Week 9 Assignment

David Russo

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10.7 #7

```
# load the data
data("USArrests")

# scale the data so that each variable has a mean of 0 and a variance of 1
usa_scaled <- as.data.frame(scale(USArrests))
# verify that the scaled data's variables all have mean 0 and variance 1
apply(usa_scaled, 2, function(x) round(c(mean(x), var(x)), 2))
```

```
##      Murder Assault UrbanPop Rape
## [1,]      0       0        0    0
## [2,]      1       1        1    1
```

```
# create data frame of combinations of rows of usa_scaled data set
# there are choose(50, 2) = 1225 ways to choose 2 observations out of 50
combs <- t(combn(nrow(usa_scaled), 2))
head(combs)
```

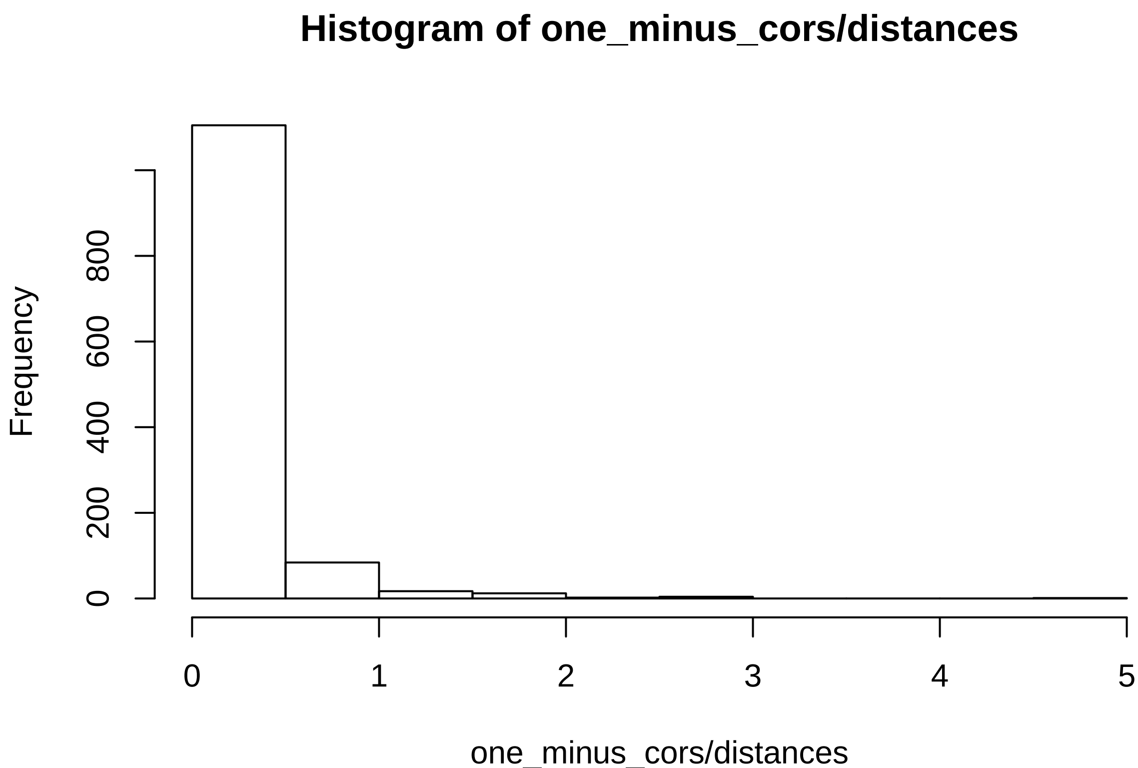
```
##      [,1] [,2]
## [1,]    1    2
## [2,]    1    3
## [3,]    1    4
## [4,]    1    5
## [5,]    1    6
## [6,]    1    7
```

```
###----------------- Determine Correlations ----------------------------

# cycle through combs to get each correlation between rows
one_minus_cors <- rep(0, nrow(combs))

# create function for getting correlations between each observation
one_minus_cor_usa <- function(x){
  res <- cor(as.numeric(usa_scaled[combs[x, 1], ]),
             as.numeric(usa_scaled[combs[x, 2], ]))
  res <- 1 - res
  res
}

# determine 1 - r_ij for each pair of observations
one_minus_cors <- sapply(1:nrow(combs), function(x) one_minus_cor_usa(x))

###----------------- Determine Squared Euclidean Distances --------------

# cycle through combs to get distance between rows
distances <- dist(usa_scaled)^2

###----------------- Determine Proportionality -------------------
```

```r
summary(one_minus_cors/distances)
```

```
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## 0.000086 0.069140 0.133900 0.234200 0.262600 4.888000
```

```r
hist(one_minus_cors/distances)
```



As can be seen from the summary above, there is no evidence of proportionality. If there were proportionality, there would only be one value. I believe the wording of this question is either wrong or simply hard to understand. The question refers to distances and correlations between the i^{th} and j^{th} observations, which would lead me to believe it is referring to the rows of the data set (i.e., 50 rows; one per state). If the authors meant to refer to the distances and correlations between i^{th} and j^{th} variables (i.e., the 4 variables of Murder, Assault, UrbanPop, and Rape), then there is constant proportionality. This is shown below.

```r
# squared distance matrix
distance_matrix <- dist(t(usa_scaled))^2
distance_matrix
```

```
##            Murder  Assault UrbanPop
## Assault  19.41642
## UrbanPop 91.18188 72.63057
## Rape     42.76927 32.80636 57.68856
```

```r
# 1 - correlation matrix
one_minus_cor_mat <- as.dist(1-cor(usa_scaled))
one_minus_cor_mat
```

```
##             Murder   Assault  UrbanPop
## Assault  0.1981267
## UrbanPop 0.9304274 0.7411283
## Rape     0.4364212 0.3347588 0.5886588
```

```r
# demonstration of proportionality
distance_matrix/one_minus_cor_mat
```

```
##          Murder Assault UrbanPop
## Assault      98
## UrbanPop     98      98
## Rape         98      98       98
```

10.7 #10

- a)

```r
# generate raw data matrix
set.seed(2017)
dat <- matrix(rnorm(60*50) + rep(c(2, -5, 7), each = 20), ncol = 50)
true_labels <- rep(c("group 1", "group 2", "group 3"), each = 20)

# get means for first 20 rows, representing the first class
mean(dat[1:20, ])
```

```
## [1] 1.990268
```

```r
# get means for the second 20 rows, representing the second class
mean(dat[21:40, ])
```

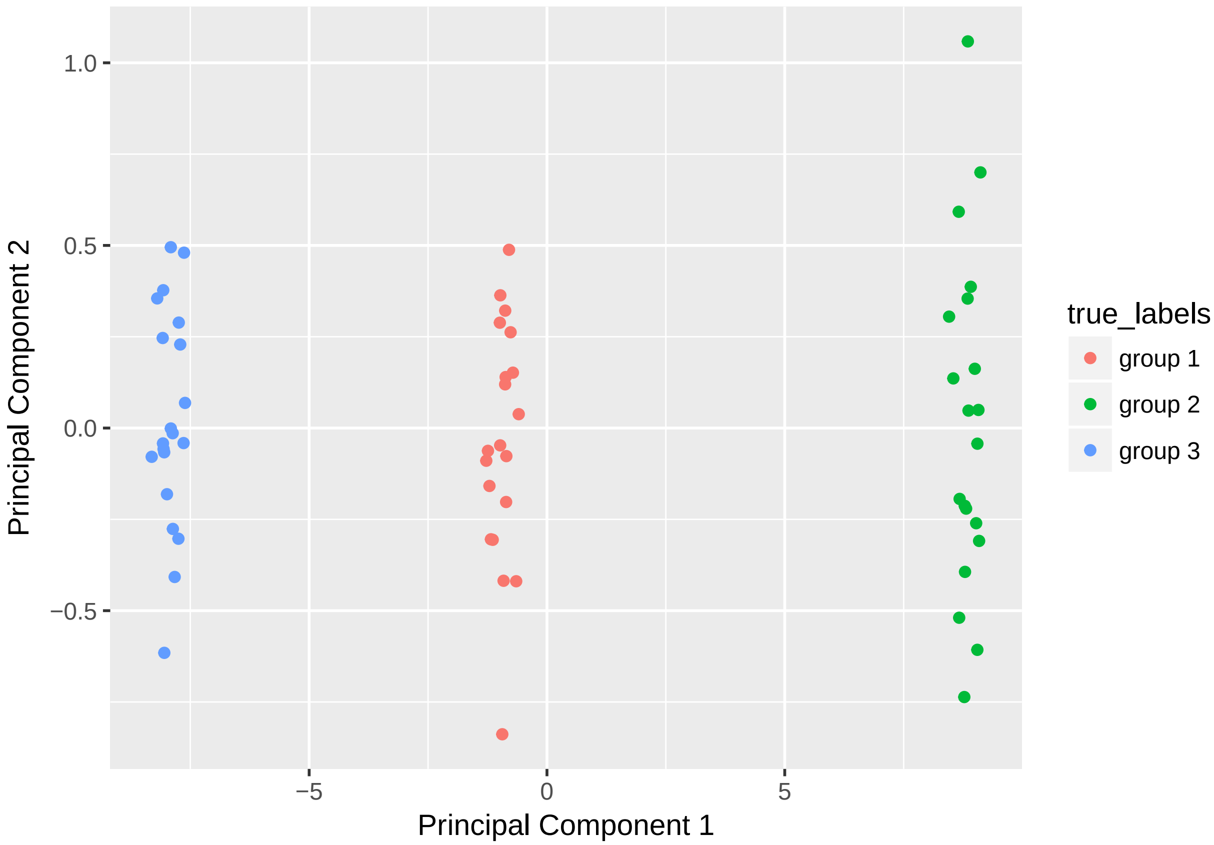
```
## [1] -5.033183
```

```r
# get means for the final 20 rows, representing the third class
mean(dat[41:60, ])
```

```
## [1] 6.998985
```

- b)

```r
# perform principal components
pr_10b <- prcomp(dat, scale = TRUE)

# plot the first two principal components
plot_dat <- as.data.frame(pr_10b$x[, c(1, 2)])
plot_dat$label <- true_labels

plot_dat %>%
  ggplot(aes(x = PC1, y = PC2)) +
  geom_point(aes(col = true_labels)) +
  xlab("Principal Component 1") +
  ylab("Principal Component 2")
```



- c)

```r
km_10c <- kmeans(dat, 3, nstart = 20)
dat_10c <- as.data.frame(dat)
dat_10c$true_label <- true_labels
dat_10c$km_label <- km_10c$cluster

table(dat_10c$true_label, dat_10c$km_label)
```

```
##
##            1  2  3
##   group 1 20  0  0
##   group 2  0 20  0
##   group 3  0  0 20
```

- d)

```r
km_10d <- kmeans(dat, 2, nstart = 20)
dat_10d <- as.data.frame(dat)
dat_10d$true_label <- true_labels
dat_10d$km_label <- km_10d$cluster

table(dat_10d$true_label, dat_10d$km_label)
```

```
##
##            1  2
##   group 1 20  0
##   group 2  0 20
##   group 3 20  0
```