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TAOP18
Supply Chain Optimization
November 30, 2016

Project Assignment 3 (v1.0)

Column Generation for Air-crew Scheduling

Excerpt from the course syllabus in the study guide:

> "The course aims to give the students an ability to model optimization problems, and an insight in how mathematical theory can be used to formulate and solve practical problems, with emphasis on applications in supply chain, distribution and transportation planning. The course also aims to give a deeper knowledge about combinatorial optimization, i.e. optimization problems with an underlying graph structure."

This assignment tries to meet these aims by practicing your ability to :

- develop specialized methods for large-scale practical supply chain problems,
- model optimization problems,
- use standard software.

In this assignment you should develop a column-generation heuristic for an Air Crew Scheduling Problem.

Please hand in a printed copy of the report, as well as submitting it in Lisam. Provide all AMPL code in an appendix. (No need to print this part though.)

Deadline: December 16th 2016

1 Air Crew Scheduling Problem

The planning of crew schedules is an important process in the operation of an airline. Crewing costs, including salaries and allowances, are second only to fuel costs in airline operations. Efficient utilization of crew resources reduces crew costs and enables the airline to offer competitive airfares while remaining economically viable.

The overall crew-scheduling problem consists of two sub-problems: the pairing problem and the rostering problem. This project is about the pairing problem that is to find cost-efficient sequences of flights for the pilots/cabin crew. The pairing problem involves the construction of feasible sequences of flights (or sectors) called Tours of Duty (ToDs), and is thus sometimes also referred to as the Tours of Duty problem. The rostering problem involves the allocation of ToDs to specific air crews, so that all the ToDs can be undertaken in the most economical or equitable way.

Generation of ToDs is performed a few times each year. Typically ToDs are constructed for each new flight timetable or schedule. The rostering problem is solved on a more regular basis for each roster period, which is typically of 14 or 28 days duration.

1.1 Sectors, Duty Periods, and Tour of Duties

A *tour of duty* (ToD) is a feasible sequence of sectors starting and ending at the same crew base. A *sector* is a specific scheduled unbroken flight between two ports with given departure and arrival times. On some sectors a crew member may pax, i.e. travel as a passenger on the flight (which may even be operated by another airline) in order to relocate to another port.

A ToD can also be considered as a sequence of one or more *duty periods*, where each duty period consists of one or more sectors. The duty periods are separated by rest periods, which can be taken in any port. In Figure 1 we illustrate flights operated by Air New Zealand. Examples of sectors, duty period and ToD are:

Sector Auckland-Sydney or Honolulu-Los Angeles.

Duty period Auckland - Nadi - Honolulu.

Tours of Duty Auckland - Honolulu - (rest period) - Honolulu - Apia (Samoa) - (rest period) - Tapu (Tonga)- Auckland.

The generation of ToDs occurs whenever a timetable of sectors for a certain aircraft type is published (which is itself another planning problem).

The partitioning into separate scheduling problems for each aircraft type is possible because air crews are permitted to operate just one particular aircraft type, for example Boeing 767, 737, 747 and 747-400. The timetable contains all the sectors that are operated during one week. It should be noted that there are no boundary conditions in time for ToDs, because the subsequent rostering problem is performed on a continuous time window. This means that a ToD could, for example, start on a Saturday and end on the following Tuesday.

A crew consists of at least a captain and a first officer. This is, of course, only part of a total aircraft crew, which would also include a cabin crew. Nevertheless, other groups can be treated quite separately from the flight-deck crew as they have different work regulations.

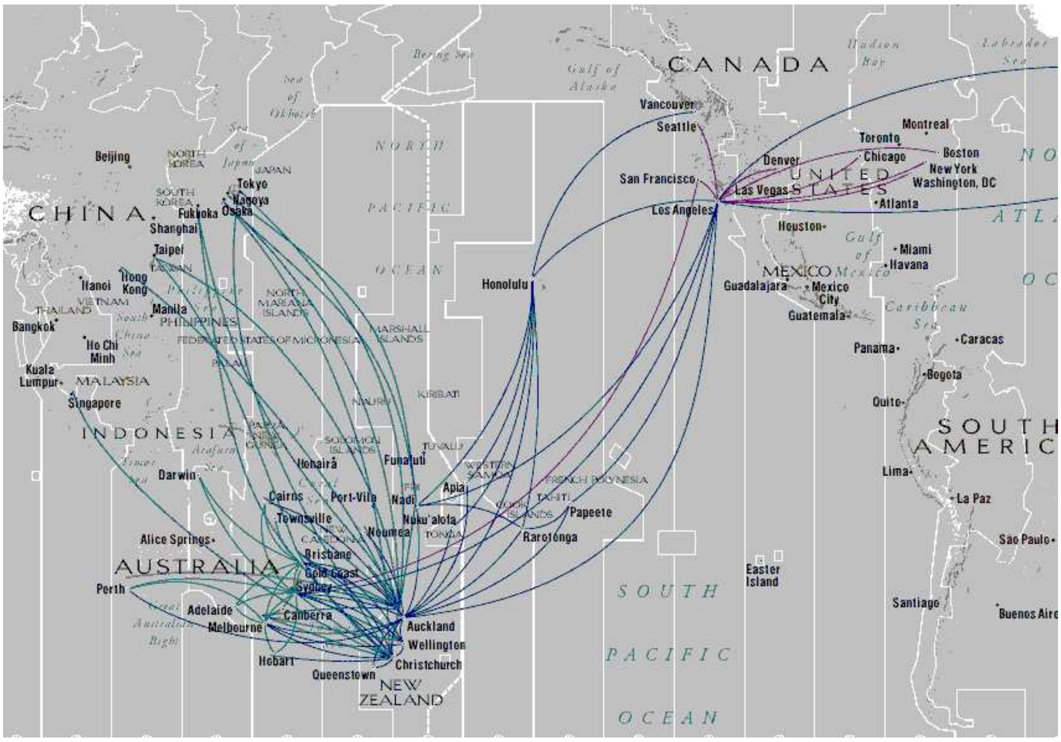


Figure 1: Map illustrating flights (sectors) operated by Air New Zealand.

1.2 Costs in the Pairing Problem

The objective of the pairing problem is to determine a set of tours of duty of minimal cost, such that each sector is part of one regular ToD. The cost of a ToD includes salary costs, rest period costs and paxing costs.

The salary cost of a ToD is based on the number of days (including part-days) it takes to complete the ToD, where days are deemed to start and end at midnight. The salary cost for each full day is based on the average salary of a crew. Rest period costs consist of crew allowances for meals and expenses and accommodation costs. These vary according to the actual port.

Paxing costs are more difficult to determine. If a pilot travels as a passenger on a flight which is fully booked the airline may lose the fare of an additional passenger. However, if there are unoccupied seats on a flight, there is effectively no cost for providing a seat for the pilot. On the other hand, if a pilot paxes on a flight operated by another airline, a cost will be incurred regardless of the occupancy level of the aircraft. Hence it is only possible to estimate paxing costs.

1.3 Airline Regulations and Restrictions

It is clear that for a sequence of sectors to be feasible it must be physically possible to follow such a flight sequence. However, rules specified both by international aviation regulations, government airline regulations and conditions of employment contracts must also be satisfied. The start and end of a duty period do not correspond to departure and arrival times, as time is needed before takeoff and after landing for checking and report writing. The time required for this depends on whether an international or domestic sector is involved.

There are also restrictions on, for example, the number of sectors, local arrival time, and nature of sectors (international or domestic), and whether or not the crew is acclimatized to local time. There are also special restrictions on the ToD itself. We have restrictions such as time limits and rest period requirements, which are imposed cumulatively over a period of time, such as one week.

2 Model Formulation

A standard approach for solving the pairing problem is to formulate and solve a set-partitioning problem (SPP). This requires a generator and a solver. This SPP is a 0-1-integer problem with as many constraints as the number of operating sectors; i.e. each sector must be assigned to a ToD exactly once. The number of columns equals the number of potential ToDs generated. This number is in many cases too large, so some filtering of the ToDs must be adopted.

It should be noted that no explicit constraints are constructed to reflect the work rules/regulations in the set-partitioning model. All of those constraints are implicitly embedded in the subset of columns generated.

2.1 Generate All Potential Columns

A crucial part of this approach is to generate all columns that are regarded as potential candidates for being in an optimal schedule. There are two alternatives for the generation: a sector based generation and a duty-period based generation.

In the latter all possible duty periods are constructed. Any construction of ToDs is then based on these duty periods. This makes it possible to take into account duty period rules before generating ToDs. The main advantage of this approach is that an optimal solution over a restricted set of columns can be guaranteed. However, there is no guarantee that an overall optimal solution can be generated from the restricted set of columns.

2.2 Generate Columns with Column Generation

Another interesting approach, which avoids generating very large numbers of columns, involves the use of column generation. Here, a small subset of the columns is initially generated. The optimal solution of the linear relaxation of the SPP over this subset of columns is then computed. Further columns with negative reduced costs (based on the optimal dual variables) are then generated, and the optimization is continued until no further columns with negative reduced costs can be generated.

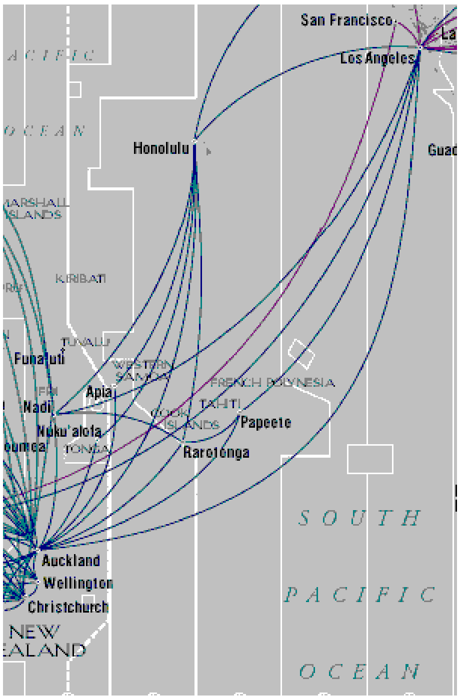
This approach would constitute an efficient method of solution for the overall problem if the generator could be made efficient in finding the additional columns.

3 Column Generation for Air New Zealand

We will now concentrate on a case from Air New Zealand. In a column generation setting, the master problem expresses that each flight must be covered at least once and a column represents a ToD. The sub-problem is a shortest path problem where each flight is represented by a node and each arc is represented by a legal rest period. The solution to the sub-problem is a ToD, which becomes a new column in the master problem.

3.1 Sectors, GMT Conversion Time and Stay-over Costs

A limited number of sectors are used in this case, concentrated to the area given in Figure 2. The flights are taken from the B767 sectors and are given in Appendix A. Data for GMT conversion time, stay-over costs etc. is given below. Please note that all flights in the data might not be shown in the figure.



Information regarding the ports used in the case including port name, GMT-conversion time and daily stay-over cost.

Port	Name	GMT time	Daily stay-over cost
AKL	Auckland	11	219
LAX	Los Angeles	-8	250
HNL	Honolulu	-11	322
APW	Apia (Samoa)	-12	180
TBU	Tapu (Tonga)	12	181
NAN	Nandi (Fiji)	11	315

Figure 2: Map illustrating area in the case study.

The GMT conversion times are such that Greenwich-Mean-Time is 0 and +11 means that Auckland is 11 hours ahead and -8 means that Los Angeles is 8 hours behind. Stay-over cost includes accommodation, meals and allowances. To find the cost for say a stay of 17 hours in Honolulu we simply take the stay-over cost times 17/24. All costs and salaries are given in $NZ.

3.2 Restrictions and Costs for Duty Periods and ToDs

In the modeling it is allowed to over-cover a flight, which essentially means that we instead solve a set-covering model. The interpretation of an over-cover is simply that one crew is paxing on that flight with no extra paxing cost. This is in fact a standard procedure at many airlines.

The restrictions imposed on duty periods and ToDs are as follows:

- Each ToD must start and end in Auckland (AKL).
- The time limit for a duty period is 11 hours.
- A duty period consists of at most two sectors.
- The time between two duty periods must be at least 16 hours.

The cost for a ToD is based on two components; Stay-over costs and salaries for the crew. Stay-over costs are calculated as the duration of the rest period multiplied by the average daily stay-over cost, and the salary is calculated as the total time of a ToD multiplied by a pilot's average salary. We use the annual salary $NZ 150000 for each pilot (there are two pilots for each flight and they follow each other throughout the entire ToD). Hence the day salary is 150000/365, and we assume that the ToD starts with the first flights start and ends with the last flights stop.

3.3 The Sub-problem

The sub-problem is, as mentioned earlier, a shortest path problem where each sector is represented by a node. Duty periods with two sectors are represented by a node including both flights. Start and end nodes are also included, see Figure 3.

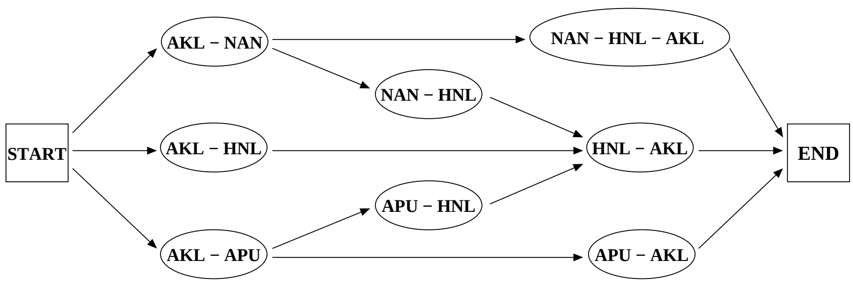


Figure 3: The sub-problem network.

Arcs from the start node to nodes (flights) departuring Aukland are also included, together with arcs from nodes arriving in Aukland to the end node. Each other arc in the sub-problem represents a resting period, and an arc is included only if the resting time is larger than 16 hours. The cost of such an arc is composed of resting and salary costs. The costs associated with a flight (node) are given to all outgoing arcs of each node, except from the start node where the arc costs are set to zero.

So given data for the flights, salary costs and restrictions, the network used to solve the sub-problem can be constructed. For this project, the sub-problem network is specified in the data file `flights.dat` and both nodes, arcs and arc costs are given.

4 Assignments

An overview of the assignments is given in Table 1. The points for each assignment is supposed to correspond to the level of difficulty and required time.

Table 1: Overview of assignments

	1	2 & 3	4	5
Topic	Modelling	Implementation	Read Paper	Summarize
Points	5	8 + 2	8	7

In order to get grade 3, assignments 1-3 shall be done with a satisfactory result. For grade 4, assignments 4 shall also be done with a satisfactory result. For grade 5, additionally solve assignment 5.

1. Give a mathematical formulation of the master problem (a set covering model) and the sub-problem (a shortest path problem). Formulate/derive an expression for the reduced cost. **(5p)**

2. Implement a column generation scheme that solves the crew-scheduling problem.

Given the script file (`flights.run`), the model file (`flights.mod`), and the data file (`flights.dat`), follow the instructions and include code for the master problem and the sub-problem in the missing parts.

Solve the small problem instance and analyze the solution. The results should at least include the total cost, the number of ToDs used and the number of ToDs generated (the number of subproblems solved). Further, please present a table with data for each of the used ToDs, including for example ToD-costs, number of duty periods and number of flights. Please also report if any paxing is used. **(8p)**

3. Solve the crew-scheduling problem given the large data file (`flightsLARGE.dat`). The results should at least include total cost, the number of ToDs used and the number of ToDs generated. Please also report if any paxing is used. **(2p)**

4. The rostering problem is the allocation of pairings (ToDs) to crew members to build a roster for each crew member. This problem is considered in the paper *Modeling and Solving the Crew Rostering Problem* by Caprara, Toth, Vigo and Fischetti. This paper can be found at Lisam. To get started, study the paper (mainly its first 3 pages) and consider the questions found in Appendix B. **(8p)**

5. Finally, study the paper carefully and summarize its contents in a small report.

Describe the problem, the suggested crew rostering model and the corresponding modeling assumptions that are made. Further, describe the principles for how lower bounds are computed. Give also a short description of how feasible solutions are constructed.

Do NOT just write down all formulas from the article without describing them. Try to do your own comments and describe the main ideas of the article. **(7p)**

5 Appendix A – Sector Data

You can download the AMPL files `flights.mod`, `flights.dat`, `flightsLARGE.dat`, and `flights.run` from the Lisam webpage.

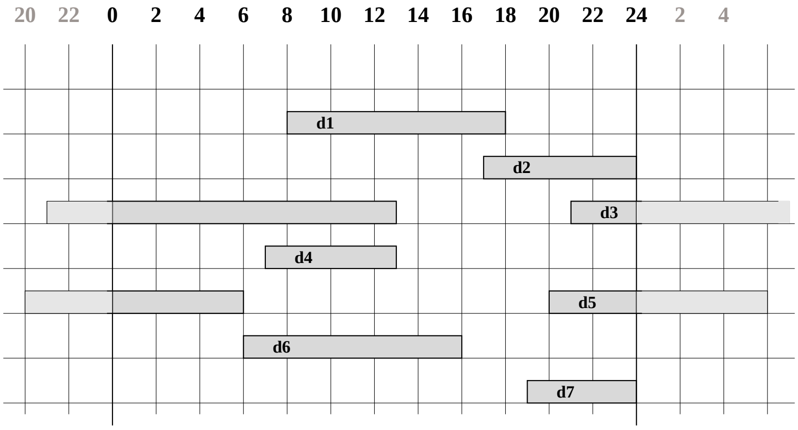
The data file (`flights.dat`) is constructed using the following sector data:

FlightNo.	Departure				Arrival				Flight
	Town	Day	Week	Time	Town	Day	Week	Time	time
1	AKL	MON	1	21.45	NAN	TUE	1	00.50	185
2	AKL	TUE	1	13.30	TBU	TUE	1	17.15	165
3	AKL	WED	1	11.45	HNL	TUE	1	22.30	525
4	AKL	THU	1	11.45	HNL	WED	1	22.30	525
5	AKL	FRI	1	10.00	NAN	FRI	1	13.05	185
6	AKL	SAT	1	11.45	HNL	FRI	1	22.30	525
7	AKL	SAT	1	15.00	NAN	SAT	1	18.05	185
8	AKL	SUN	1	15.00	NAN	SUN	1	18.05	185
9	LAX	SUN	1	23.00	HNL	MON	2	02.00	360
10	LAX	MON	2	21.30	HNL	TUE	2	00.30	360
11	LAX	TUE	2	21.30	HNL	WED	2	00.30	360
12	HNL	MON	2	11.30	LAX	MON	2	20.05	335
13	HNL	SAT	1	04.50	LAX	SAT	1	13.25	335
14	HNL	SUN	1	04.50	LAX	SUN	1	13.25	335
15	HNL	TUE	2	01.50	NAN	WED	2	06.40	410
16	HNL	TUE	2	04.45	APW	TUE	2	09.10	325
17	HNL	TUE	2	23.45	AKL	THU	2	07.05	560
18	HNL	WED	2	23.45	NAN	FRI	2	04.35	410
19	HNL	THU	2	23.45	AKL	SAT	2	07.05	560
20	HNL	FRI	2	23.45	AKL	SUN	2	07.05	560
21	HNL	SUN	1	01.50	NAN	MON	2	06.40	410
22	HNL	MON	2	03.20	NAN	TUE	2	08.10	410
23	APW	MON	2	21.00	HNL	TUE	2	03.15	315
24	APW	TUE	2	10.25	TBU	WED	2	11.55	90
25	TBU	WED	1	13.00	AKL	WED	1	15.05	185
26	TBU	TUE	2	18.15	APW	MON	2	19.45	90
27	TBU	THU	2	13.00	AKL	THU	2	15.05	185
28	NAN	TUE	2	01.50	HNL	MON	2	10.15	385
29	NAN	FRI	1	14.05	HNL	THU	1	22.30	385
30	NAN	SAT	1	19.05	HNL	SAT	1	03.30	385
31	NAN	SUN	1	19.05	HNL	SUN	1	03.30	385
32	NAN	WED	2	07.40	AKL	WED	2	10.45	185
33	NAN	FRI	2	05.35	AKL	FRI	2	08.40	185
34	NAN	MON	3	07.40	AKL	MON	3	10.45	185
35	NAN	TUE	3	09.10	AKL	TUE	3	12.15	185

6 Appendix B

Consider the seven duties given below.

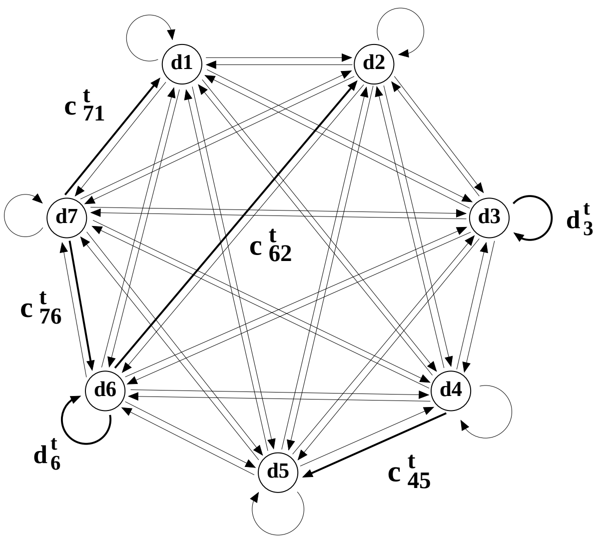
Duties		
d_i	s_i	p_i
1	8	10
2	17	7
3	21	16
4	7	6
5	20	10
6	6	10
7	19	5



a) Make a schedule (roster) without any resting time, which means that a new duty can start directly after finishing the previous duty. Since all duties have to be made every day, it doesn't matter which duty we start with. Try to find a roster over as few days as possible! **(1p)**

b) Make a schedule with 18 h rest between the duties (22 h rest if both duties pass overnight). Start with duty 4. **(1p)**

c) For the given graph below, use the formulas given in the paper to calculate the costs c_{ij}^t and d_i^t, for the indicated arcs and loops. Notice that each arc in fact represents multiple arcs, one for each value of t. **(2p)**



d) Draw the solution from question **b)** in a new graph and calculate the cost for this solution. Include only the active arcs in your graph to make it easier to interpret. **(2p)**

e) Introduce an idle day every $4th$ day (it is 6 days in the paper). Make a new roster and calculate the cost. **(2p)**