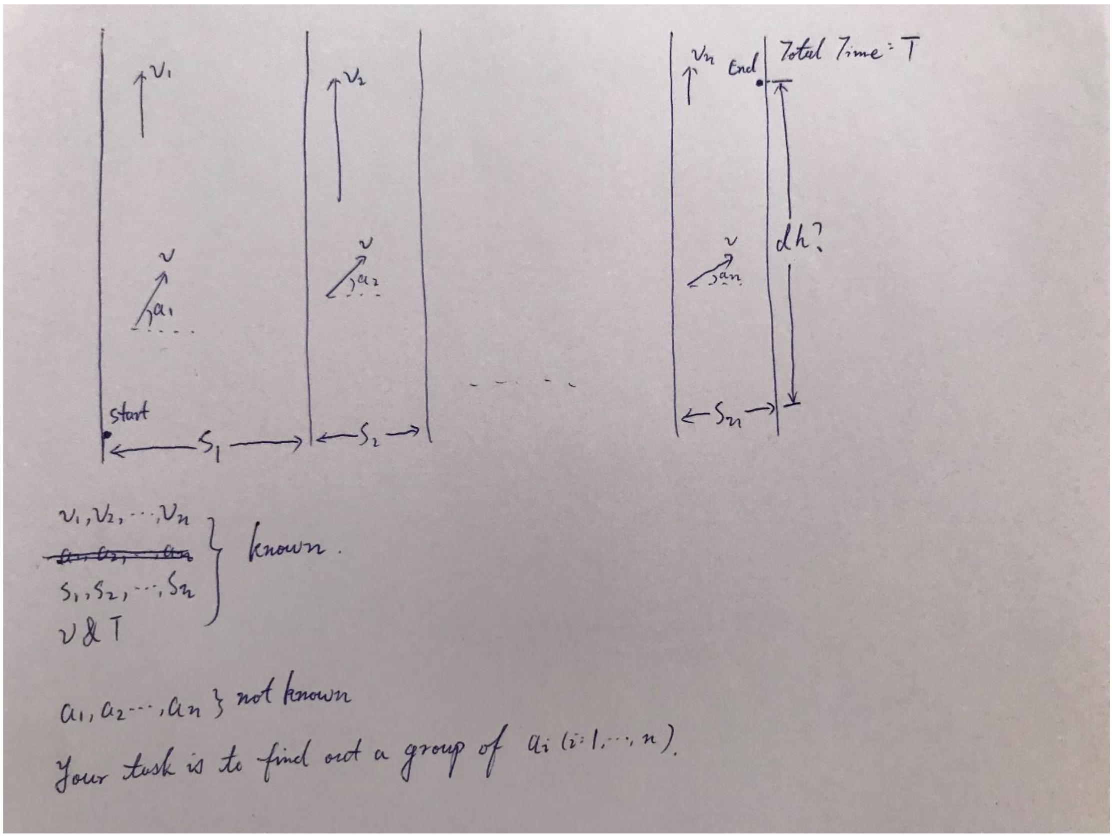

A person is swimming across several rivers.

Speeds of those rivers are different: v1, v2, …, vn. To simplify this problem, we only consider the speed in vertical direction.

The person’s speed is v. It’s a constant, no way to change that. And the angle of the person’s velocity to horizontal line is a1, a2, …, an.

The total time for swimming is T. And, the person must pass those rivers.

Your task is:

Find out an equation to determine by choosing what angles (a1, a2, …, an) the person can get maximum distance in vertical direction (That is to say, please maximize *dh* by determining *a1, a2, …, an*) under the total time *T*. 【You are not required to give out concrete angle numbers, a “cost function” that can be derived from is enough】

Tips: For this question, a mathematical tool you may need is called “Lagrangian Multiplier”. Which means, when you provide a formula, say *E*, which still need to satisfy some more conditions, say *a > 1*, for the convenience of calculating, we can write those 2 parts (formula *E* and condition *a > 1*) together as one new formula. Here the new formula will be: E – λ(a - 1).

Good luck to your math trip.

【Note】This is a real interview question.

解：

每过一条河所用时间：$t_i = \frac{s_i}{v \cdot cos\alpha_i}$

总共时间为T, 可以得到一个约束条件：$\mathrm{T} = \sum_{i=1}^{n} t_i = \sum_{i=1}^{n} \frac{s_i}{v \cdot \cos\alpha_i}$

沿着河流方向的距离为：$\mathrm{dh} = \sum_{i=1}^{n}(v_i + v \cdot \sin\alpha_i)$
引入拉格朗日乘子：λ
引入方程：

$$\mathrm{F}(\alpha_1, \alpha_2, \ldots, \alpha_n, \lambda) = \sum_{i=1}^{n}(v_i + v \cdot \sin\alpha_i) + \lambda\left(\sum_{i=1}^{n} \frac{s_i}{v \cdot \cos\alpha_i} - T\right)$$

要求$\alpha_1, \alpha_2, \ldots, \alpha_n$使得dh在约束条件下达到最大值，即求$\mathrm{F}(\alpha_1, \alpha_2, \ldots, \alpha_n, \lambda)$关于$\alpha_1, \alpha_2, \ldots, \alpha_n, \lambda$的极值，即：

$$\frac{\partial \mathrm{F}}{\partial \alpha_i} = v\cos\alpha_i + \lambda \frac{s_i \sin\alpha_i}{v(\cos\alpha_i)^2} = 0$$

$$\frac{\partial \mathrm{F}}{\partial \lambda} = \sum_{i=1}^{n} \frac{s_i}{v \cdot \cos\alpha_i} - T = 0$$

解得：$\alpha_1, \alpha_2, \ldots, \alpha_n, \lambda$