Name____

ME 786/886
Fall 2022
Prof. I. Tsukrov

Homework #1

(Due 09/13/2022, bring to class)

A. Homework expectations

Late homework will not be accepted. The solutions must be submitted according to the following requirements:

Include this assignment sheet as the first page of your report. Each problem should begin on a separate sheet of paper. Use the engineering graph or white paper with your name and a problem number clearly labeled at the top. Do not use the back of the pages. Staple the homework in the upper left hand corner. Present your work in a neat and orderly fashion. To receive credit for a given problem, you must show all of your work leading to the final answers.

B. Reading (Text):

4th edition: 1.1-1.3 – Introduction; 2.1, 2.2 – Matrix Algebra and Gaussian Elimination
5th edition: 1.1, 1.2 – Introduction; 2.1-2.3 – Matrix Algebra and Gaussian Elimination

C. Problems

C.1 (20 pts) Given the matrices

$$\mathbf{A}=\begin{pmatrix}-1 & 5 & 2 & 1\\ 7 & 3 & 8 & 2\\ 6 & -4 & -7 & 9\end{pmatrix},\ \mathbf{B}=\begin{pmatrix}-6 & 8 & -2\\ 1 & -3 & 5\\ 5 & -9 & 5\end{pmatrix},\ \mathbf{x}=\begin{pmatrix}4\\ 3\\ 7\\ 1\end{pmatrix},\ \mathbf{y}=\begin{pmatrix}6\\ -7\\ 2\\ 9\end{pmatrix}.$$

Find (a) $\mathbf{A}\cdot\mathbf{x}$, (b) $\mathbf{B}\cdot\mathbf{A}$, (c) $\mathbf{A}\cdot\mathbf{B}^{\mathbf{T}}$, (d) $\mathbf{B}\cdot\mathbf{x}$, (e) $2\cdot\mathbf{x}\cdot\mathbf{y}^{T}-5\cdot\mathbf{I}$, (f) $\mathbf{y}\cdot\mathbf{x}^{T}$, (g) $\det(\mathbf{y})$, (h) $\det(\mathbf{A}\cdot\mathbf{B})$, (i) $\det(\mathbf{A})$ by hand calculation (calculators are allowed). Write "nonsense" if the operation cannot be performed.

C.2 (20 pts., Problem 2.1 modified). Given that

$$\mathbf{A}=\begin{pmatrix}4 & -5 & 1\\ -5 & 2 & 3\\ 1 & 3 & 5\end{pmatrix},\ \mathbf{d}=\begin{pmatrix}1\\ 13\\ 7\end{pmatrix}$$

Find (a) $\mathbf{d}\cdot\mathbf{d}^{\mathbf{T}}-3\cdot\mathbf{I}$, (b) $\det(2\cdot\mathbf{A})$, (c) solution to $\mathbf{A}\cdot\mathbf{x}=\mathbf{d}$ ($\mathbf{x}$ is a vector-column of unknowns) using Gaussian elimination by hand calculation (calculators are allowed).

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C.3 (10 pts). Given that $\mathbf{N} = \begin{bmatrix} -5 \cdot \xi + 2 & 7 \cdot \xi^3 \end{bmatrix}$ find

(a) $\displaystyle\int_{-1}^{1} \mathbf{N}\, d\xi$;   (b) $\displaystyle\int_{-1}^{1} \mathbf{N}^T \mathbf{N}\, d\xi$ (c) $\dfrac{d\left(\mathbf{N}^{\mathbf{T}}\mathbf{N}\right)}{d\xi}$.

C.4 (10 pts) Show that $(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$ using $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

C.5 (10 pts, Text, Problem 2.5, modified).

Express $q = x_1^2 + 3 \cdot x_2 - 5 \cdot x_1 \cdot x_2 + 22 \cdot x_2^2$ in the matrix form $\frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} + \mathbf{c}^{\mathrm{T}}\mathbf{x}$.

Hint: you are expected to find the symmetric square matrix $\mathbf{Q}$ and column-vector $\mathbf{c}$ for $\mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$.

C.6 (30 pts) Solve the SLE below. Apply both Jacobi and Gauss-Seidel methods, beginning with the initial vector (0, 0, 0). Switch the rows if needed. Keep 3 significant digits after decimal point. Comment on convergence. Use either a calculator or a computer (Mathcad, Matlab, Excel, etc) for your calculations.

$$\begin{cases} 7x_1 - 3x_2 + 2x_3 = -4 \\ -x_1 - x_2 - 6x_3 = 11 \\ x_1 + 5x_2 - 3x_3 = 12 \end{cases}$$