Stress Analysis in Varied Plates with Circular Hole Defects Using Finite Element Analysis

Author: David Luby

Date: December 1st, 2022

ME 786

HW 9

# Introduction

Circular hole defects cause stress concentrations in plates near the hole. The limits of the magnitude of stress in an infinite plate with a hole defect in uniaxial tension at the 1) edge of the hole defect and 2) edge of the plate approach 1) three times and 2) one times the magnitude of stress in a plate with no defect. These conjectures are illustrated in Figure 1.

Diagram

Description automatically generated

**Figure 1:** Plate in uniaxial tension (left) and infinite plate with hole defect in uniaxial tension (right). Magnitude of stresses indicated by multiples of *p*, the stress caused by uniaxial tension in a plate with no defect.

Finite Element Analysis (FEA) can be used to illustrate this phenomenon via nodal analysis of an infinite, square plate with a hole defect. Exploiting symmetries in the plate yields a fully defined analysis using just a quarter of the plate.

Diagram

Description automatically generated

**Figure 2:** Symmetry-enabled reduction of plate analysis with boundary conditions.

The accuracy of the stress at the top edge of the hole defect and the stress distribution along the centerline of the plate increase as 1) the resolution of the FEA mesh and 2) the side length of the plate increase.

# Methodology

FEA analysis in support of these conjectures were performed for three different plate side lengths, *l* = 40 mm, *l* = 60 mm, and *l* = 80 mm, and for various FEA mesh resolutions.

Figure 3, Figure 4, Figure 5, and Figure 6 show the boundary conditions and FEA meshes used for the 40 mm side length simulations.

Diagram

Description automatically generated

**Figure 3:** Boundary conditions for 40 mm side length plate with 10/10 FEA mesh divisions.

A picture containing text, measuring stick

Description automatically generated

**Figure 4:** Boundary conditions for 40 mm side length plate with 12/12 FEA mesh divisions.

Diagram

Description automatically generated

**Figure 5:** Boundary conditions for 40 mm side length plate with 14/14 FEA mesh divisions.

Diagram

Description automatically generated

**Figure 6:** Boundary conditions for 40 mm side length plate with 20/20 FEA mesh divisions.

Figure 7, Figure 8, and Figure 9 show the boundary conditions and FEA meshes used for the 60 mm side length simulations.

Diagram

Description automatically generated

**Figure 7:** Boundary conditions for 60 mm side length plate with 10/10 FEA mesh divisions.

Diagram

Description automatically generated

**Figure 8:** Boundary conditions for 60 mm side length plate with 12/12 FEA mesh divisions.

Diagram

Description automatically generated

**Figure 9:** Boundary conditions for 60 mm side length plate with 14/14 FEA mesh divisions.

Figure 10, Figure 11, and Figure 12 show the boundary conditions and FEA meshes used for the 80 mm side length simulations.

A picture containing chart

Description automatically generated

**Figure 10:** Boundary conditions for 80 mm side length plate with 10/10 FEA mesh divisions.

Diagram

Description automatically generated

**Figure 11:** Boundary conditions for 80 mm side length plate with 12/12 FEA mesh divisions.

Diagram

Description automatically generated

**Figure 12:** Boundary conditions for 80 mm side length plate with 14/14 FEA mesh divisions.

# Results

Figure 13 shows the horizontal stress at the top of the circular hole defect as a function of changing plate side lengths and FEA mesh resolution. Only one analysis was performed for a mesh resolution with 20/20 divisions.

Chart, bar chart

Description automatically generated

**Figure 13:** Stress at top of hole defect as a function of plate side length and mesh resolution.

Figure 14 shows the vertical stress along the 40 cm plate centerline plotted against the horizontal location of the respective node for varying mesh resolutions.

Chart, line chart

Description automatically generated

**Figure 14:** Vertical stress along 40 cm plate centerline as a function of horizontal position (where the left edge of the defect is located at x = 12 cm and the right edge of the plate is located at x = 40 cm).

Figure 15 shows the vertical stress along the 60 cm plate centerline plotted against the horizontal location of the respective node for varying mesh resolutions.

Chart, line chart

Description automatically generated

**Figure 15:** Vertical stress along 60 cm plate centerline as a function of horizontal position (where the left edge of the defect is located at x = 12 cm and the right edge of the plate is located at x = 60 cm).

Figure 16 shows the vertical stress along the 80 cm plate centerline plotted against the horizontal location of the respective node for varying mesh resolutions.

Chart, line chart

Description automatically generated

**Figure 16:** Vertical stress along 80 cm plate centerline as a function of horizontal position (where the left edge of the defect is located at x = 12 cm and the right edge of the plate is located at x = 80 cm).

Figure 17 is a graphical demonstration of the Gauss quadrature data collected from the simulation.

A picture containing letter

Description automatically generated

**Figure 17:** Gauss quadrature data for the vertical stress at the nodes in the 40 cm plate with a hole defect.

Figure 18 shows the maximum vertical stress for a three-dimensional simulation of the 40 cm plate with a hole defect.

Diagram

Description automatically generated

**Figure 18:** Three-dimensional vertical stresses at each node in the 40 cm plate with a hole defect.

# Discussion

Figure 13 shows a decrease in stress at the top-center of the hole defect as the length of the plate increases. There is an additional trend that shows an increase in stress at the top-center of the hole defect as the mesh resolution increases. Both the former and the latter are likely the result of increases in accuracy of the simulations as the plate length approaches infinity and as the resolution of the FEA mesh increases.

Figure 14, figure 15, and figure 16 support the conjecture that the stress at the edge of the hole defect in the plate is three times as high as the stress at the edge of the plate because the stress plotted from the simulation decreases as it approaches the edge of the plate. Similarly, the stress values line up with the three and one times pressure due to uniaxial tension.

The same three figures demonstrate increased accuracy as the length of the side of the plate increases because the stress values approach their expected ones as the length of the plate increases, or approaches infinity.

Further, the figures show that an increase in resolution results in more accurate simulations, as stress values tend closer to those which are expected as the resolution increases.

The Gauss quadrature data from the output file is equal to the data retrieved from visual inspection of the nodes.

Figure 17 confirms the results from the two-dimensional, 40 cm simulation, showing stress values of 732 MPa, 305 MPa, 91 MPa, and -228 MPa, which, when compared to the 40 cm plate data in figure 14, lines up closely with the four curves.