Montana Wiring

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Described here is a fourier law based theory for deriving lengths of wires needed to properly thermalize wires into montana fridge. I found that as long as you wrap a few times around the bobbins ~ 10 cm and the wires from each stage are reasonable, ~ 1 cm we should not have a thermalization problem. To limit power transfer from 30K to 4K to 10 mW, we need at least 1 cm of wire. Improving base temperature can be achieved by increasing length of the wires between stages.

I. QUICK NUMBERS

Thermal Conductivity ($\mathrm{Wm^{-1}K^{-1}}$):

Copper	385	300K
Copper	560	4K
Stycast	1.3	300K
Stycast	0.064	4K
Polyesterimide	0.4	300K
Polyesterimide	0.1	30K
Polyesterimide	0.05	4K
N Grease 10 μm	0.1	5K
N Grease 10 μm	0.5	30K
Constantan	21	?

Dimensions (m):

Bobbin Diameter	14.5×10^{-3}
Wire Diameter	79.9×10^{-6}
Insulation Thickness	17.8×10^{-6}
Bobbin Spacing (guess)	2×10^{-3}

II. DESCRIPTION OF PROBLEM

We have wires at room temperature that need to be cooled to 4K. We can thermalize at a 30K plate. We wish to find the following lengths of wire:

- Length from 300K to bobbin at 30K (L_1)
- Length that wraps around bobbin at 30K (L_2)
- Length that goes from 30K bobbin to 4K bobbin (L_3)
- Length that wraps around bobbin at 4K (L_5)

We consider the power from thermal energy that can be transfered by these wires, labeling them $P_1, \cdots, P_4$ that are associated to the lengths $L_1, \cdots, L_4$. We can think about the powers like this:

P_1: Power transfered from room temperature to the 30K bobbin

P_2: Power that the 30K bobbin can dissipate
AND
Power that the 30K bobbin can source

P_3: Power transfered from 30K to 4K bobbin

P_4: Power that the 4K bobbin can dissipate
AND
Power that the 4K bobbin can source

From this, we conclude that the following relations must hold in order for the wires to not raise in temperature, i.e., the net power into the wires is less than the net power out of the wires:

$$P_1 < P_2$$
$$P_2 > P_3$$
$$P_3 < P_4$$
$$P_3 < 10 \text{ mW}$$

The first and fourth of these relations is obvious: power into the bobbin from the wires must be less than the power the bobbin is able to dissipate else the wires will be heated. The second is to make sure that the bobbin at 30K has much larger heat dissipation to prevent heating the next stage of wires. This may not be necessary. The last relation is from Brian's test of the montana system, where he measured the cooling power at ~ 4 K to be about 10 mW.

I think this is all we need to do. If the power we can dissipate is greater than the power into that node, then the temperature of the wires should not rise.

III. THEORY

I looked up some theory on heat transfer. It seems like the classic way of thinking about this in engineering is through *Fourier's Law*. Wikipedia has a good description at `https://en.wikipedia.org/wiki/Thermal_conduction`. Basically, Fourier's law of thermal conduction states that

$$\boldsymbol{q} = -k\nabla T$$

where q is the local heat flux density in $\mathrm{Wm^{-2}}$, k is the thermal conductivity in $\mathrm{Wm^{-1}K^{-1}}$, and ∇T is the temperature gradient. Integrating over the total surface,

$$P = -k \oint_S \nabla T \cdot d\boldsymbol{A} = -kA\frac{\Delta T}{\Delta x}$$

for a homogenous material with endpoints at constant temperature.

This is all we need for the wires. In the following simulations, we just plug in different values for Δx. We know the cross sectional area A of the wires and the thermal conductivity by reading the data sheets. We take ΔT to be the largest possible value.

For the bobbins, we need to think a little harder. It will turn out that we do not have to think any more than the stycast thermal conductivity because stycast sucks at low temperature, but I go through the calculations in case I made some mistake.

For the bobbins, I assume a flat geometry. I do not care about the heat in the bobbin itself so I think this is a good approximation. Wikipedia tells us that thermal resistance is additive with multilayers. In other words, conductance is like capacitance here. So,

$$\frac{1}{U_{tot}} = \frac{1}{U_1} + \frac{1}{U_2} + \cdots$$

where $U_1, \cdots$ are the conductance where conductance

$$U = k/\Delta x$$

. I make a small leap and say that I can just roll the area into it and say

$$\frac{1}{U_{tot}A'} = \frac{1}{U_1A_1} + \frac{1}{U_2A_2} + \cdots$$

where A' is some effective area that we never have to think about again. This may be a problem. I'm not entirely sure but it makes sense from an electrical view. Moving on...

For the bobbins, we can identify the following layers:

1. Constantan $\rightarrow$ stycast through insulation (polyesterimide)
2. insulation $\rightarrow$ copper bobbin through stycast
3. Copper bobbin top $\rightarrow$ Copper bobbin bottom through copper
4. copper bobbin $\rightarrow$ plate through N grease.

$$\begin{aligned}
(U_1A_1)^{-1} &= \left(\frac{\text{Insulation Thickness}}{k_{\text{insulation}}}\right)\frac{1}{A_{\text{wire}}} \\
&= \frac{18\ \mu\text{m}}{0.1\ \text{W/mK}}\frac{1}{A_{wire}} \\
(U_2A_2)^{-1} &= \left(\frac{\text{Distance from wire to bobbin}}{k_{\text{stycast}}}\right)\frac{1}{A_{\text{wire}}} \\
&= \frac{n \times 2\ \text{mm}}{0.064\ \text{W/mK}}\frac{1}{A_{wire}} \\
(U_3A_3)^{-1} &= \left(\frac{\text{Bobbin Height}}{k_{\text{Copper}}}\right)\frac{1}{A_{\text{bobbin}}} \\
&= \frac{20\ \text{mm}}{385\ \text{W/mK}}\frac{1}{A_{bobbin}} \\
(U_4A_4)^{-1} &= \left(\frac{\text{Grease Thickness}}{k_{\text{N grease}}}\right)\frac{1}{A_{\text{bobbin}}} \\
&= \frac{10\ \mu\text{m}}{0.1\ \text{W/mK}}\frac{1}{A_{bobbin}}
\end{aligned}$$

The $A_{bobbin} = \pi(17\ \text{mm}/2)^2$ is easy to calculate. For the wrapping wire, I use a slightly more sophisticated method

$$A_{\text{wire}} = \left(\frac{l_{\text{wire}}}{\pi * d_{\text{bobbin}}} - (n-1)\right)(\pi d_{\text{bobbin}})(\pi d_{\text{wire}})$$

but for here let's take only 1 loop around the bobbin so $n = 1$ and a length of wire l_{wire} long enough to wrap completely around the bobbin once. For our bobbins, that gives us a prefactor 3 because our bobbin can support rougly 3 turns. When we plug in numbers, we get

$$A_{\text{wire}} = 3(\pi)^2(80 \times 10^{-6}\ \text{m})(14 \times 10^{-3}\ \text{m}) \sim 3 \times 10^{-5}\ \text{m}^2$$

and

$$A_{\text{bobbin}} = 2 \times 10^{-4}\ \text{m}^2.$$

Plugging into the conductances, we get

$$\begin{aligned}
(U_1A_1)^{-1} &\sim 6 \\
(U_2A_2)^{-1} &\sim 1000 \\
(U_3A_3)^{-1} &\sim 0.25 \\
(U_4A_4)^{-1} &\sim 0.5
\end{aligned}$$

We see very easily that the conduction through the stycast dominates the total conduction completely. For completness, our simulation takes into account both the stycast and the insulation. Perhaps we will find a better material than stycast or perhaps I pulled the wrong value for the thermal conduction of stycast.

I used 2 mm for the thickness of the stycast to simulate the very likley case that we will not be able to pull the wires completely taunt against the copper bobbin.

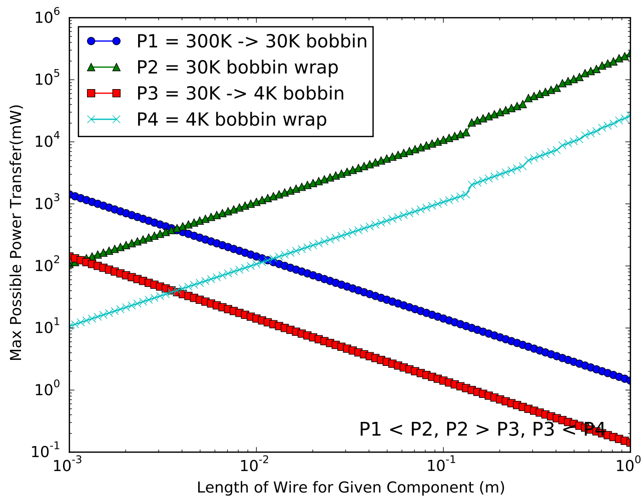

FIG. 1 Power Transfer or Dissipated for each component. The powers are all multiplied by 50 to simulate 50 wires being brought into the cold

IV. SIMULATION

I simulated these powers for wire length from 1 mm to 1 m in figure 1.

First, note that the bobbin wraps positively sloped. This makes sense because the more you wrap the wires around the bobbin, the more heat the bobbin can take away from the wires. Similarly, the slope of the lengths of wire is negative as the longer wires will not transfer as much power to the cold. On the bobbin wraps, note the little steps at high wire length. This is because when one starts wrapping in different layers, weird things start happening. **TODO: I'm not sure about this... I think it should step down a little. I'll have to look more into this.** However, this is not essential to our dialog, as the difference is very small.

Looking at the levels of the plots, we can see that $P_1 < P_2$ after 3 mm of each wire. We know that P_1 should have at least 10 cm of wire just by geometry of our measurement setup so that puts us in a safe region. Then, we note that for $P_3 < P_4$ we need about 3 mm. Again, this is not a problem. The bobbins will probably have centimeters of wire around them. We are home free in power transfer. That means that bobbins with stycast with a few turns ($<$ 10 centimeter of total wire length around the bobbin) will be sufficient to thermalize the wires.

Lets consider the heat load at $4K$. Our goal is to limit the heat as much we can. We see that any heat load will not be from 300K, but from 30K. So, we need to make the wires from the 30K to 4K long. Wires $\sim$ 1 cm will give 10 mW of heating, wires $\sim$ 10 cm will give us 2 mW of heating. The bobbin at the 4K stage is pretty inconsequential when the power flowing into 4K is so low.

V. CONCLUSION

I did a preliminary model of power transfer into the montana cryostat and found that we do not need that much wire. If we get a few good loops around the bobbins, they should thermalize properly. However, to limit power transfer to 4K, we need to make the wires from the 30K to 4K stage long.

Also, it is never bad to make the 300K to 30K wires longer, as that will probably improve our base temperature as it will limit the amount of cooling power required at 30K.