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TAOP18
Supply Chain Optimization
November 2, 2016

Project Assignment 1 (v1.0)

Optimization Models for a Supply Chain Distribution Problem

1 Purpose

Excerpt from the course syllabus in the study guide:

> "The course aims to give the students an ability to model optimization problems, and an insight in how mathematical theory can be used to formulate and solve practical problems, with emphasis on applications in supply chain, distribution and transportation planning. The course also aims to give a deeper knowledge about combinatorial optimization, i.e. optimization problems with an underlying graph structure."

This assignment tries to meet these aims by practicing your ability to :

- model optimization problems,
- use standard software,
- develop specialized methods for large-scale practical supply chain problems.

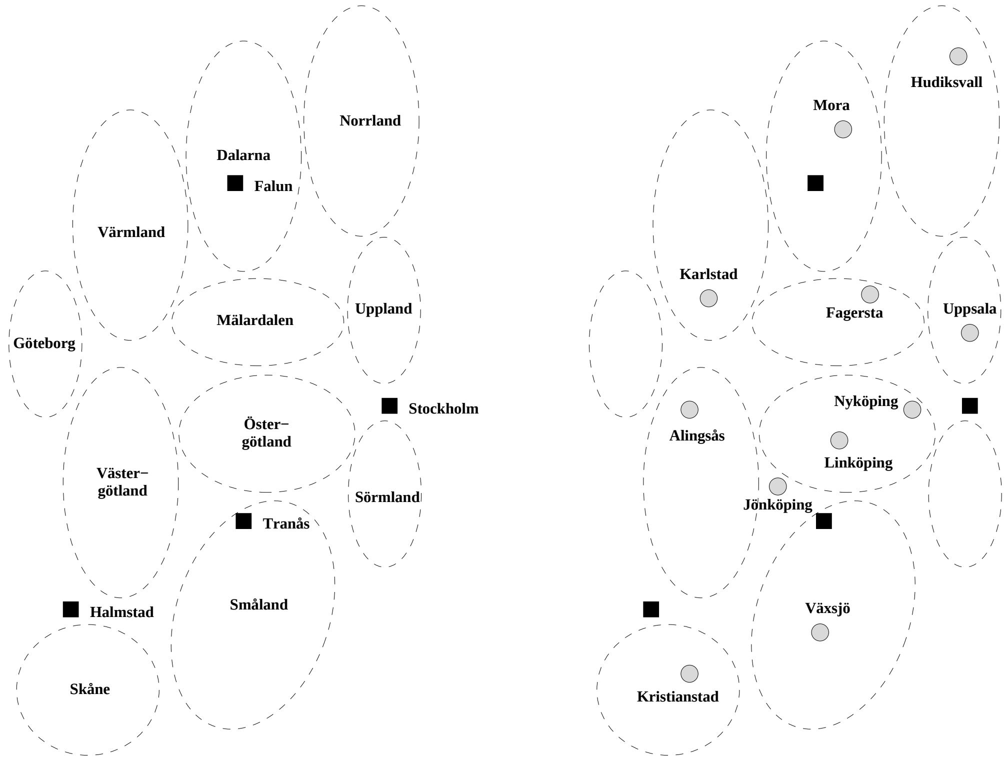
In this assignment you should develop an optimization model for a supply chain problem with several products and time periods. You should also develop solution methods for this problem that utilize the underlying structure in the problem, and the connections to other relevant to supply chain optimization problems.

Please hand in a printed copy of the report, as well as submitting it in Lisam. Provide all AMPL code in an appendix. (No need to print this part though.)

Deadline: November 18th 2016

2 Problem Description

A company has four factories located in Halmstad, Tranås, Stockholm and Falun, that produce a set of products. The customers that buy the assembled products are grouped into a number of customer areas, based on their geographical locations. These areas are illustrated in Figure 1a.



(a) Locations of the factories.

(b) Locations for the distribution centers.

Figure 1: Geographical description of factories, customer areas and potential distribution center locations.

Today, the factories both produce components and then assemble them into end products. The assembled products are bulky (they require large space), especially compared to the components that make up the product. This means that transportation costs for all components of a product is lower than the cost for transporting the assembled product. The company therefore wants to investigate the potential of assembling the products closer to the customers. The solution they consider is to only produce the components at the factories, and then assemble the end product from these components at distribution centers. The set of potential locations for the distribution centers are shown in Figure 1b. All factories can produce all components, but can only provide a limited supply of components. If needed, the factories can buy more components from a subcontractor (to an unfavourable price).

If a distribution center is used, then the company pays a rent for the facility. Each end product assembled at a distribution center incur both an assembling cost and an assembling time. The available time for assembling products at a distribution center is limited. In the first assignments, the factories are also able to assemble the end products (to a cost). We assume unlimited assembling time at the factories.

The aggregated *minimum* demand for each end product in a customer area is known, and this minimum demand must be fulfilled in order for the company to stay on the market. However, there is usually a higher demand than the known minimum and the company is using an elaborate sales model following certain rules. Everything sold in excess (e) of the known minimum demand (d) is sold at a reduced price:

Table 1: Rules for excess sales.

Price reduction	Intervall
10%	$0 < e \leq 0.20 \cdot d$
30%	$0.20 \cdot d < e \leq 0.50 \cdot d$
50%	$0.50 \cdot d < e \leq 1.00 \cdot d$

As seen in Table 1, an upper limit on the excess sales is also applied, set at twice the minimum demand. Example: with sales price 20, minimum demand 100, and sold units 160, the profit is $100 \cdot 20 + 20 \cdot (0.9 \cdot 20) + 30 \cdot (0.7 \cdot 20) + 10 \cdot (0.5 \cdot 20) = 2880$.

Your assignment is to help the company with this distribution planning problem and give them recommendations on how to use distribution centers (if at all). All transportation costs are estimated by Euclidean distances multiplied by a constant that depends on which transportation alternative that is used. The objective is to maximize the total profit while fulfilling minimum customer demands.

3 Overview

An overview of the assignments is given in Table 2. The points for each assignment is supposed to correspond to the level of difficulty and required time.

Table 2: Overview of the assignments

	1	2	3	4	5	6
Topic	Modelling	Modelling	Modelling	Heuristics	Rolling	Lagrange
Points	5	2	8	3	6	6

The assignments cover different planning aspects. The first and second assignment concerns strategical planning (long term planning), and the company would like to know whether or not distribution centers should be utilized.

The third assignment concerns strategical planning (short term planning), and the company is now interested in a time expanded version of the problem, in order to plan for multiple time steps. This allows for inventories at the distribution centers, and it is possible to store *both* components and products.

The last assignments concern implementation of different heuristical methods. In Assignment 4 you should implement a straightforward greedy constructive heuristic, while Assignment 5 concerns a more advanced heuristic based on the Rolling Horizon technique. Lastly, a Lagrangean relaxation scheme is considered in Assingment 6.

Strategical planning

The first two assignments concern strategical planning (long term planning), and the company would like to know firstly whether assembly of end products should take place at the factories or if distribution centers should be utilized, and secondly (if DCs seems useful) which distribution centers to use.

Assignment:

1. Develop a mathematical model for the distribution problem stated in Section 2.

The model should include both the alternative to assemble end products at the factories and transporting them directly to the customer areas, as well as to transport components to distribution centers for assembly before transporting end products to the customer areas.

For this assignment, assume that only one component is needed in order to assemble a product. Further, assume that end products can be assembled both at the factories and at the distribution centers, and that the assembling cost depends on where the final manufacturing is done. This implies that the model can be formulated by using only products (we have a one-to-one relationship between components and products).

To do:

Implement the model in AMPL to compare the following costs.

a) Products must be assembled at factories.

b) Products must be assembled at distribution centers.

c) Products can be assembled at both factories and distribution centers.

Analyze the results carefully and make a suggestion to the company whether products should be assembled at distribution centers or not. **(5p)**

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2. The company has now decided that final assembling must take place at the at the distribution centers. It is still not decided, though, which DCs to utilize.

In this assignment, the end products are composed of several components, and actually needs to be assembled. A number of sets, given in the data file, specify which products that a component is included in (there are never duplicates of the same component in a product).

To do:

Develop and implement a mathematical model in AMPL for this problem. **(2p)**

Tactical planning

At this point, a strategical decision has been made by the company. All assembly should take place at distribution centers, and a fixed set of distribution centers is now available (hence whether a DC should be used or not is no longer variable).

The DCs to be used must be specified in `set USED_DC` in the data file, preferably based on the outcome of Assignment 2.

Assignment:

3. The company is now interested in a time expanded version of the model from Assignment 2 in order to plan for multiple time steps. This makes it possible to make use of inventories at the distribution centers. It is possible to store *both* components and products at the distribution centers, which incur holding costs. All inventory levels are initially zero.

Since the planning problem is multi-period, the supply and demand can differ between time periods. This also holds for the assembling costs.

Although it is known which distribution centers to utilze, it is now possible to vary the assembling capacity between time steps (by using time limited contracts for the workers). A set of different capacity levels are given in the data file together with their respective costs. Inventories at the DCs are still functional (components can arrive and products can be sent) even if the assembly capacity is set to zero in a time step.

Note that the capacity levels are set individually for each DC.

To do:

a) Develop an AMPL model for this multi-period distribution problem.

Hint: If the model takes forever to solve, you might need to adjust the cplex option `mipgap` to a more reasonable accuracy level.

b) It turns out that the parent company has a distribution planning problem that is structurally identical to the company's problem, but scaled up. Their problem involves more customers and they have a longer planning horizon.

We assume that the Parent company choose to utilize the same DCs as the company. Can your model solve this problem as well?

(8p)

Make sure to analyze the results carefully, try to understand and visualize the flows, does everything seem reasonable? The objective value itself isn't that interesting!

Heuristics

Assignment:

4. Describe a constructive heuristic that can be used to find a feasible solution to the problem in Assignment 1b). Easier subproblems, such as a network flow problem, can be used inside the heuristic framework.

To do:

Implement your heuristic using AMPL. Evaluate the objective function value of the solution generated by your heuristic by using AMPL. **(3p)**

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5. Describe a constructive rolling-horizon heuristic that can be used to find feasible solutions to the problem in Assignment 3.

To do:

a) Implement your heuristic using AMPL. Evaluate the objective function value of the solution generated by your heuristic.

b) Also apply your heuristic to the parent company's problem. Compare this result with the best found objective in Assignment 3. **(6p)**

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6. Consider once again the problem in Assignment 1b). Now develop a Lagrangian relaxation scheme, where the capacity constraints in the distribution centers are relaxed with Lagrangian multipliers λ.

To do:

a) Formulate the dual function $h(\lambda)$. Which subproblem(s) occurs?

b) Implement the Lagrangian scheme in AMPL. Use it to solve the Lagrangian relaxation in order to evaluate an upper bound $h(\lambda^*)$ to the optimal value. (Use the objective function value from the feasible solution obtained in Assignment 4 as $\underline{z}$ in the Polyak formula.)

c) Improve your solution scheme by also using a Lagrangian heuristic. **(6p)**

Good luck!

Appendix

```
#-------------------------------------------------------------------------------------
# Modelfile for Project 1 -- Assignment 1
#
# Remember that in Assignment 1, since only one component is needed in order to
# manufacture one product, the model can be formulated by using only products!
#-------------------------------------------------------------------------------------
param Tcost_F2C;                            # Transportation cost Factory to Customer
param Tcost_F2D;                            # Transportation cost Factory to DC
param Tcost_D2C;                            # Transportation cost DC to Customer

set FACTORIES;                              # Set of Factories
set D_CENTERS;                              # Set of Distribution Centers
set CUSTOMAREAS;                            # Set of Custom Areas
set PRODUCTS;                               # Set of Products

set EXCESSLEVEL := 1..3;                    # Set of excess sales levels

param RevScale{EXCESSLEVEL};                # Excess sales revenue reduction
param ExcessLimit{EXCESSLEVEL};             # Excess sales intervals

param AssemblyCost_F{FACTORIES, PRODUCTS};   # Assembly costs at Factories
param AssemblyCost_DC{D_CENTERS, PRODUCTS};  # Assembly costs at DCs
param CompCost{FACTORIES, PRODUCTS};         # Component prices
param Revenue{CUSTOMAREAS, PRODUCTS};        # Selling reward

param Supply{FACTORIES, PRODUCTS};           # Supply at Factories
param Demand{CUSTOMAREAS, PRODUCTS};         # Demand at Customer Areas
param AssemblyTime{D_CENTERS, PRODUCTS};     # Assembly time for one product at DCs

param DC_Setup{D_CENTERS};                   # Setup costs and capacities
param DC_Capacity{D_CENTERS};                # for Distribution Centers


#-------------------------------------------------------------------------------------
# Coordinates for Factories, Customer areas and Distribution centers
#-------------------------------------------------------------------------------------
param CoordFactory_x{FACTORIES};
param CoordFactory_y{FACTORIES};

param CoordDCenter_x{D_CENTERS};
param CoordDCenter_y{D_CENTERS};

param CoordCustomer_x{CUSTOMAREAS};
param CoordCustomer_y{CUSTOMAREAS};


#-------------------------------------------------------------------------------------
# Euclidian distances between Factories, DCs and Customers are calculated
#-------------------------------------------------------------------------------------
param distFC{i in FACTORIES, j in CUSTOMAREAS} :=
sqrt( (CoordFactory_x[i]-CoordCustomer_x[j])^2 + (CoordFactory_y[i]-CoordCustomer_y[j])^2 );

param distFD{i in FACTORIES, j in D_CENTERS} :=
sqrt( (CoordFactory_x[i]-CoordDCenter_x[j])^2  + (CoordFactory_y[i]-CoordDCenter_y[j])^2 );

param distDC{i in D_CENTERS, j in CUSTOMAREAS} :=
sqrt( (CoordDCenter_x[i]-CoordCustomer_x[j])^2 + (CoordDCenter_y[i]-CoordCustomer_y[j])^2 );
```

Additional parameters for Assignment 2

```
#-----------------------------------------------------------------------------------
# Modelfile for Project 1 -- Assignment 2
#
# In Assignment 2 and 3, multiple components are used in order to manufacture
# one product. Hence we must also consider components!
#-----------------------------------------------------------------------------------

set COMPONENTS;                             # Set of Components
set PROD_USING_COMPONENTS{COMPONENTS};      # Set of Products using Components

param CompCost{FACTORIES, COMPONENTS};      # Component prices

param Supply{FACTORIES, COMPONENTS};        # Supply at Factories
```

Additional parameters for Assignment 3

```
#-----------------------------------------------------------------------------------
# Modelfile for Project 1 -- Assignment 3
#
# In Assignment 3, we now consider a planning horizon over multiple time steps.
# This makes it possible to model inventories of both components and products.
#
# It is also decided which DCs to utilize.
#-----------------------------------------------------------------------------------

param T;                                    # Number of time periods

param HoldCost_Comp;                        # Inventory holding costs Components
param HoldCost_Prod;                        # Inventory holding costs Products

set USED_DC    within D_CENTERS;            # Set of opened DCs
set CAPACITY   := 0..3;                     # Set of Assembly Capacity Levels

param AssemblyCost{D_CENTERS, PRODUCTS, 1..T};
param CompCost{FACTORIES, COMPONENTS, 1..T};
param Revenue{CUSTOMAREAS, PRODUCTS, 1..T};

param Supply{FACTORIES, COMPONENTS, 1..T};
param Demand{CUSTOMAREAS, PRODUCTS, 1..T};

param AssemblyTime{D_CENTERS, PRODUCTS, 1..T};

param DC_CapCost{D_CENTERS,CAPACITY};       # Capacity costs and capacities
param DC_Capacity{D_CENTERS,CAPACITY};      # for Distribution centers
```