Examining the Kinematics of a Multi-Linkage Assembly

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Introduction:

The objective of this report is to analyze the kinematics of a multi-linkage assembly. The analyzed assembly consisted of three rods. Two of the rods were connected by a pin support to a static ground position on one end. The opposite ends of each rod were attached by pin connections to the opposite ends of the third rod. One rod rotated about the origin with a constant angular velocity of 5 radians per second (rad/s). Fig. 1 visually illustrates how the assembly was joined together and its initial condition. Returning to the initial objective—to analyze the kinematics of the entire assembly given θ_o and the angular velocity and length of rod AB. This report aims to outline how the system moved based on the initial conditions and to conclude why it functioned in such a way.

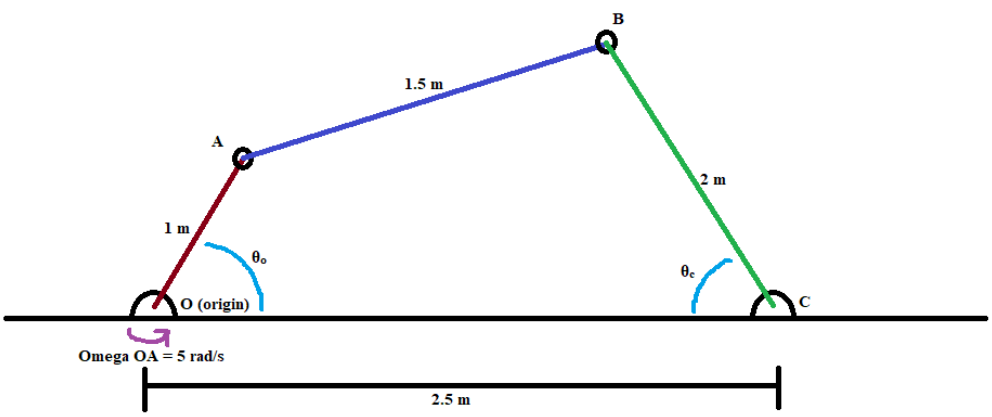


Figure 1: Graphical representation of the analyzed system. Joints O and C fixed pin supports. Joints A and B secured by pin connections. AO = 1 m. BA = 1.5 m. BC = 2 m. CO = 2.5 m. Omega OA = 5 rad/s. $\theta_o = 0 : 2\pi$. θ_c was unknown.

Background:

Equations of motion were instrumental to the kinematic analysis of this system because they mathematically described how the system moves. This mathematical representation provided the values with which, plots and diagrams of the motion were constructed and analyzed.

Universally, the position of any rigid body can be described using an equation unique to that rigid body. The change in the position equation with respect to the time represents the equation of the velocity of a rigid body with respect to time. Furthermore, the change in the velocity with respect to time represents the acceleration with respect to time. For the analysis of this system, the equations of motion depended on the position equations for joints A and B along with their first—and second—time derivatives (Elert, 2020).

Solution Strategy:

In order to analyze the motion of the system, expressions for the position functions of joint A and joint B were derived. Matlab was used to differentiate each function with respect to time for the velocity of each joint. Once more, Matlab was used to differentiate both velocity functions with respect to time to find the acceleration of each joint.

The position function for joint A is an expression that relates θ_o and the length of AO to the x and y coordinates of joint A's position. Equation 1 (Eq. 1) is listed.

Eq. 1: $posA(\theta_o) = AO*\cos(\theta_o)\ i + AO*\sin(\theta_o)\ j$

Derivation of the position equation for link BC required the use of the laws of sines and cosines in order to solve for θ_c. These equations govern the relationships between the sides and angles of non-right triangles. The imaginary line labeled L in Fig. 2 allowed for the derivation of θ_c as the sum of the angles θ_1 and θ_2 (Eq. 2) (Joyce, 1997).

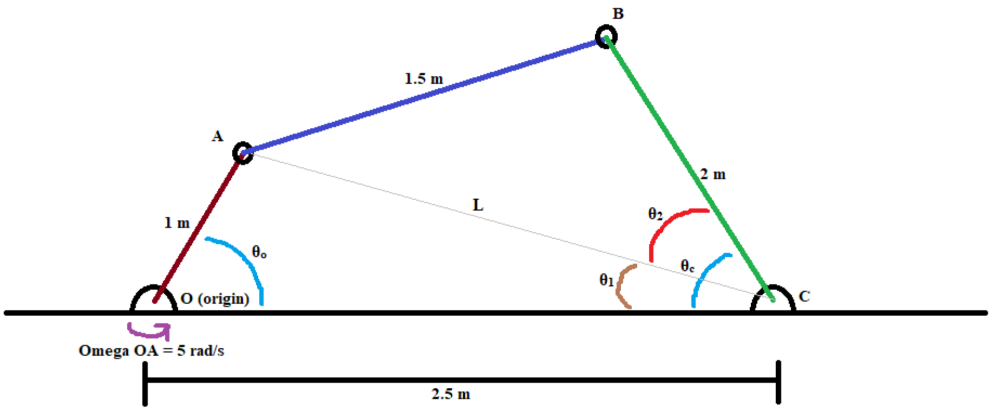


Figure 2: Schematic used to obtain θ_c using the laws of sines and cosines to find the vectorized position function for joint B.

Law of sines:

- $a / \sin(\theta_a) = b / \sin(\theta_b) = c / \sin(\theta_c)$ (Joyce, 1997)

Law of cosines:

- $c^2 = a^2 + b^2 - 2*a*b*\cos(\theta_c)$ (Joyce, 1997)

Derivation of θ_c:

- $L^2 = AO^2 + CO^2 - 2*AO*\cos(\theta_o)$
    - $L = \sqrt{(AO^2 + CO^2 - 2*AO*\cos(\theta_o))}$

- $AO / \sin(\theta_1) = L / \sin(\theta_o)$
    - $\theta_1 = \arcsin(AO*\sin(\theta_o) / L)$

- $BA^2 = L^2 + BC^2 - 2*L*BC*\cos(\theta_2)$
    - $\theta_2 = \arccos(L^2 + BC^2 + BA^2 / (2*L*BC))$

Eq. 2: $\theta_c = \theta_1 + \theta_2 = \arcsin(AO*\sin(\theta_o) / L) + \arccos(L^2 + BC^2 + BA^2 / (2*L*BC))$

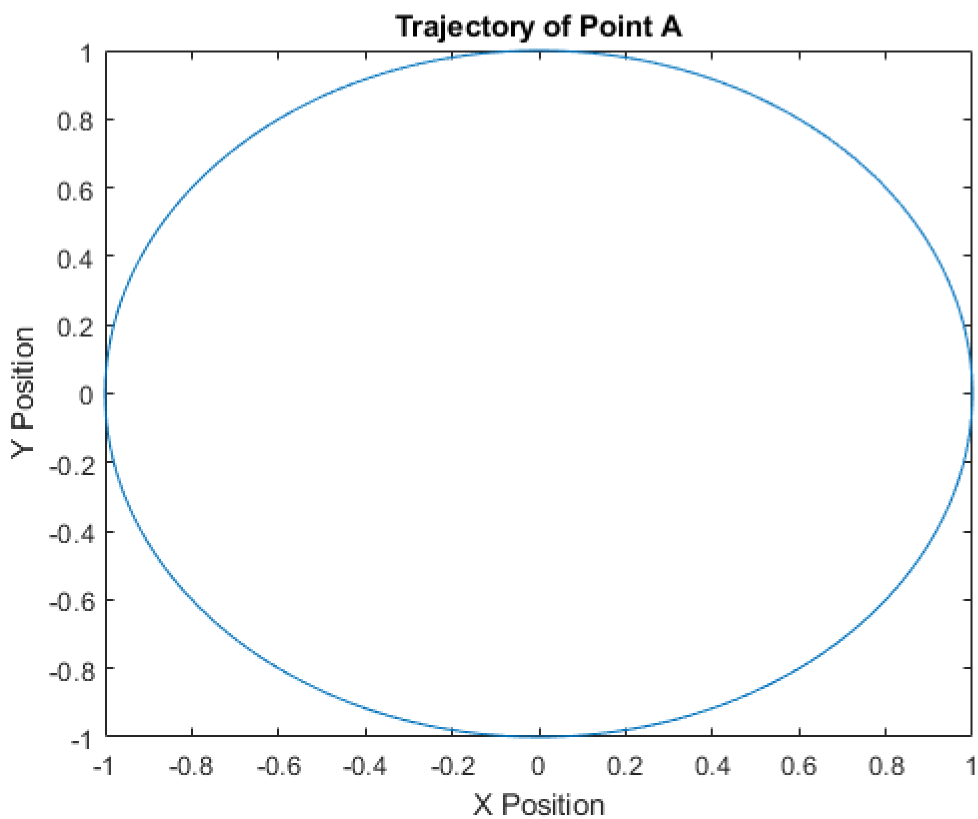
θ_c was used to calculate the vector function of position for point B by relating the length of BC and θ_c to the x and y coordinates of joint B's position in Equation 3.

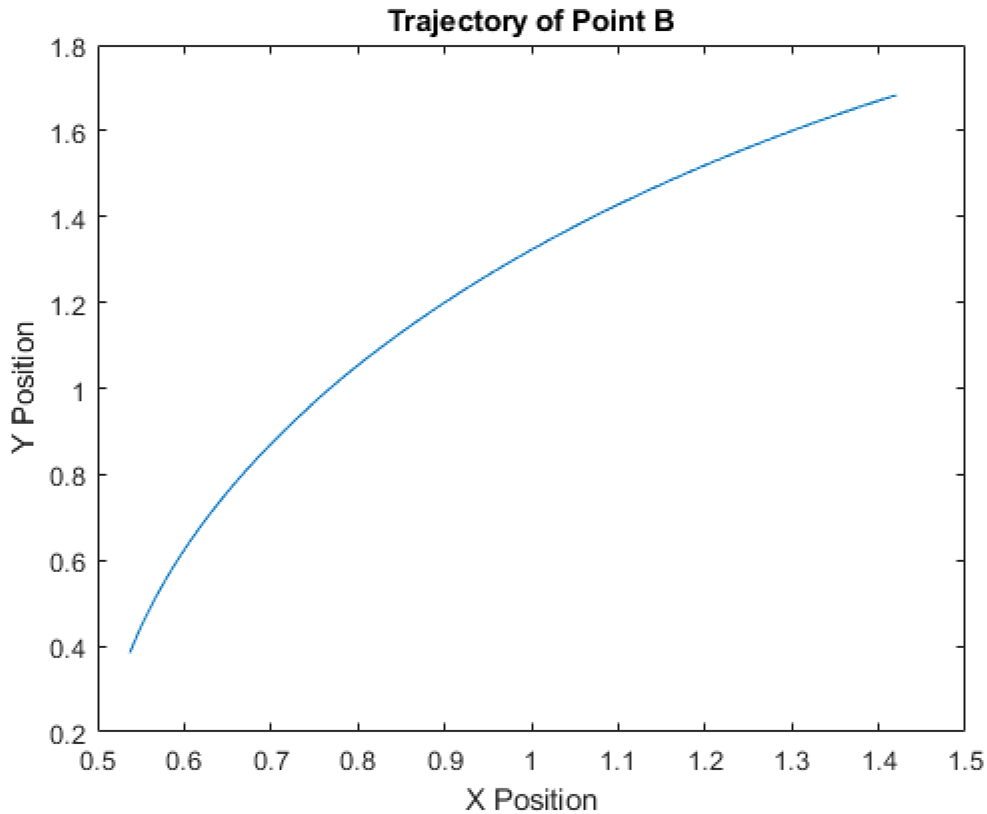
Eq. 3: posB(θ_c) = CO – BC*cos(θ_c) i + BC*sin(θ_c) j

Both functions of position were entered into Matlab and solved for each value of θ_o and θ_c over the duration of 4 seconds, respectively. The results were stored in matrices and plotted in figures demonstrating the paths followed by each point (Fig. 3, Fig. 4). The following step utilized Matlab to differentiate both functions' values with respect to time once for the x and y components of velocity. X and y component values of velocity for each joint were stored in vectors and plotted (Fig. 4, Fig. 5) with respect to time. Acceleration of each joint was calculated using Matlab to differentiate the x and y component values of velocity with respect to time. Similarly, the results were stored in vectors to be plotted with respect to time (Fig.6, Fig. 7).

Results:

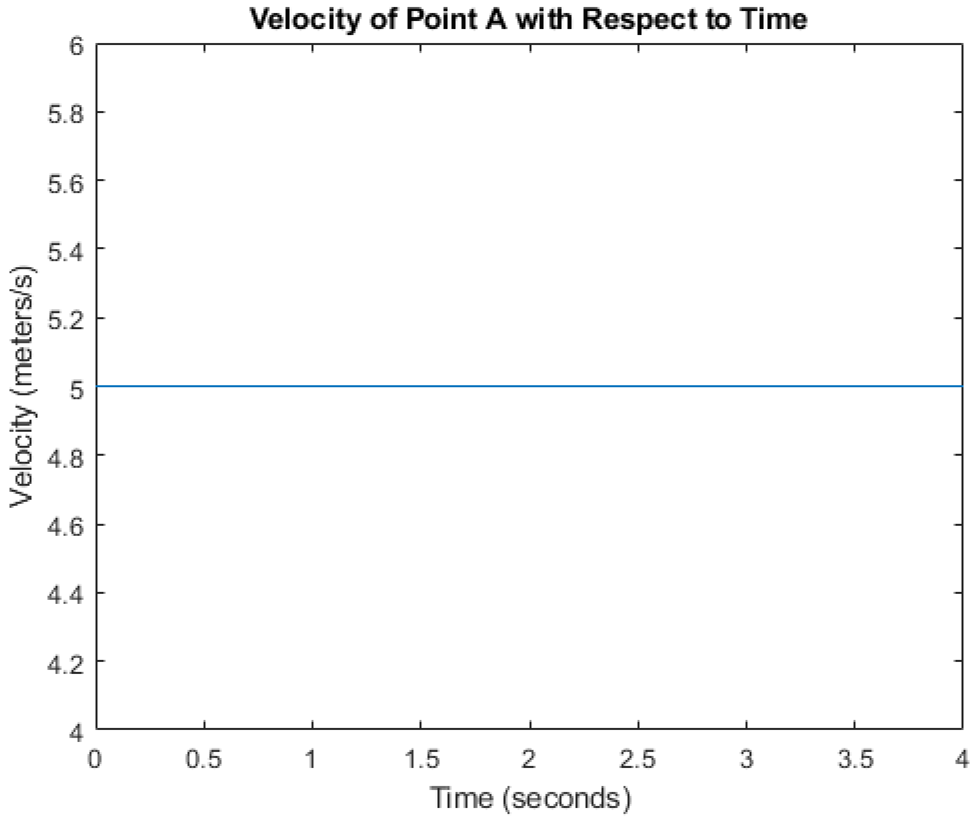
It is shown in Fig. 3 that joint A followed a circular path of radius 1 m. In Fig. 4, joint B is seen going back and forth along a parabolic path as it moved with respect to joint A.

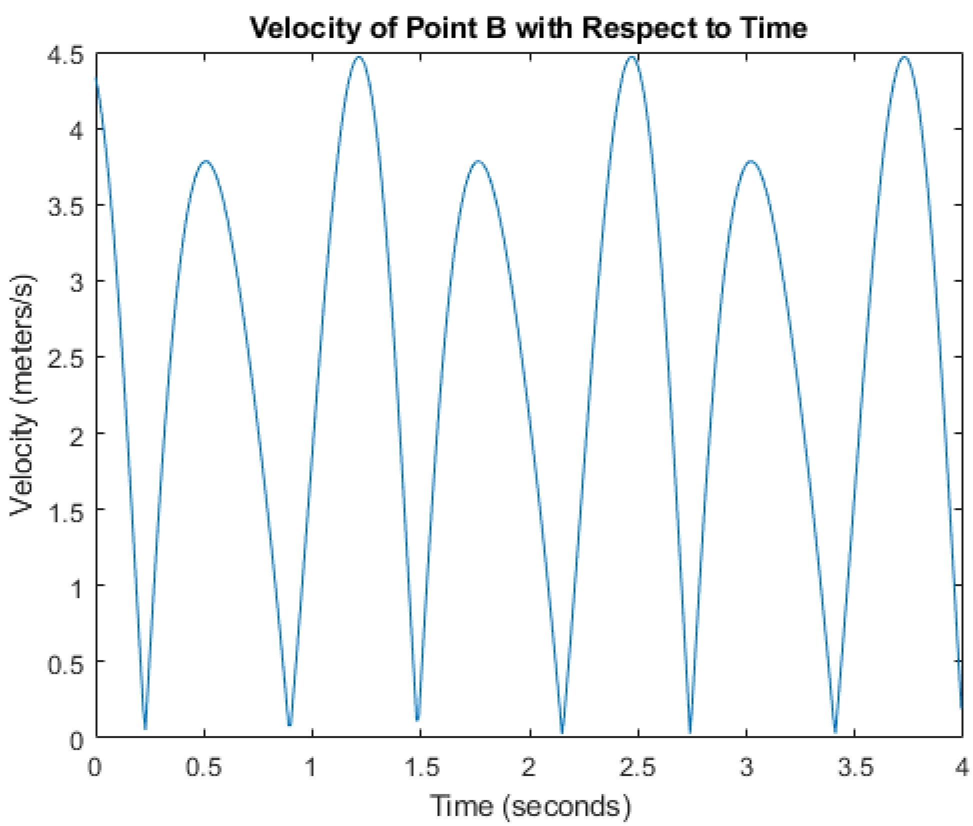




Figures 3 and 4: The trajectory of point A (top) and point B (bottom).

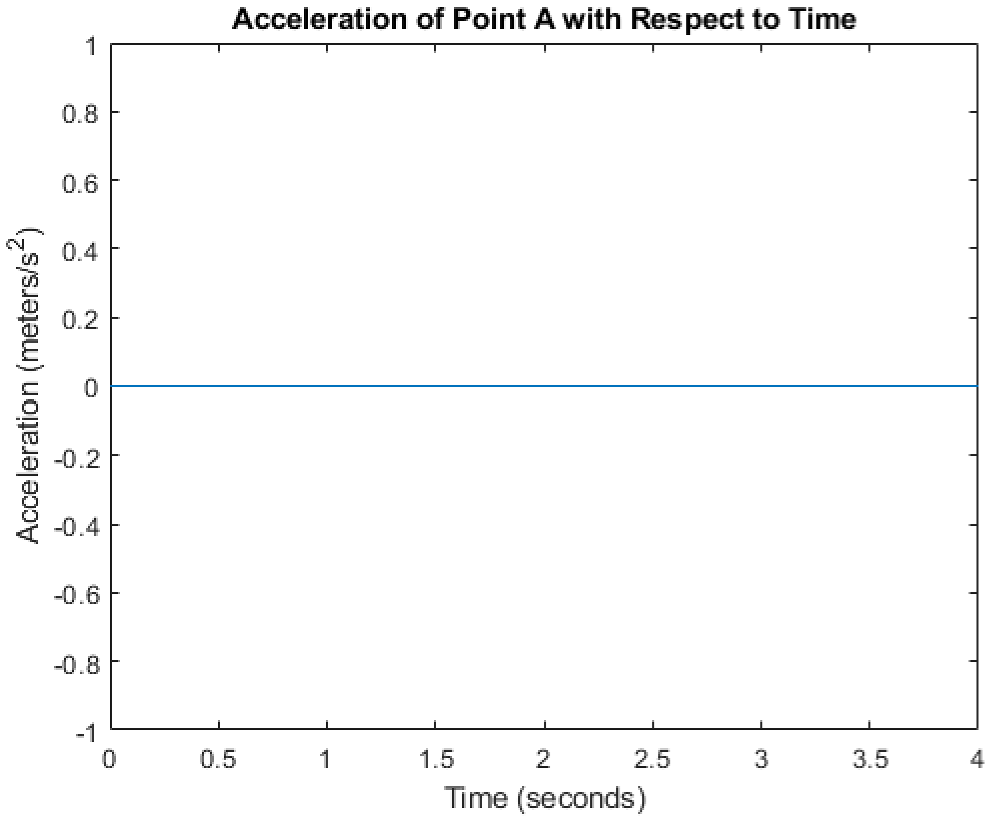
Fig. 5 shows that joint A had a constant velocity of 5 meters per second (m/s). The maximum velocity of joint B was approximately 4.5 m/s and the minimum velocity was approximately 0.026 m/s (Fig. 6).

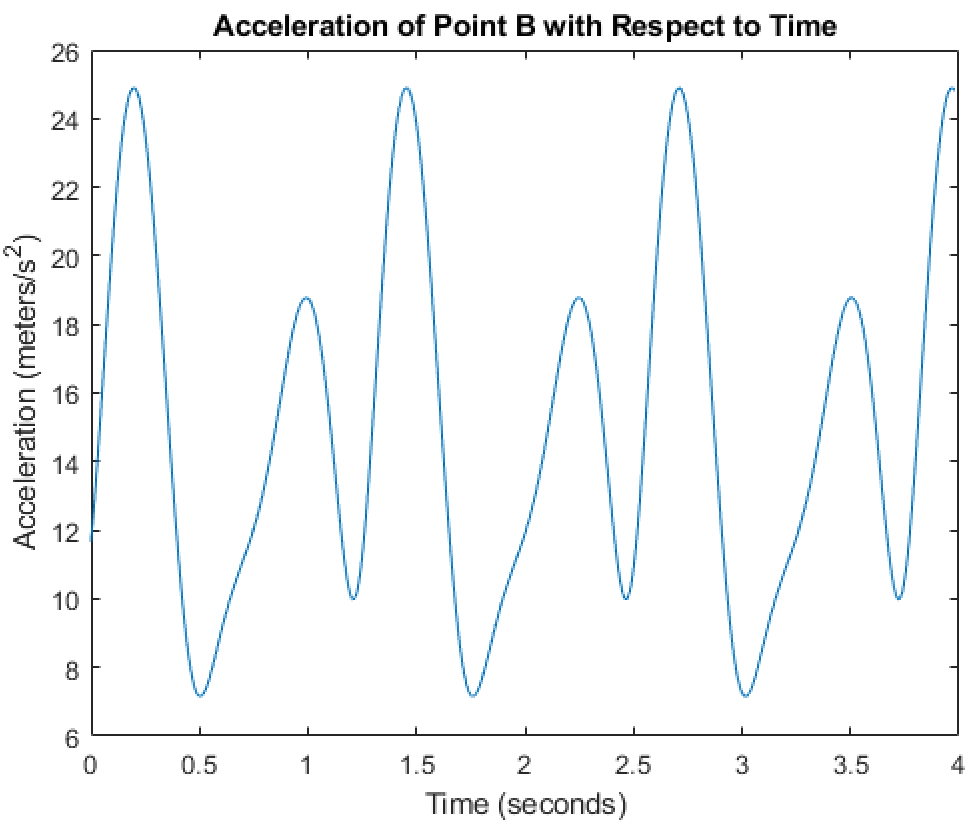

Velocity of Point A with Respect to Time
Velocity (meters/s)
Time (seconds)
6
5.8
5.6
5.4
5.2
5
4.8
4.6
4.4
4.2
4
0
0.5
1
1.5
2
2.5
3
3.5
4


Velocity of Point B with Respect to Time
Velocity (meters/s)
Time (seconds)
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
3.5
4

Figures 5 and 6: The velocity of joints A (top) and B with respect to time (bottom).

The acceleration of joint A was a constant 0 m/s^s and the maximum and minimum acceleration values for joint B were 25 m/s^2 and 7.2 m/s^2, respectively.


Acceleration of Point A with Respect to Time
Acceleration (meters/s^2)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.5
1
1.5
2
2.5
3
3.5
4
Time (seconds)


Acceleration of Point B with Respect to Time
Acceleration (meters/s^2)
26
24
22
20
18
16
14
12
10
8
6
0
0.5
1
1.5
2
2.5
3
3.5
4
Time (seconds)

Figures 7 and 8: The acceleration of joints A (top) and B (bottom) with respect to time.

Conclusion:

In examining the results of the data, the overall motion of this system was variable and cyclical. Even though the position of joints A and B varied with respect to time, both joints completed cycles, returning to their original state by the end of each cycle. The same was true for the velocity of joint B. It dropped to nearly 0.0 m/s during its cycle and increased to approximately 4.5 m/s again by the end of its cycle. The acceleration followed a similar pattern, starting at approximately 12 m/s^2, dropping to approximately 7.2 m/s^2, and returning to its initial acceleration.

Even though the velocity and acceleration of joint A were constant, the velocity and acceleration of joint B varied greatly. As evidenced in Fig. 6 and Fig. 8, the velocity and acceleration of joint B followed a graphically bumpy path. This is represented in the parabolic position vector for joint B. B moved and accelerated slowly closer to the flattened portion of the curve, while the opposite was true for the steep area. This was because link AB acted as both a buffer and an intensifier to the velocity and acceleration of B depending on their orientation.

References:

1. David E. Joyce, 1997, The Law of Cosines and Sines, https://www2.clarku.edu/faculty/djoyce/trig/laws.html, 12/17/20, Department of Mathematics and Computer Science, Clark University.
2. Glenn Elert, 2020, The Physics Hypertextbook, Glenn Elert, Ch 1, Section 8.