Homework 6, Problem 3

```
% David Lim
% A16398479
% 03/06/25
clear
```

Step 1: State space representation

```
% Define parameters
N = 40;
T = 10;
mu = 0.1;
L = 5;
c = L^2*mu/(N^2*T);

% Construct A matrix
A = zeros(2*N);
for i = 1:N  % string element index
    k = 2*i-1;  % odd number index
    A(k,k+1) = 1;
    if i > 1; A(k+1,k-2) = 1/c; end
    A(k+1,k) = -2/c;
    if i < N; A(k+1,k+2) = 1/c; end
end

% Construct C matrix
C = zeros(N,2*N);
for i = 1:N
    C(i,2*i-1) = 1;
end

% Display results for publishing
A
```

```
A = 80×80
          0          1          0          0          0          0 ···
     -12800          0       6400          0          0          0
          0          0          0          1          0          0
       6400          0     -12800          0       6400          0
          0          0          0          0          0          1
          0          0       6400          0     -12800          0
          0          0          0          0          0          0
          0          0          0          0       6400          0
          0          0          0          0          0          0
          0          0          0          0          0          0
```

```
C
```

```
C = 40×80
     1     0     0     0     0     0     0     0     0     0     0     0 ···
     0     0     1     0     0     0     0     0     0     0     0     0
     0     0     0     0     1     0     0     0     0     0     0     0
     0     0     0     0     0     0     1     0     0     0     0     0
     0     0     0     0     0     0     0     0     1     0     0     0
     0     0     0     0     0     0     0     0     0     0     1     0
     0     0     0     0     0     0     0     0     0     0     0     1
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
```

```
eig(A)
```

```
ans = 80×1 complex
10^2 ×
  -0.0000 + 1.5988i
  -0.0000 - 1.5988i
  -0.0000 + 1.5953i
  -0.0000 - 1.5953i
  -0.0000 + 1.5894i
  -0.0000 - 1.5894i
   0.0000 + 1.5812i
   0.0000 - 1.5812i
  -0.0000 + 1.5707i
  -0.0000 - 1.5707i
```

Step 2: Modal decomposition

```
% IMPORTANT (excerpt from MATLAB documentation, learned the hard
way):
% By default eig does not always return the eigenvalues and
eigenvectors in
% sorted order. Use the sort function to put the eigenvalues in
ascending
% order and reorder the corresponding eigenvectors.

[V,D] = eig(A);  % diagonalizes A
% WRONG (assumed eig always ordered eigenvalues):
% V = fliplr(V);  % reorders eigenvalues from least to greatest
magnitude
```

```
% D = rot90(D,2);  % reorders eigenvectors
% RIGHT (copied from MATLAB documentation):
[d,ind] = sort(diag(D));
D = D(ind,ind);
V = V(:,ind);

X = zeros(size(V));  % preallocate transformation matrix
Anew = zeros(size(D));  % preallocate block diagonal matrix

% Construct block diagonal matrix Anew
for i = 1:N
    k = 2*i-1;

    p1 = (V(:,k)+V(:,k+1))/2;
    p2 = (V(:,k)-V(:,k+1))/2j;
    a1 = real(D(k,k));
    b1 = imag(D(k,k));

    X(:,k:k+1) = [p1 p2];
    Anew(k:k+1,k:k+1) = [a1 b1; -b1 a1];
end

% Display results for publishing
X
```

```
X = 80×80
    0.0000    0.0027    0.0000   -0.0027    0.0000    0.0027   -0.0000    0.0027 ···
    0.0167    0.0000   -0.0336   -0.0000    0.0502   -0.0000    0.0666    0.0000
    0.0000    0.0054    0.0000   -0.0054    0.0000    0.0053   -0.0000    0.0052
    0.0333   -0.0000   -0.0664    0.0000    0.0979   -0.0000    0.1270   -0.0000
    0.0000    0.0081    0.0000   -0.0080    0.0000    0.0076   -0.0000    0.0072
    0.0497    0.0000   -0.0977   -0.0000    0.1403   -0.0000    0.1755    0.0000
    0.0000    0.0107    0.0000   -0.0103    0.0000    0.0096   -0.0000    0.0085
    0.0658   -0.0000   -0.1266   -0.0000    0.1754   -0.0000    0.2077    0.0000
    0.0000    0.0133    0.0000   -0.0125    0.0000    0.0110   -0.0000    0.0090
    0.0815    0.0000   -0.1526   -0.0000    0.2012   -0.0000    0.2205         0
     ⋮
```

```
Anew
```

```
Anew = 80×80
   -0.0000   -6.1284         0         0         0         0         0         0 ···
    6.1284   -0.0000         0         0         0         0         0         0
         0         0    0.0000  -12.2479         0         0         0         0
         0         0   12.2479    0.0000         0         0         0         0
         0         0         0         0   -0.0000  -18.3493         0         0
         0         0         0         0   18.3493   -0.0000         0         0
         0         0         0         0         0         0   -0.0000  -24.4239
```

```
         0         0         0         0         0         0   24.4239   -0.0000
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
    ⋮
```

```
Cnew = C*X
```

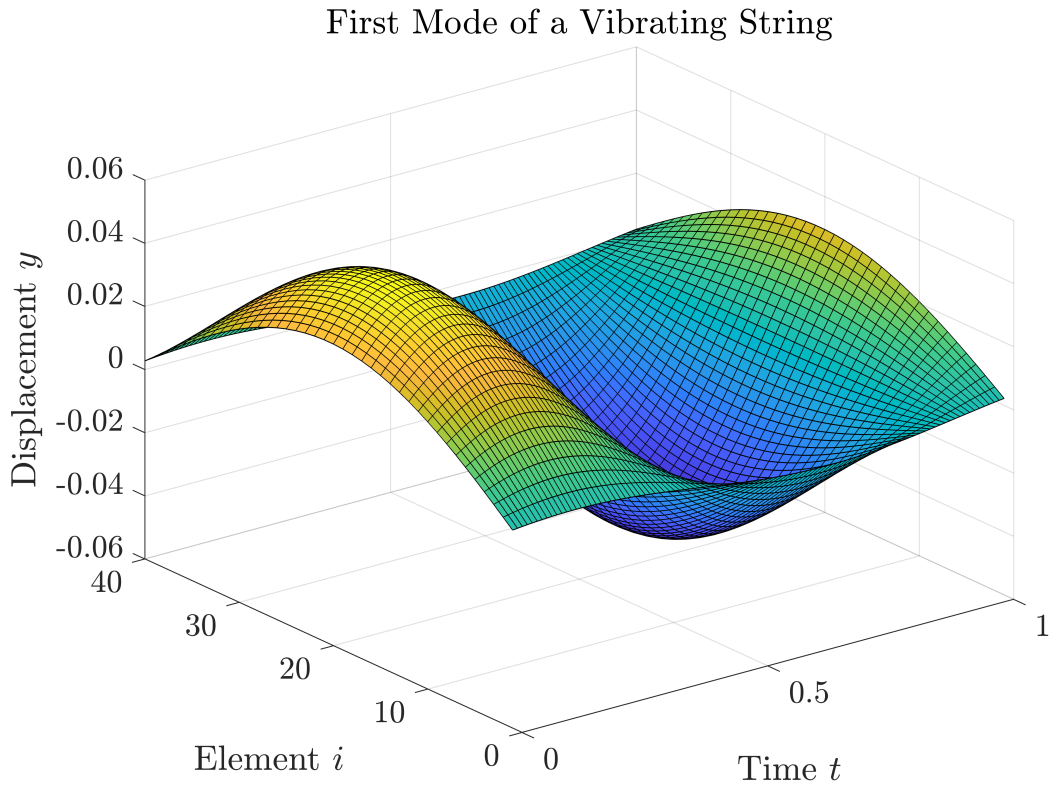
```
Cnew = 40×80
    0.0000    0.0027    0.0000   -0.0027    0.0000    0.0027   -0.0000    0.0027 ···
    0.0000    0.0054    0.0000   -0.0054    0.0000    0.0053   -0.0000    0.0052
    0.0000    0.0081    0.0000   -0.0080    0.0000    0.0076   -0.0000    0.0072
    0.0000    0.0107    0.0000   -0.0103    0.0000    0.0096   -0.0000    0.0085
    0.0000    0.0133    0.0000   -0.0125    0.0000    0.0110   -0.0000    0.0090
    0.0000    0.0158    0.0000   -0.0143    0.0000    0.0118   -0.0000    0.0087
    0.0000    0.0182    0.0000   -0.0158    0.0000    0.0120   -0.0000    0.0076
    0.0000    0.0205    0.0000   -0.0169    0.0000    0.0116    0.0000    0.0057
    0.0000    0.0226    0.0000   -0.0176    0.0000    0.0106    0.0000    0.0034
    0.0000    0.0247    0.0000   -0.0180    0.0000    0.0090    0.0000    0.0007
    ⋮
```

Step 3: First mode

```
q01 = zeros(2*N,1);
q01(1:2) = [1;1];  % initial condition

t = 0:0.01:1;
M = length(t);
q1 = zeros(2*N,M);
q1(:,1) = q01;
for j = 2:M
    q1(:,j) = expm(Anew*t(j))*q1(:,1);
end
y1 = Cnew*q1;

figure(1)
surf(t,1:N,y1);
xlabel('Time $$t$$','Interpreter','latex')
ylabel('Element $$i$$','Interpreter','latex')
zlabel('Displacement $$y$$','Interpreter','latex')
title('First Mode of a Vibrating String','Interpreter','latex')
set(gca,'FontSize',16,'TickLabelInterpreter','latex')
```

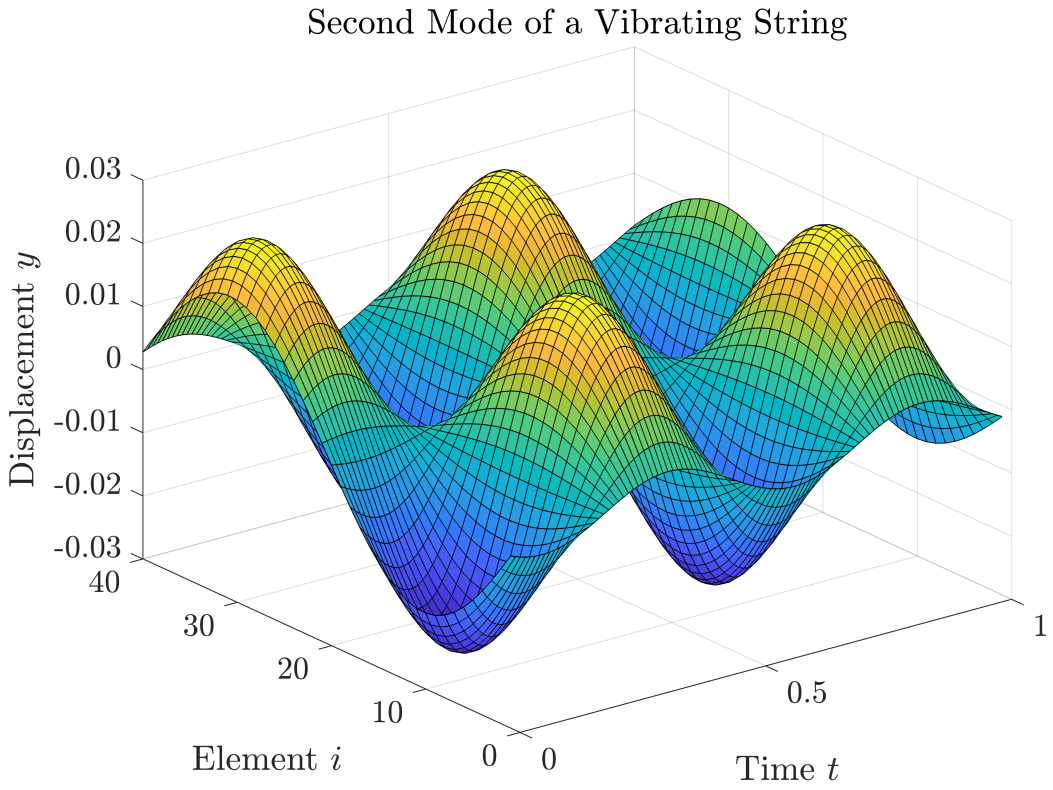


Step 4: Second mode

```
q02 = zeros(2*N,1);
q02(3:4) = [1;1];  % initial condition

q2 = zeros(2*N,M);
q2(:,1) = q02;
for j = 2:M
    q2(:,j) = expm(Anew*t(j))*q2(:,1);
end
y2 = Cnew*q2;

figure(2)
surf(t,1:N,y2);
xlabel('Time $$t$$','Interpreter','latex')
ylabel('Element $$i$$','Interpreter','latex')
zlabel('Displacement $$y$$','Interpreter','latex')
title('Second Mode of a Vibrating String','Interpreter','latex')
set(gca,'FontSize',16,'TickLabelInterpreter','latex')
```

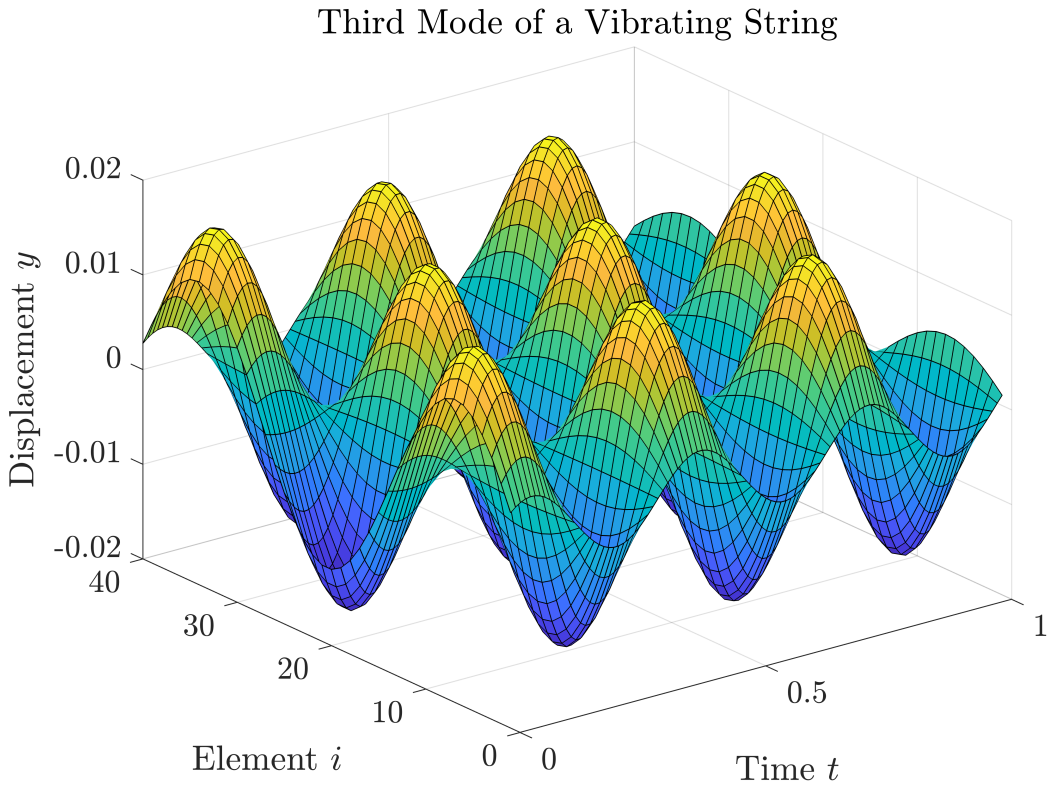


Step 5: Third mode

```
q03 = zeros(2*N,1);
q03(5:6) = [1;1];  % initial condition

q3 = zeros(2*N,M);
q3(:,1) = q03;
for j = 2:M
    q3(:,j) = expm(Anew*t(j))*q3(:,1);
end
y3 = Cnew*q3;

figure(3)
surf(t,1:N,y3);
xlabel('Time $$t$$','Interpreter','latex')
ylabel('Element $$i$$','Interpreter','latex')
zlabel('Displacement $$y$$','Interpreter','latex')
title('Third Mode of a Vibrating String','Interpreter','latex')
set(gca,'FontSize',16,'TickLabelInterpreter','latex')
```


Third Mode of a Vibrating String
Displacement y
0.02
0.01
0
-0.01
-0.02
40
30
20
10
0
Element i
0
0.5
1
Time t