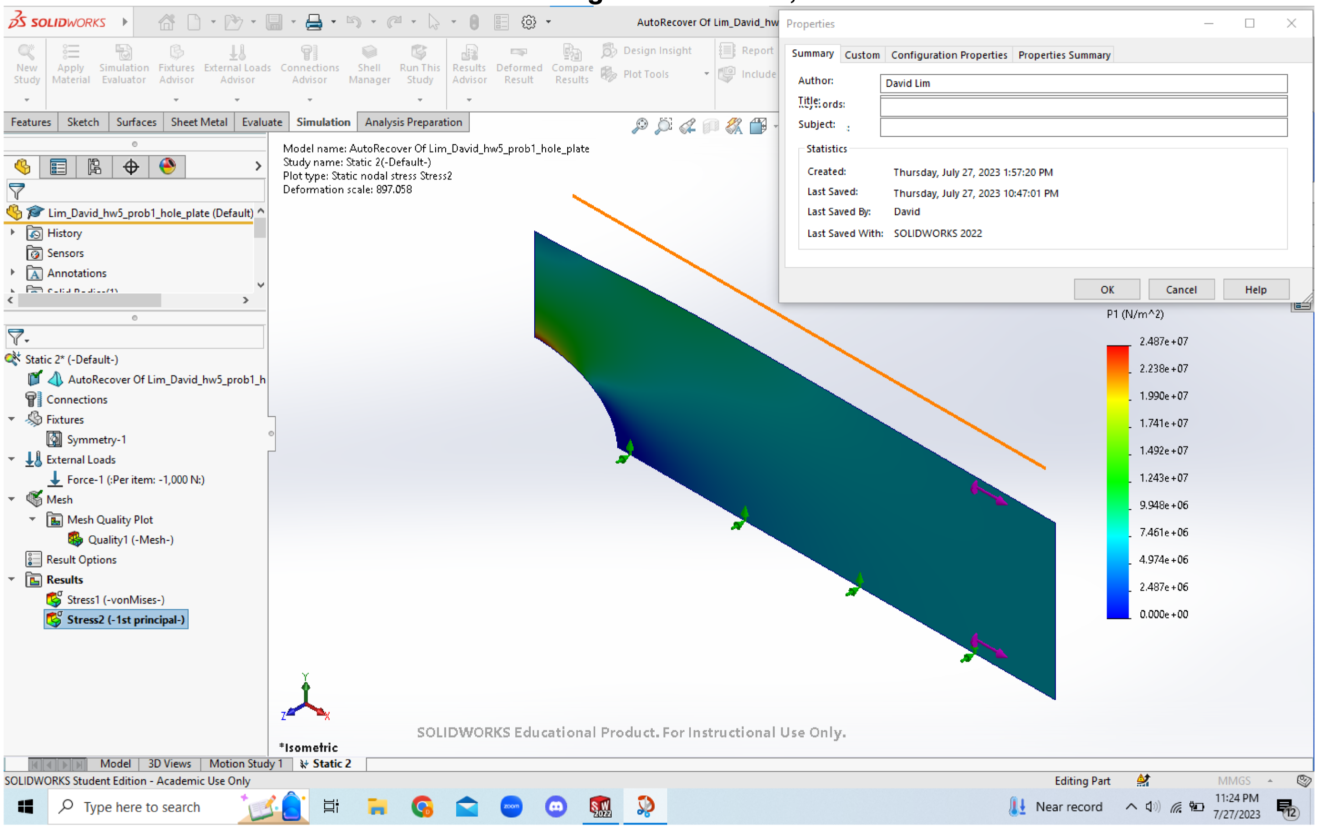
Stress of Large Hole in Plate, Mesh 2



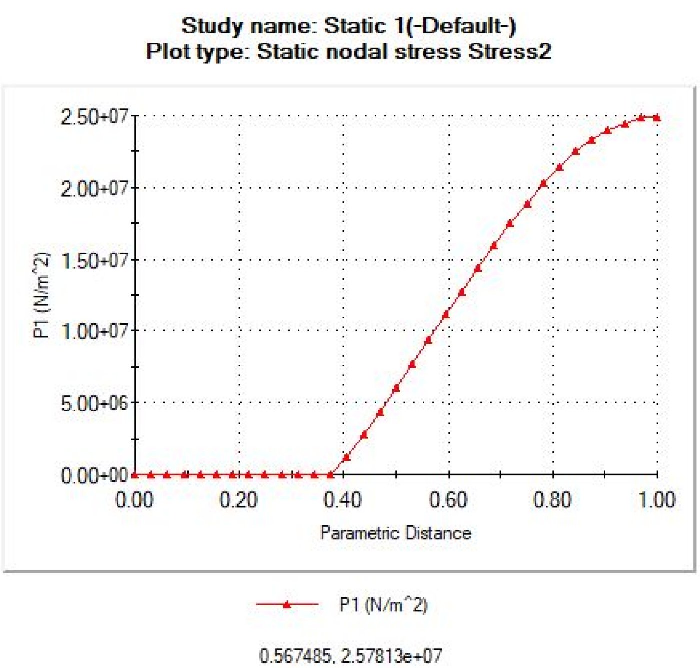
Stress at Edge Nodes of Large Hole, Mesh 2



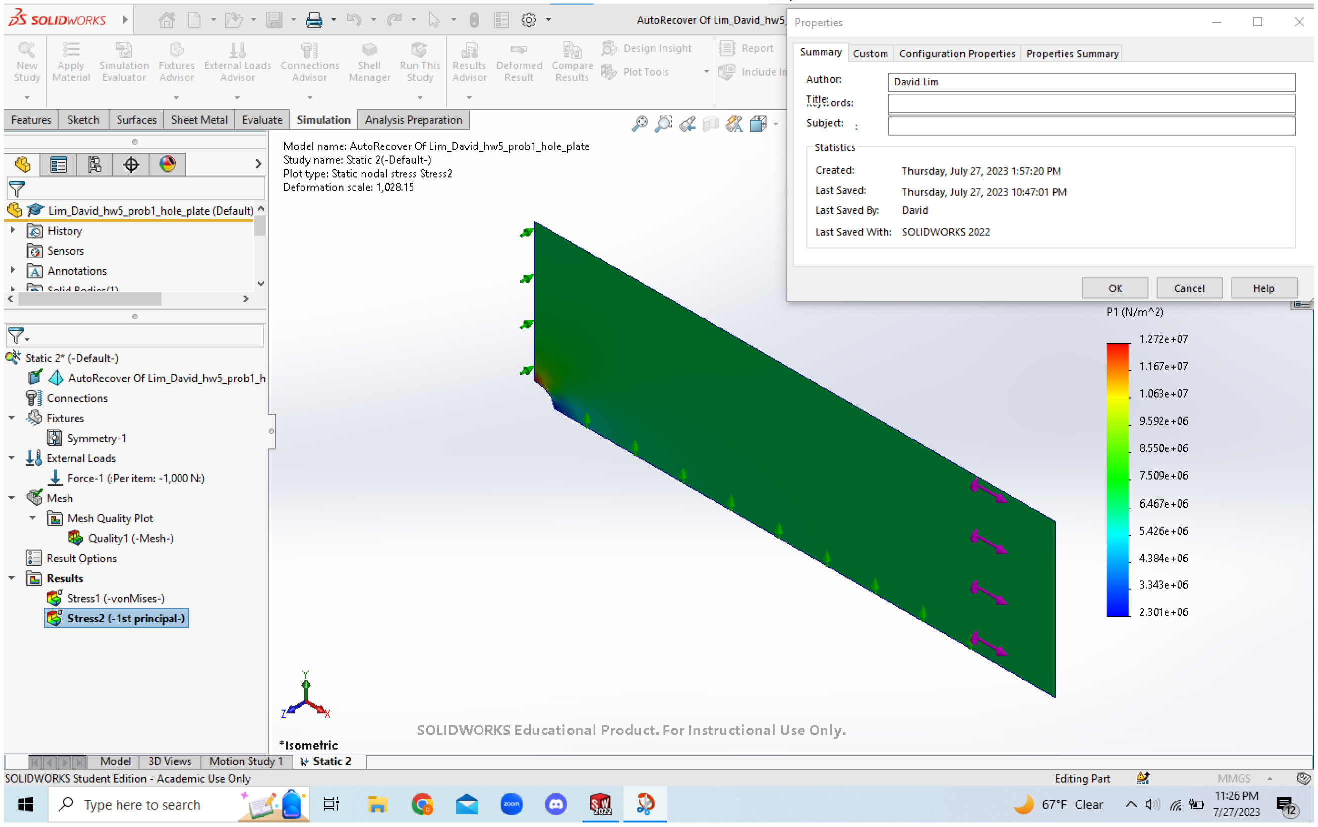
Stress of Large Hole in Plate, Mesh 3



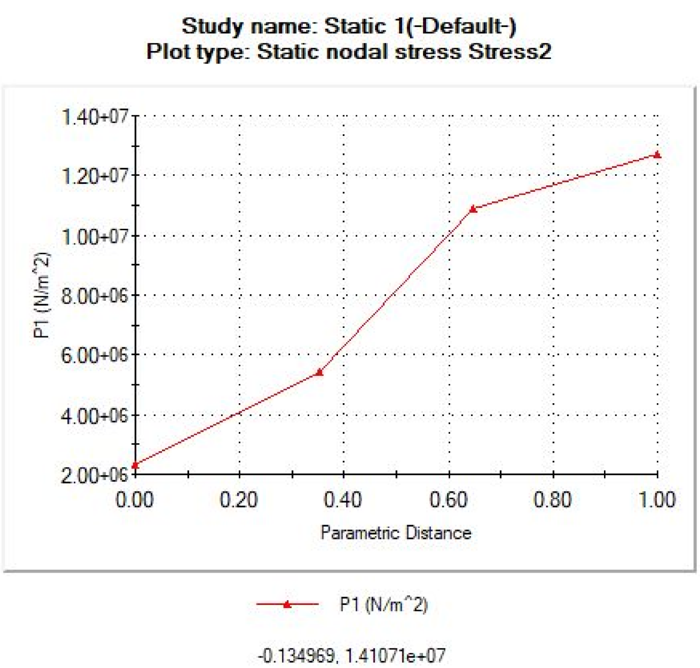
Stress at Edge Nodes of Large Hole, Mesh 3



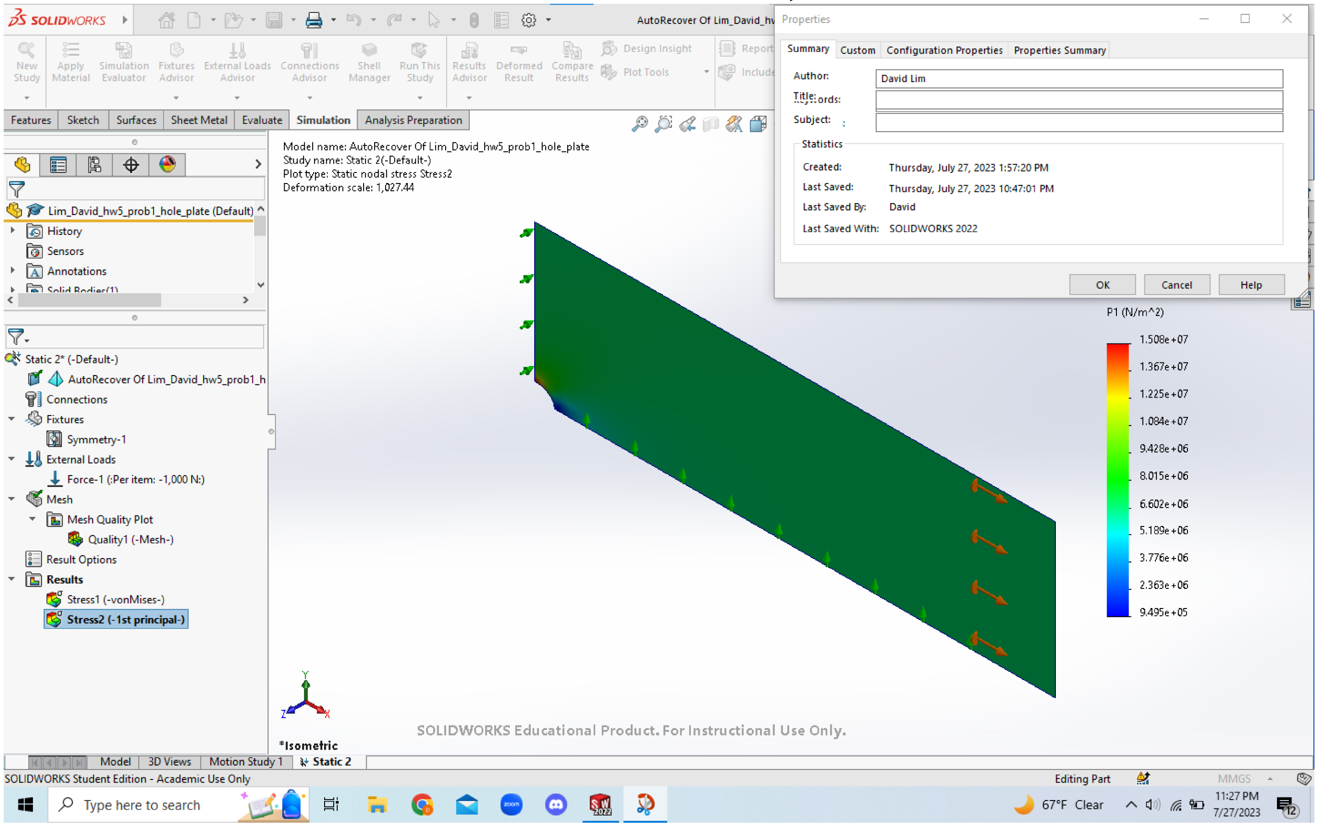
Stress of Small Hole in Plate, Mesh 1



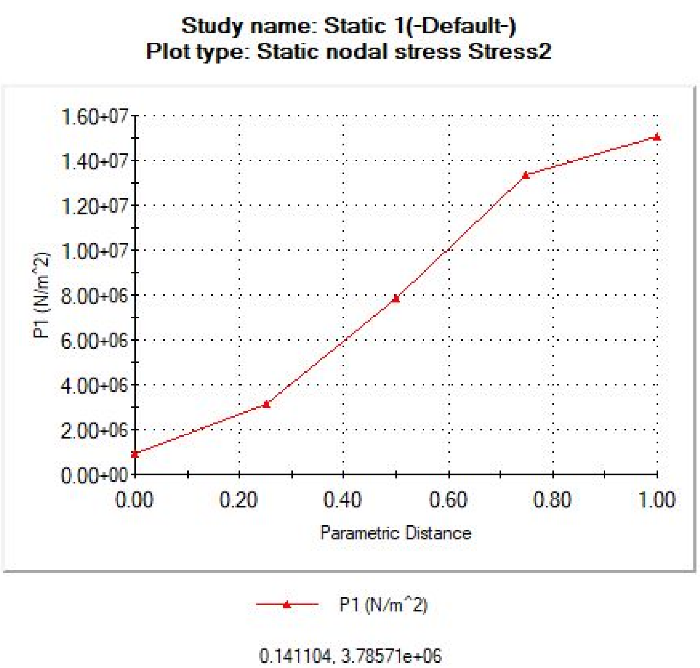
Stress at Edge Nodes of Small Hole, Mesh 1



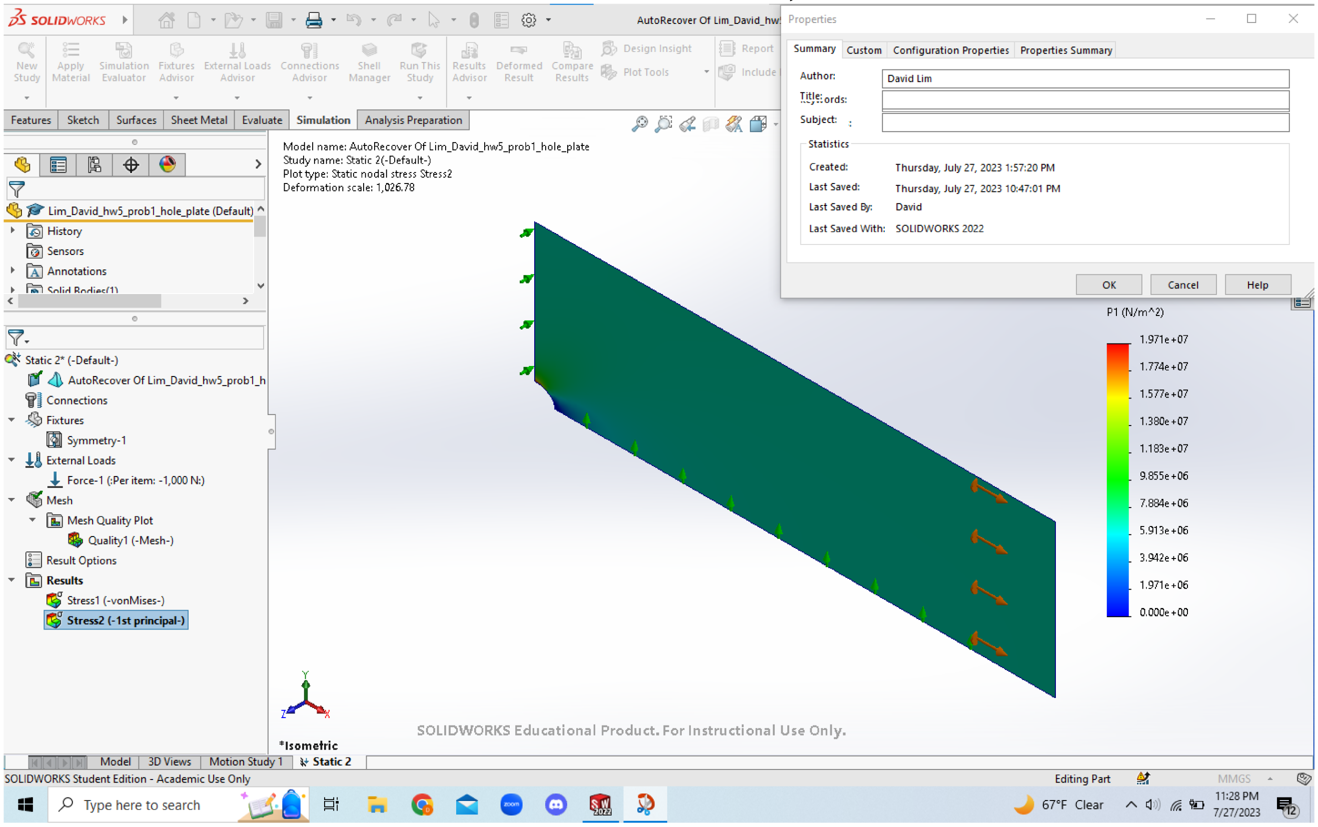
Stress of Small Hole in Plate, Mesh 2



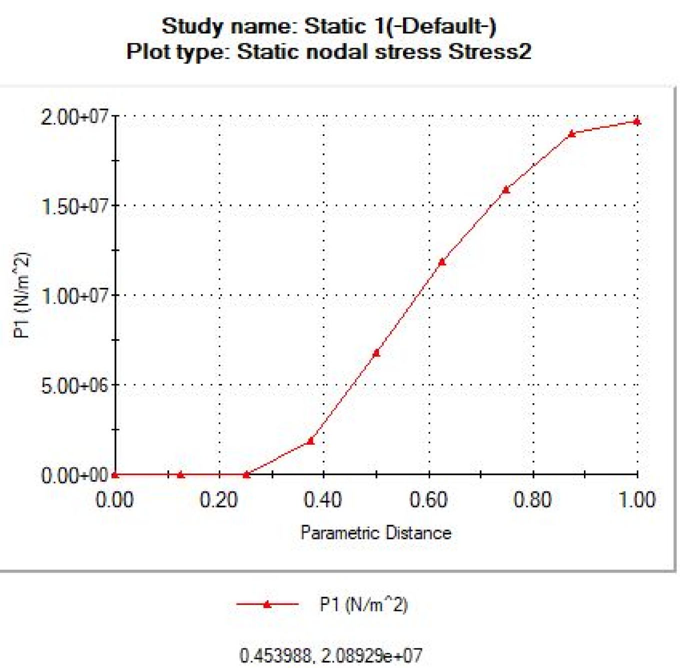
Stress at Edge Nodes of Small Hole, Mesh 2



Stress of Small Hole in Plate, Mesh 3



Stress at Edge Nodes of Small Hole, Mesh 3



1)

The cross-sectional area of the plate without the hole is A = 2*h*t = 2 * 0.025 (m) * 0.006 (m) = 0.0003 m^2, and the applied force is F = 2000 N.

The nominal stress far from the hole is F/A = 2000 (N) / 0.003 (m^2) = 6.66 * 10^6 Pa.

2)

The stress at the top of the small hole should be 3 * 6.66 MPa = 20 MPa.

Comparison of the FEA to the analytical solution:

Mesh Setting	Stress (MPa)	Error (%)
1	12.72	36.40
2	15.08	24.60
3	19.71	1.45

3)

The FEA stress concentration factor of the 10 mm radius hole is about 2.24, which is smaller than the factor for the 2.5 mm radius hole. As the radius a approaches the length h, the stress concentration factor likely approaches a constant positive value (which appears to be 2), since the factor should not be zero or negative.

Table of Contents

MAE 150 HW 5 Problem 2

```
clear
close all

% Parameters
L = 1; % m
r = 0.02; % m
I = pi*r^4/4; % 2nd moment of area of circular cross section
E = 200*10^9; % Pa
P = 100; % N
```

Part (a): Vertical deflection at free end

```
delta_L = P*L^2*(3*L-L)/(6*E*I);

fprintf('Part(a)\n\n')
fprintf('delta(x=L) =\n\n')
disp(delta_L)
fprintf('(Units: m)\n')
```

Part (b): FEM of beam

```
m = 5; % number of elements
n = m + 1; % number of nodes

l = L/m; % length of each element

% Initialize arrays in loop
Ke = cell(m,1);
KG = zeros(2*n);

% Loop through elements
for el = 1:m
    i = el;
    j = el + 1;

    Ke{el} = E*I/l^3*[ 12   6*l  -12   6*l;
                      6*l 4*l^2 -6*l 2*l^2;
                      -12  -6*l   12  -6*l;
```

```
                      6*l 2*l^2 -6*l 4*l^2];

    Kt = zeros(2*n);
    Kt([2*i-1,2*i,2*j-1,2*j],[2*i-1,2*i,2*j-1,2*j]) = Ke{el};
    KG = KG + Kt;
end

% Define load vector
F = zeros(n,1); % N
F(1) = nan;
F(2) = nan;
F(3) = 0;
F(4) = 0;
F(5) = 0;
F(6) = 0;
F(7) = 0;
F(8) = 0;
F(9) = 0;
F(10) = 0;
F(11) = -100;
F(12) = 0;

% Indexing array for reduction (displacement = 0 at reaction forces)
redux = ~isnan(F);

% Reduce KG and F
Kr = KG(redux,redux);
Fr = F(redux);

% Solve for unknown displacements
dr = Kr\Fr;

% Define complete displacement vector
d = zeros(2*n,1);
d(redux) = dr;
d(2:2:end) = d(2:2:end)*180/pi; % convert angles to degrees

% Reduce KG (reduce opposite rows)
Kr = KG(~redux,redux);

% Solve for unknown loads
Fr = Kr*dr;

% Define complete load vector
F(~redux) = Fr;
```

Print results for Part (b)

```
fprintf('\n\n\n')
fprintf('Part(b)\n\n')
fprintf('K_e1 =\n\n')
disp(Ke{1})
fprintf('K_e2 =\n\n')
```

```
disp(Ke{2})
fprintf('K_e3 =\n\n')
disp(Ke{3})
fprintf('K_e4 =\n\n')
disp(Ke{4})
fprintf('K_e5 =\n\n')
disp(Ke{5})
fprintf('(Units: N/m)\n')

fprintf('\n\n\n')
fprintf('K_G =\n\n')
disp(KG(:,1:10))
disp(KG(:,11:12))
fprintf('(Units: N/m)\n')

fprintf('\n\n\n')
fprintf('d =\n\n')
disp(d)
fprintf('(Units: m & deg)\n')

fprintf('\n\n\n')
fprintf('F =\n\n')
disp(F)
fprintf('(Units: N & N*m)\n')
```

Part (c): Plotting

```
x = 0:0.01:L;
y = -P*x.^2.*(3*L-x)/(6*E*I);

d(redux) = dr; % back to angles in radians
y_FEM = zeros(1,length(x));
for el = 1:m

    w1 = d(2*el - 1);
    th1 = d(2*el);
    w2 = d(2*el + 1);
    th2 = d(2*el + 2);

    c1 = (2*w1 + l*th1 - 2*w2 + l*th2)/l^3;
    c2 = (-3*w1 - 2*l*th1 + 3*w2 - l*th2)/l^2;
    c3 = th1;
    c4 = w1;

    idx = x > (el-1)*l & x <= el*l;
    y_FEM(idx) = c1*(x(idx)-(el-1)*l).^3 + c2*(x(idx)-(el-1)*l).^2 +
 c3*(x(idx)-(el-1)*l) + c4;
end

fprintf('\n\n\n')
fprintf('Part(c)\n\n')

linewidth = 2;
```

```
fontsize = 14;
color1 = 'b';
color2 = 'r';

close all
figure
clf
plot(x,y,'-','Color',color1,'Linewidth',linewidth);
hold on
plot(x,y_FEM,'--','Color',color2,'Linewidth',linewidth);
plot(0:l:L,d(1:2:end),'o','Color',color2,'Linewidth',linewidth);
hold off
grid on
title('Beam Under External Load')
xlabel('X axis (m)')
ylabel('Y axis (m)')
set(gca,'FontSize',fontsize)
lgd = legend('Analytical','FEM');
title(lgd,'Solution Methods')
```

Part (f)

```
A = pi*r^2; % m^2
rho = 7850; % kg/m^3

f1 = 1.875^2/(2*pi*L^2)*sqrt(E*I/(rho*A));
```

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